Statistical Thermodynamics for Engineers
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Lecture 06
Macrostates and Microstates

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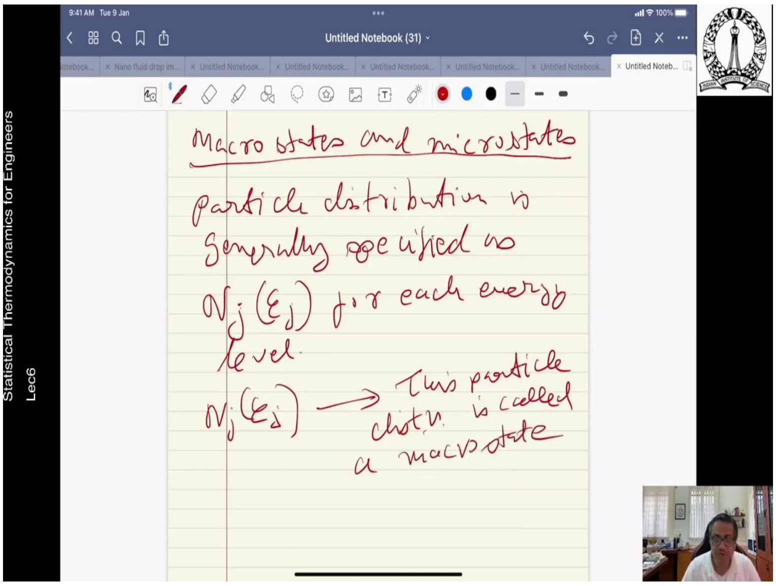

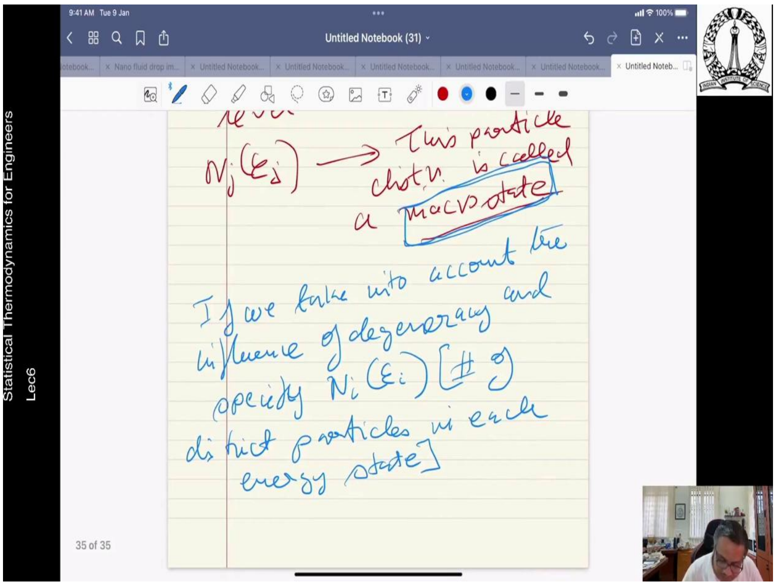

So, welcome to lecture six of the Statistical Thermodynamics for Engineers. So, we have covered a little bit of what is called fermions what is called bosons. Now, we are going to look at first the definition of macrostates and microstates in this particular problem. So, let us put it this way macrostates and microstates. So, as we already saw in the previous lecture that the particle distribution is generally specified as Nj Ej.

So, this is the number of particles or energy level for each energy level so, this particle distribution is Nj Ej. This is a particle distribution this is the number of particles, this is Nj number of particles that Ej energy level or the jth energy level. This particle distribution is called a microstate, this particle distribution is called a microstate.

As the name suggests that this is a macrostate distribution of particles. Now, if we take into account, the influence of degeneracy and specify Ni Ei, so, this is the number of distinct particles in each energy state.

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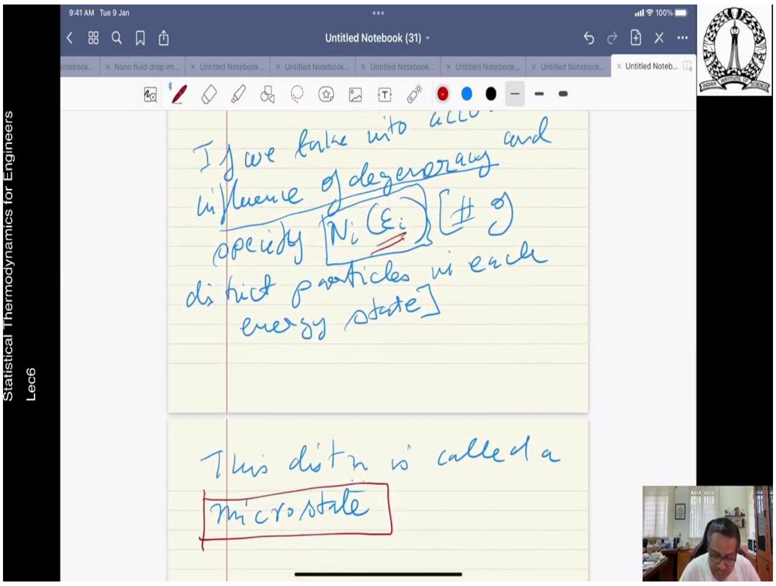

