Statistical Thermodynamics for Engineers
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Lecture 21
Supplementary Video 6 Coordinate System 1

Welcome, everyone to another session of supplementary videos and today we will be discussing the basics of coordinate systems in a generic fashion. So, that is the today's topic. So, let us start the topic of today's coordinate systems.

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