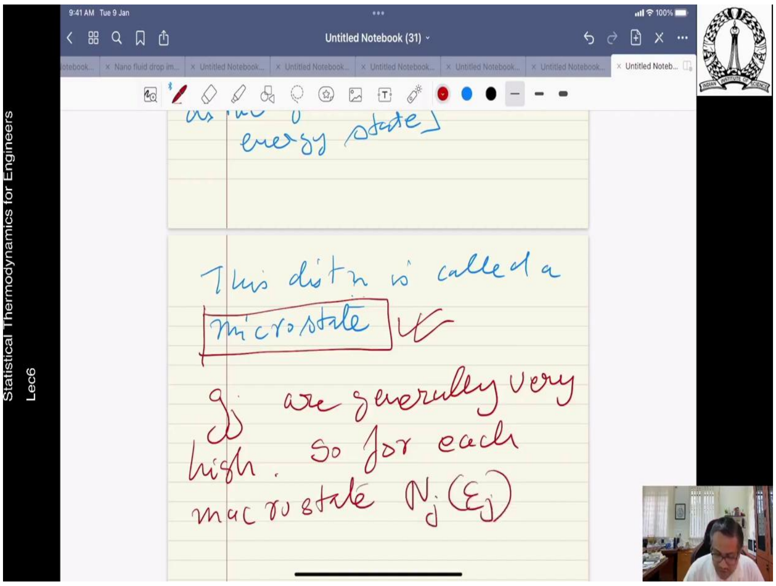

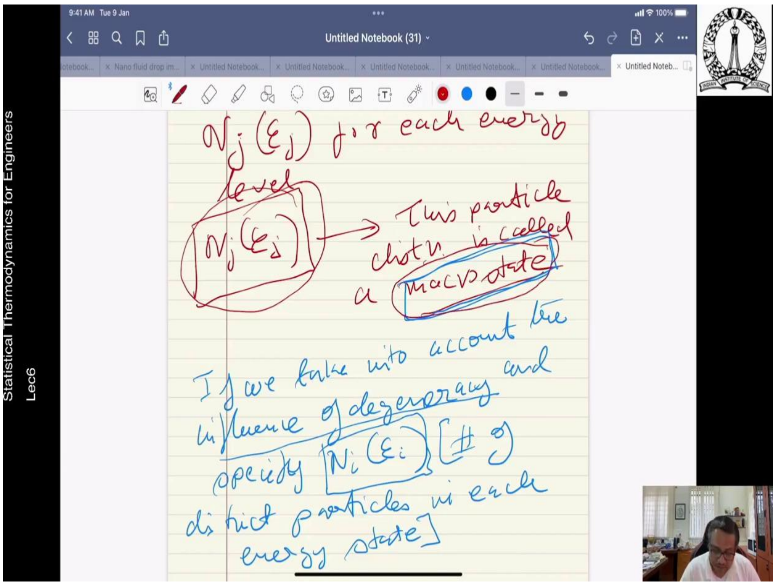

So, as you can see if we take into account the degeneracy and we specify the number of particles in each energy state, then that particular distribution, this distribution is called a microstate. So, what will happen over here is that Nj Ej which is basically the number of particles in each energy level that is called that distribution once I give you this, then I am providing you and account for the macrostate of the system.

If I provide you this, which is basically the number of particles into each energy state, number of distinct particles in each energy state then what I am giving you is called the microstate of the system, the system's microstate. So, gj values are generally very high. So, for each macrostate microstate again recall each macrostate Nj Ej.

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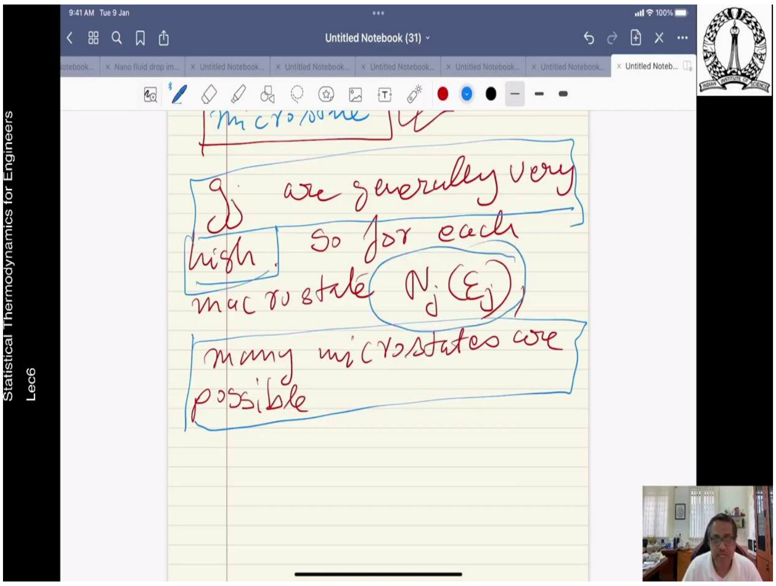

For each macrostate many microstates are possible. So, you can understand that many microstates are possible for each macrostates gj values are very high. So, naturally each macrostate when I provide a macrostate definition there will be many microstates that will be possible.

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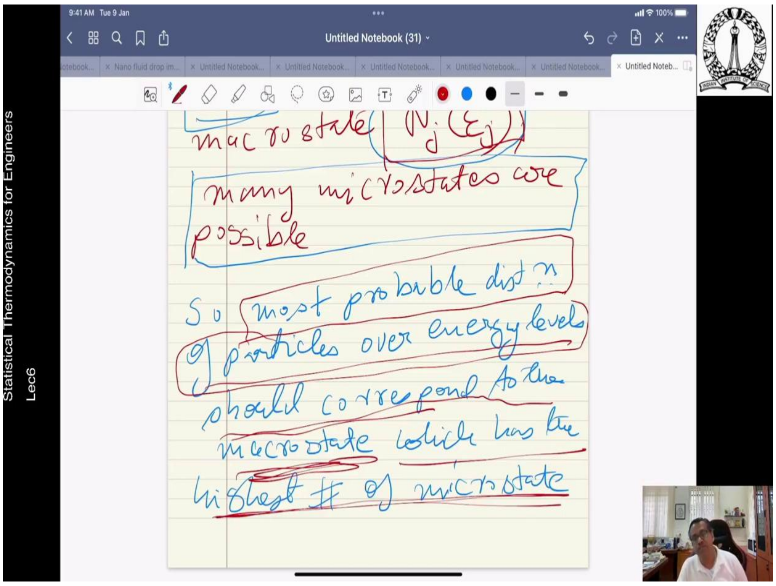

So, most probable distribution because we already saw the most probable distribution is what gives you the equilibrium distribution, the most probable distribution of particles over energy levels should correspond then to the macrostate which has got the highest number of microstates. So, each macrostate has got many microstates try to understand this each macrostate has got many microstates.

Now, the most probable distribution as we saw over the energy levels that means, out of all these macrostates, one macrostate will be the most probable one and as we saw that macrostate is probably the one that determines the equilibrium condition like as we said that for example, internal energy has a well-defined value for an isolated system. So, the most probable distribution of particles over energy levels should correspond to the macrostate which has got the highest number of microstates, so, that energy, so the macrostate that means, the distribution of particles among different energy levels that macrostate is the most probable one if it has got the highest number of microstates associated with it.

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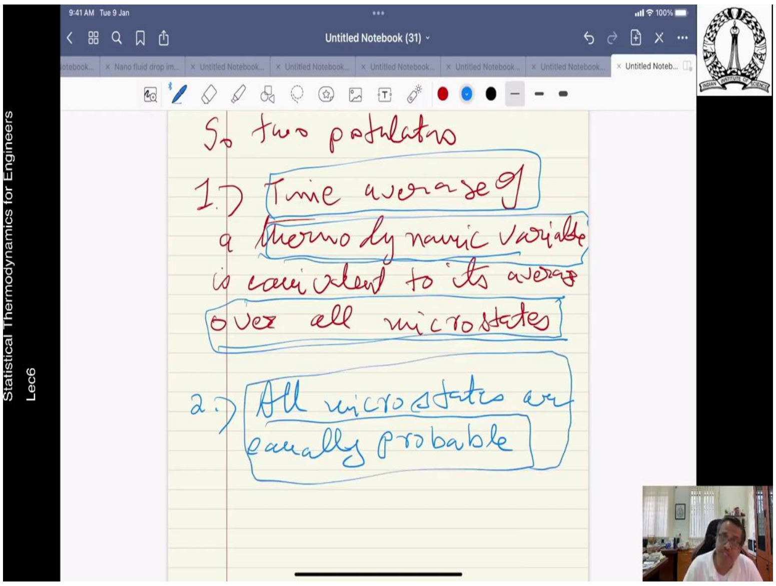

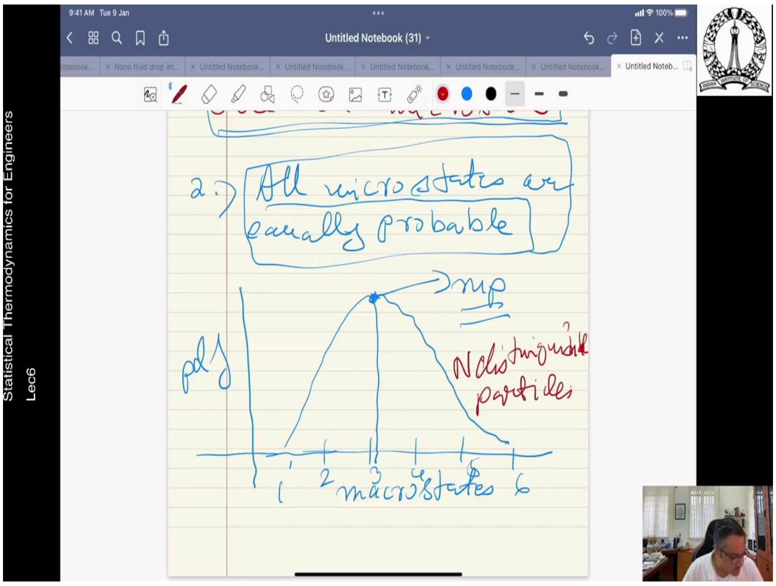

So, the two postulates are, so two postulates here that we can say one is the time average of a thermodynamic variable whatever the thermodynamic variable maybe, variable is equivalent to its to its average over all microstates. So, the two postulates are that the time average of a thermodynamic variable is equivalent to its average over all microstates number.

Two is that, all microstates are equally probable. So, the thermodynamic average, the time average of a thermodynamic variable is equivalent to its average over all microstates and all microstates are equally probable not the macrostates, because the energy levels are not equally probable as we saw a number of distributions. So, this is like what a pdf should look like, if we just plot it. So, this is the pdf is these are set of microstates, this is 1 2 3 4 5 6 whatever, and you have a graph which surprisingly looks something like this without any undulations (())(08:22), so the freehand graph, so, this one is the most probable microstate because that is mp and you should note here the N particles are all this is N indistinguishable particles.

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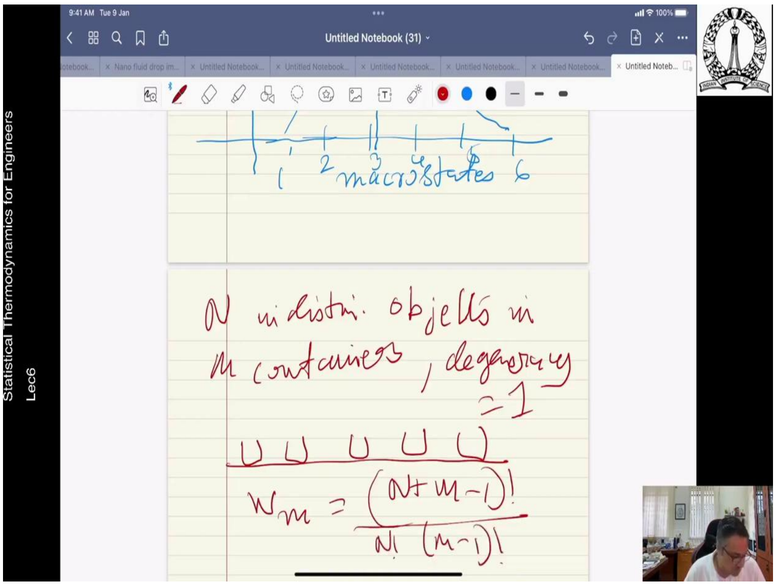

So, we already let us just recall a little bit that if we had N indistinguishable particles objects in m containers as you mean degeneracy is 1, that means one particle , So, that means, this is the level these are the different j's. So, we already saw that the most probable recall, this is what we already found out.

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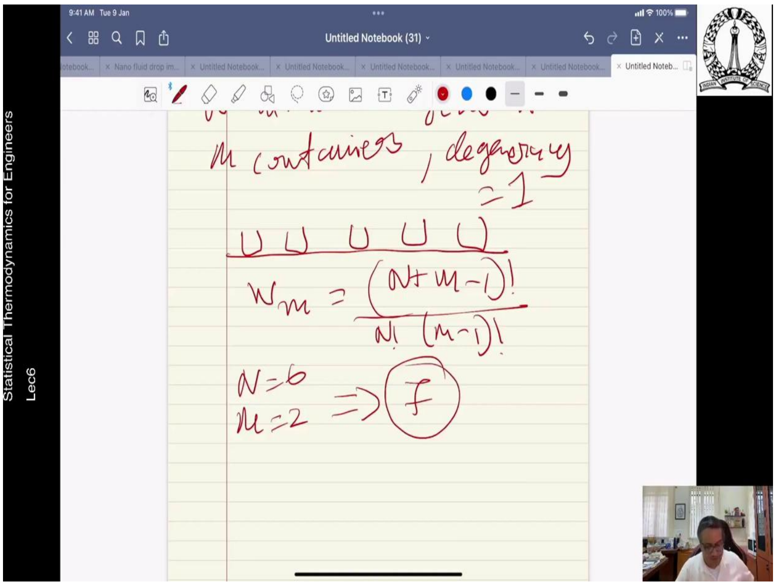

Now, if N is equal to 6, M is equal to 2 this leads to a number like 7. So, this we already did if you recall in the last class or degeneracy 1 for all energy.

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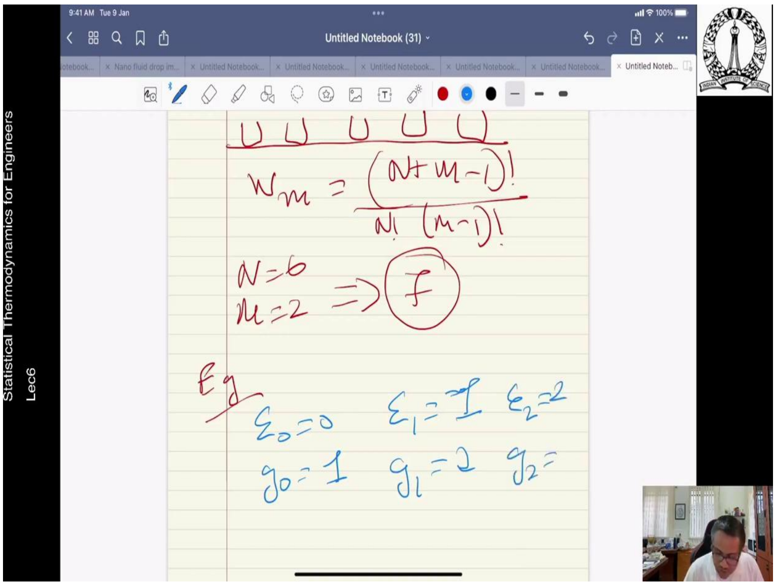

Now, let us look at an example, once again as an example that will give you a more clearer idea say E naught is equal to zero, g naught which is associated degeneracy is 1, E1 is equal to say 1, g1 is equal to 2, E2 is equal to 2, g2 is equal to 3 and we have the total number of particles is 2 and the total energy is 2 as well, energy will be conserved.

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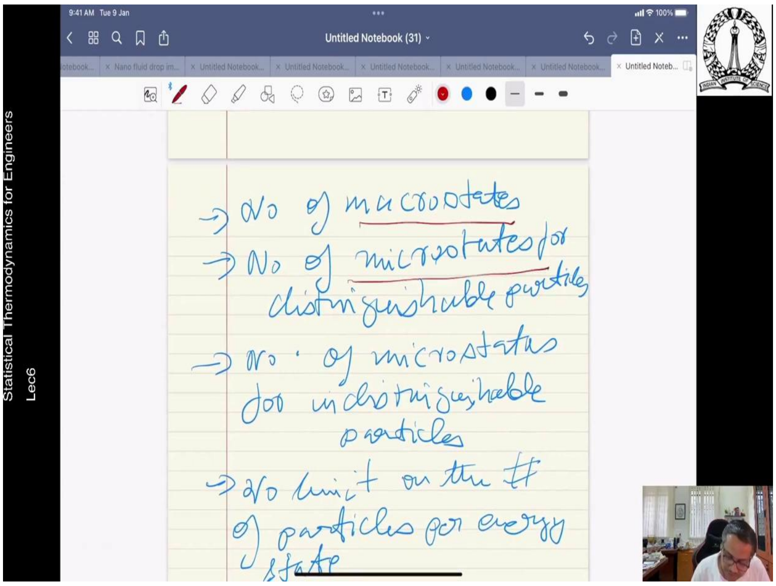

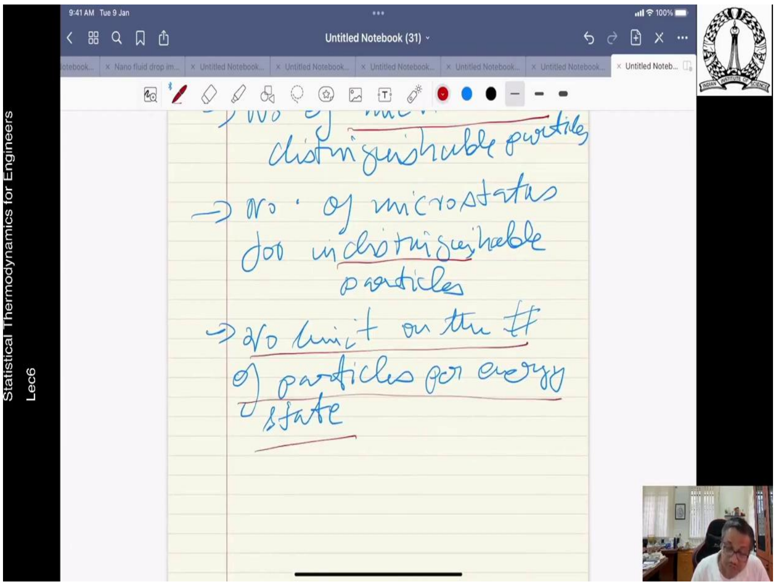

So, firstly determine what are the number of macrostate and then the number of microstates for distinguishable objects, distinguishable particles and the number of microstates for indistinguishable particles.

So, there is no limit and then no limit on the number of particles per energy state. So, let us look at, so this is an interesting example we can do it in a nice little way. So, as you can see, so, what we have to determine this is what is given and we need to find out the total number of macrostates the number of microstates but distinguishable indistinguishable objects and we are putting no limit on the number of particles per energy state that is this is a (())(12:14).

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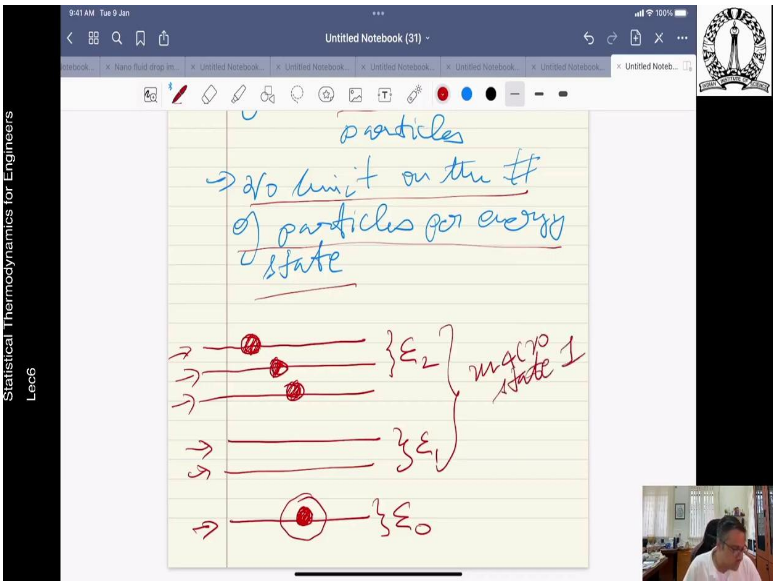

So, let us take the first thing. So, E2 has got degeneracy of 3, so, there are three possibilities, you can booked because all of them has energy of two all of them are energy. So, this is the three particles you can place the particle here here here in whatever way you want then of course, you have energy level 1 and then you have energy level zero. Zero has degeneracy of 1, this has got degeneracy 2. So, therefore, there are two energy states, this has got one energy state there are three energy states here.

So, the first macrostate will be will look something like this. So, you place a particle in 1 and 0 and then you can put the other particle in either one of these boxes for 2, because this has got energy of two, this has got an energy of zero. So, this gives you the first microstate, microstate 1.

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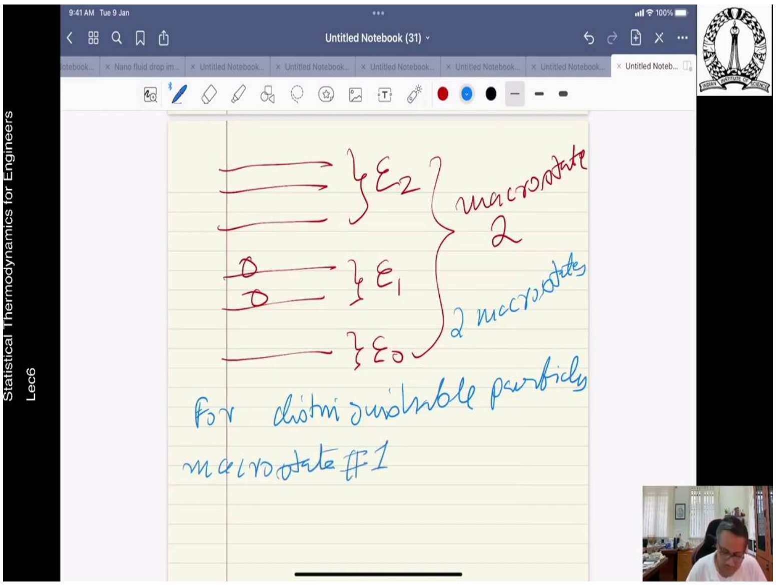

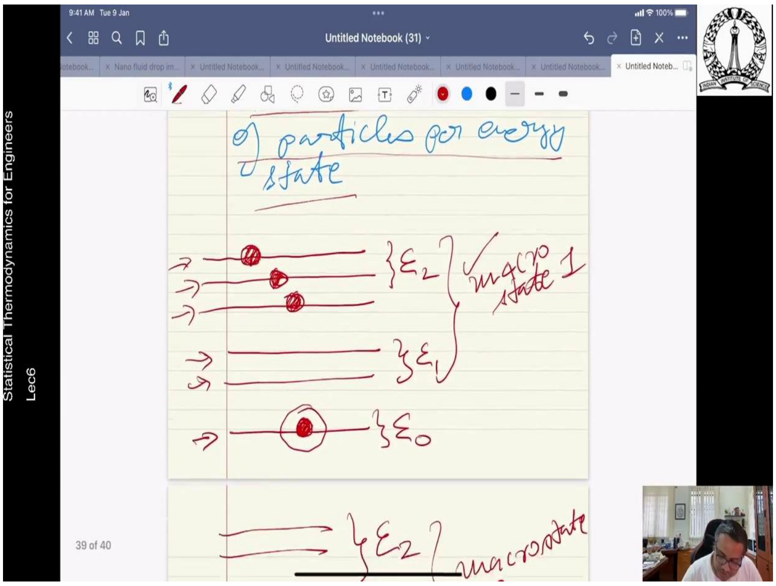

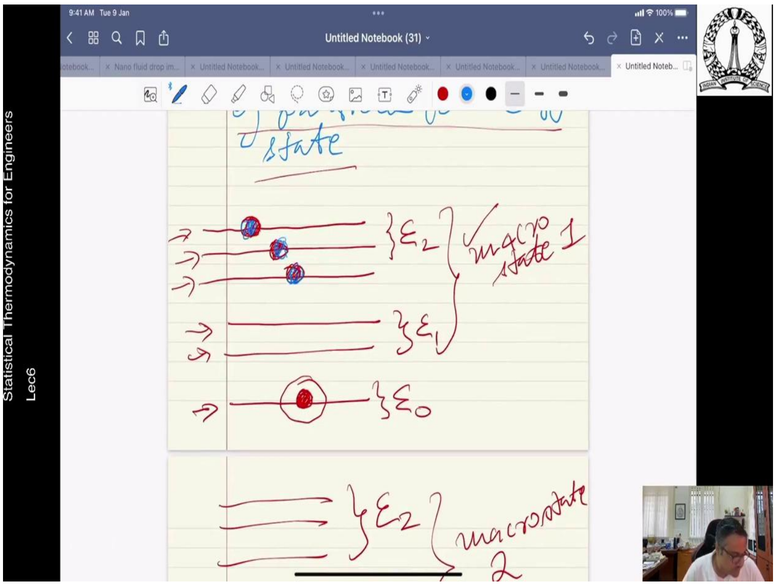

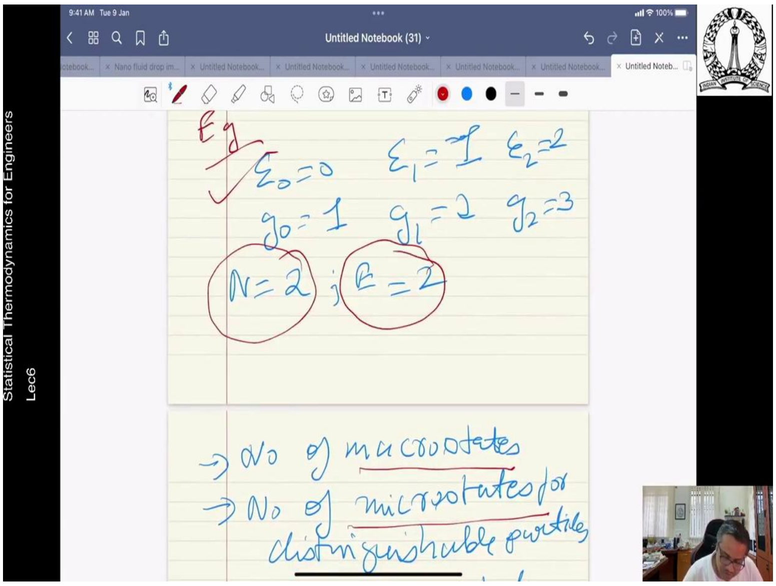

Let us look at what about other ones. So, again this is like energy level two, this is like energy level one this is like energy level zero. So, here you can now have two particles you can place them all in energy level one because then two particles will carry you know energy of two because this is one unit. So, this is like a microstate, microstate 2. So, now, as you can see, so, these are the two macrostates that are possible, which satisfies N equal to 2 and E equal to 2, this criteria. So, it satisfies that.

And this is the few arrangements that you can have a system can have this arrangement, and yet each it will be able to fulfill its energy requirement and the conservation of mass. So, for distinguishable particles, if you take for distinguishable particles, let us look at that. So, there are two microstates that we found out for this very simple system, so for distinguishable particles. So, let us look at microstate, macrostate number 1.

So, macrostate number 1 can have actually 6 microstates because you have these three other possible arrangements and for each of them you can have a distinct arrangement of E naught. So, there are basically 6 microstates.

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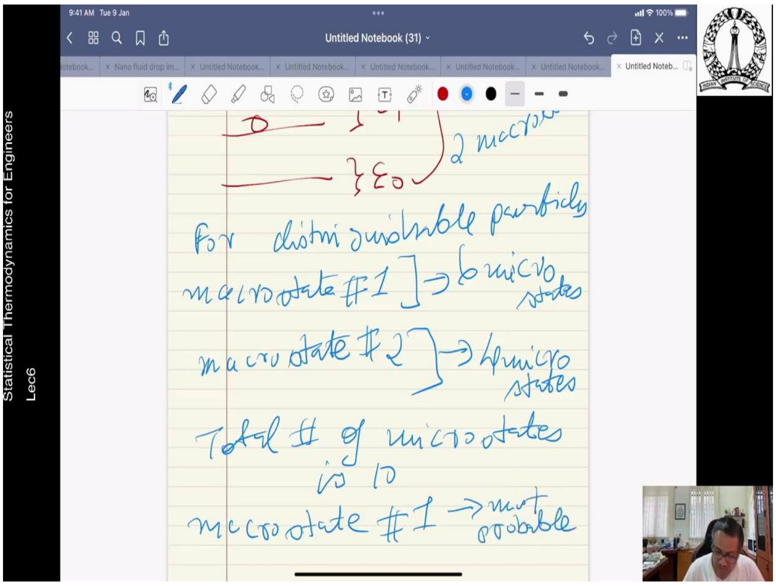

Now, the macrostate number 2 on the other hand it can just have for this is for distinguishable objects mind you, so it can have 4 microstates. So, the total number of microstates for distinguishable objects, total number of microstates is 10. And out of that macrostate 1 which has got the highest number of microstates to six is the most probable one. This is not distinguishable object, that is probable one.

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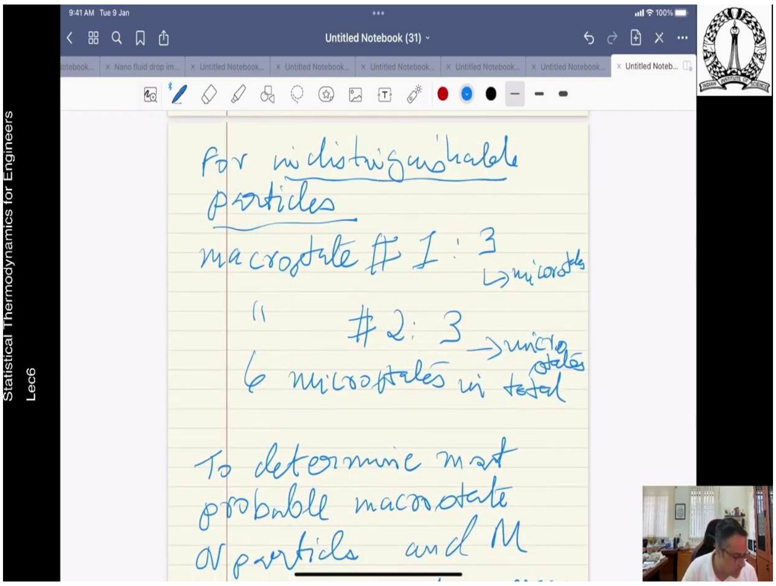

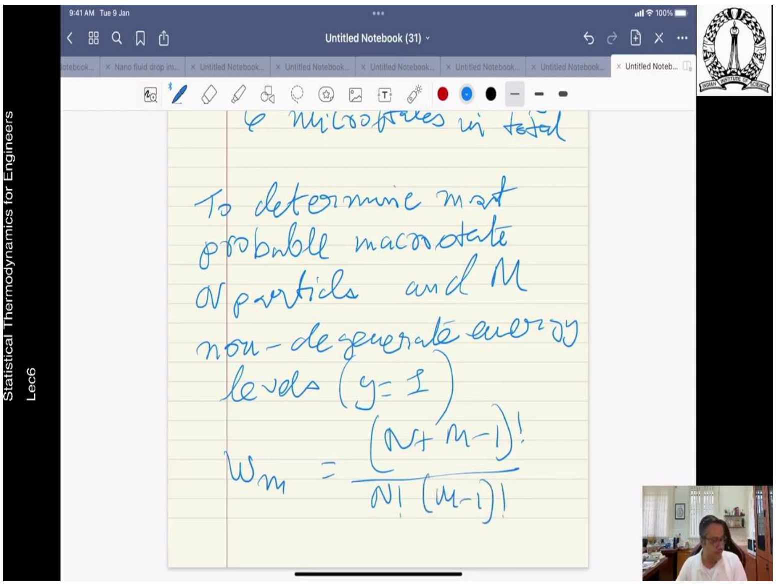

It should be, this is very simple example which gives you an idea of what is going on. Now, for if the particles are for example, for indistinguishable particles where there is no limit so, microstate 1 gives you 3 microstates you can check it out yourself why it is 3. And, macrostate number 2 also gives 3 number of microstates. So, there is altogether 6 microstates in total.

Now, to determine the most probable microstate suppose, so, to determine. So, you can see how the process works to determine the most probable microstate, suppose, we have N particles, so, this is generic now and M non degenerate energy levels that means g is equal to 1. So, in order to find the, how to find the possible particle distributions or the microstate, wm we already know this we did in the last lecture it is N plus M minus 1 factorial divided by N factorial M minus 1 factorial. So, we assumed we assumed in this particular case indistinguishable particles.

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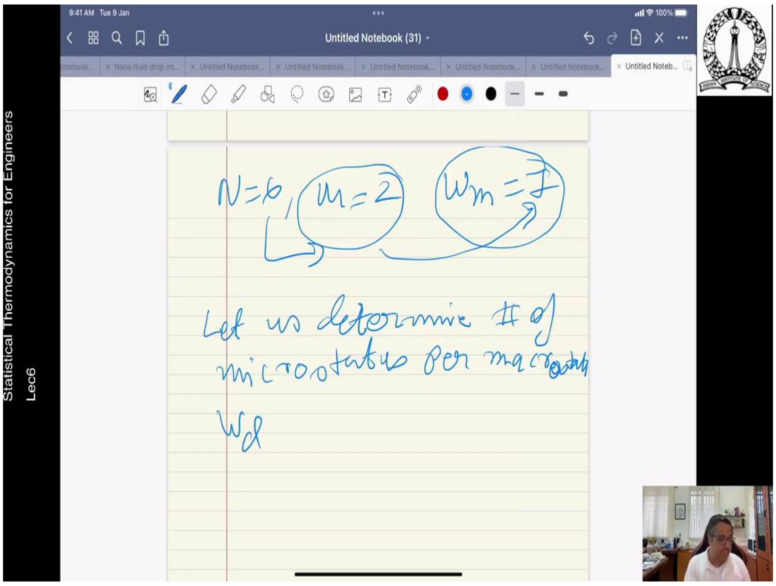

Now, if we just plug in the numbers over here see if N is equal to 6, M equal to 2, wm is equal to 7, this we already do. So, 7 macrostates are possible. So, 6 particles distributed in 7 macrostates in two energy levels, so energy levels are two and it is distributed in 7 macrostate 7 possible macrostates there, these are non-degenerate that means degeneracy is equal to 1 at each particular level.

Now, if we want to determine the number of microstates per macrostate let us determine the number of microstates per macrostate microstate, we have to determine that particular way. So, the number of microstates per macrostate which is say Wd, if we recall it, so, this depends on, so if there are now N distinguishable objects placed in M different containers such that Ni objects occupies the ith container this also we know.

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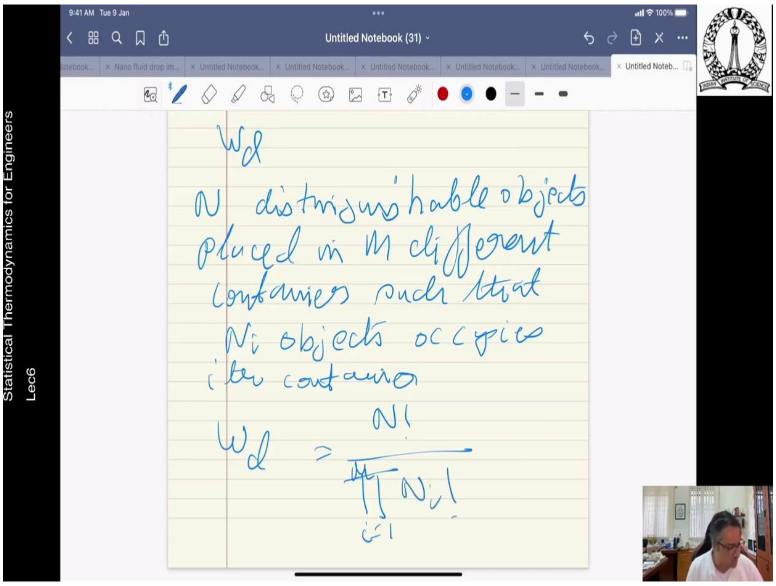

So, say for example. So, this can be cast now, you can see how the previous lectures are coming into the picture. So, this is equivalent casting problem that N distinguishable objects placed in M different containers, this is the question that we are asking, containers such that Ni objects occupies ith container. So, therefore, this Wd the number of microstates now will be given as N factorial divided by the multiplicative M i equal to 1 to Ni factorial.

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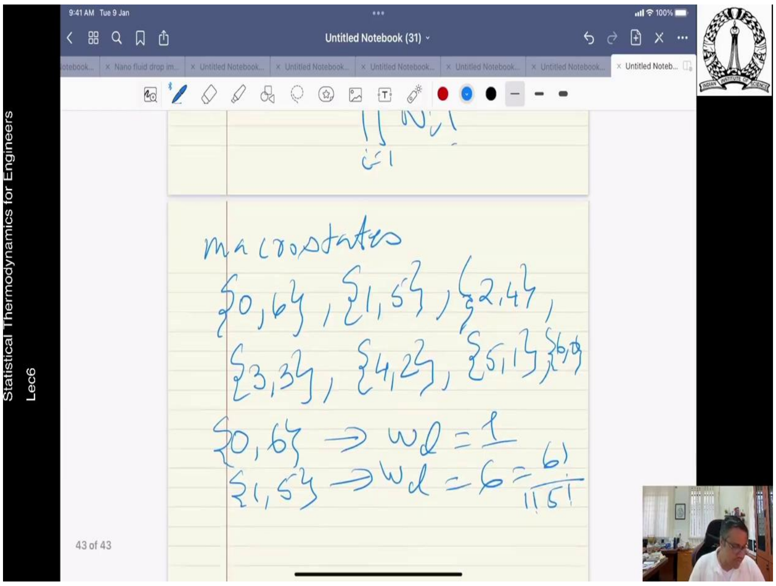

So, the macrostates for example, for that old problem that we had where N is equal to 6, M equal to 2 this particular problem. So, say the macrostates just to give you a brief example the microstates in this case are where 0,6 1, 5 2, 4 3, 3 4, 2 5,1 and 6, 0. So, say for macrostate 0 comma 6 Wd is equal to 1 for macrostate 1 comma 5, Wd is equal to 6. 6, how does it come? If 6 factorial divided by 1 factorial 5 factorial.

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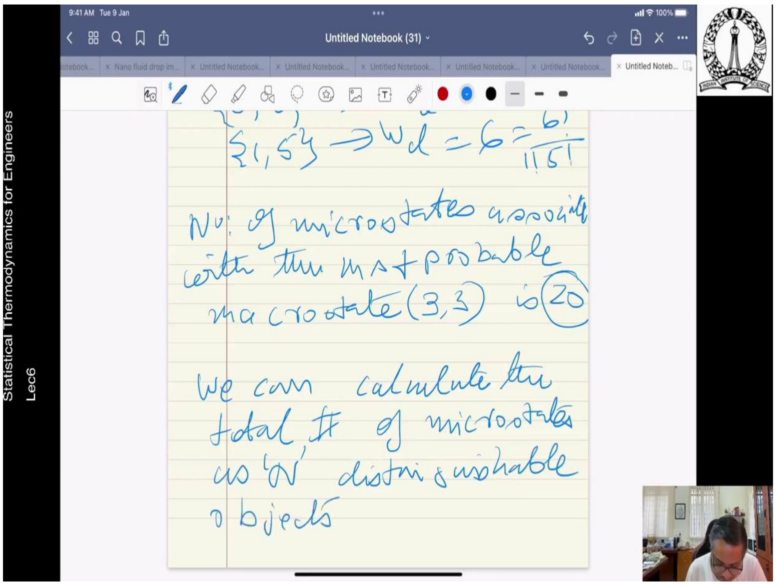

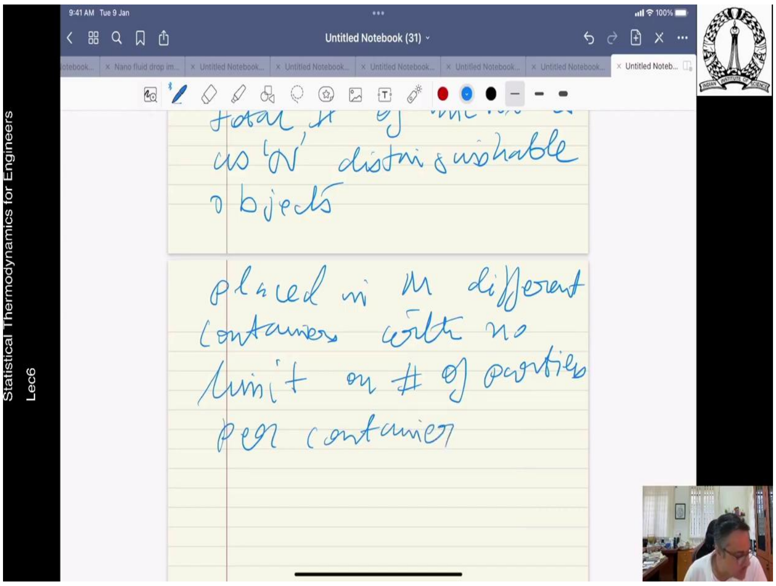

Hence, the number of microstates associated with the most probable microstate is so, therefore, the number of microstates associated with the most probable microstate is 3, 3 is 20. So, you can get an idea that what we are aiming at over here. So, you can also consider you can also ask this problem as the, we can also write this problem also like this, we can also calculate the total number of microstates as in distinguishable objects placed in M different containers with no limitation, no limit on the number of particles per container this also we have seen earlier.

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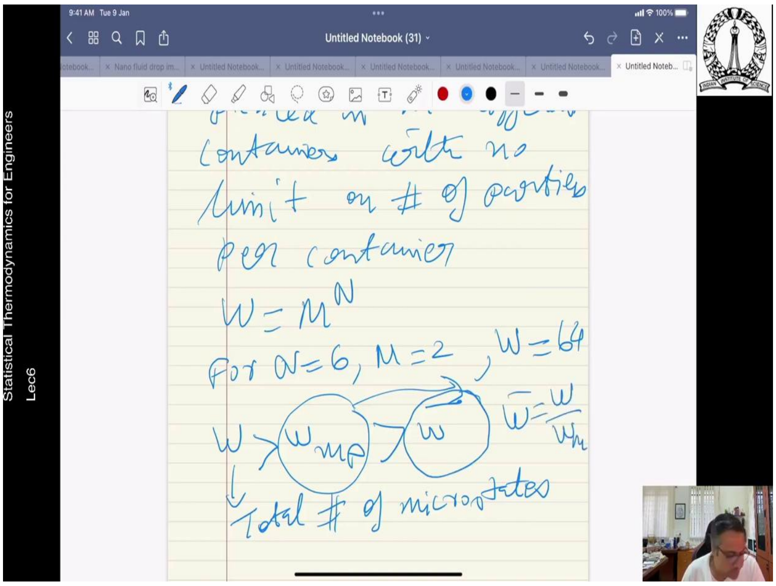

So, then W becomes M raised to the power N. So, for again the problem N equals to 6, M equal to 2, W becomes 64 this is the total number of microstates. We can also say that W the total number of microstates is greater than the most probable microstate and is therefore greater than the mean. So, this is the total number of microstates and this is the microbe number of microstates corresponding to the most probable microstate and this is the average number of microscopes. So, average is basically W by W.

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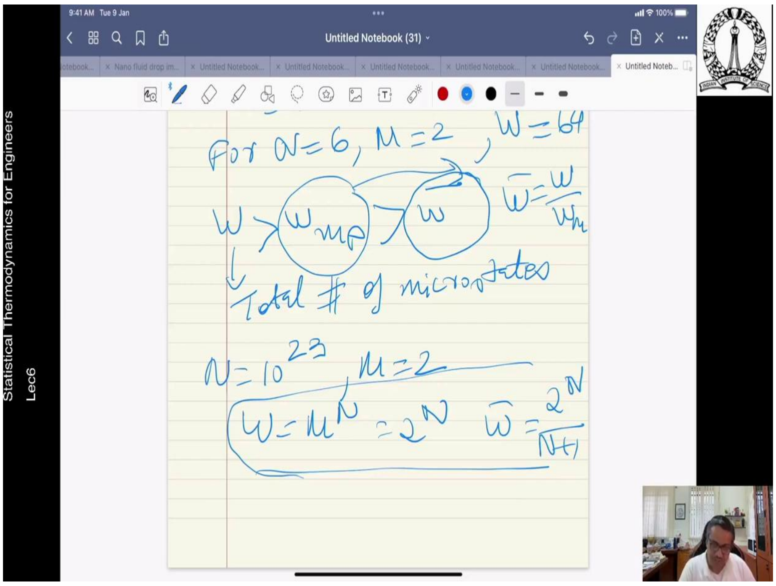

So, if this number N is very large, so the N 10 to the power 23, N equal to say 2, then W equal to M raised to power N which is 2 into N and then W bar will be equal to 2 to the power of N divided by N plus 1. So, we will look at some of these things in a little bit more details in the next class, because we have to now look at some of the other features of this macroscopic equilibrium.

So, right now, what you saw in this particular lecture is that you can you can follow a procedure to calculate the total number of microstates and the associated number of microstates with that macrostate and the total number of microstates as well and, there can be several microstates which are possible microstates are all equally probable, but the macrostates are not. So, the macrostate which has got the highest number of microstates is the most probable microstate and that is actually related to the equilibrium condition that is how you have a single value or a well-defined value of internal energy and stuff like that.

So, in the next class, we are going to see how this translates to some of the known distributions and how we can see that what is the most probable microstate, how, what are the number of microstates that are associated with it. So, in the next class, we are going to cover these things in a little bit more detail. So, here we end our lecture 6. So, Lecture 7 we will continue this conversation of macrostate and microstates. Thank you.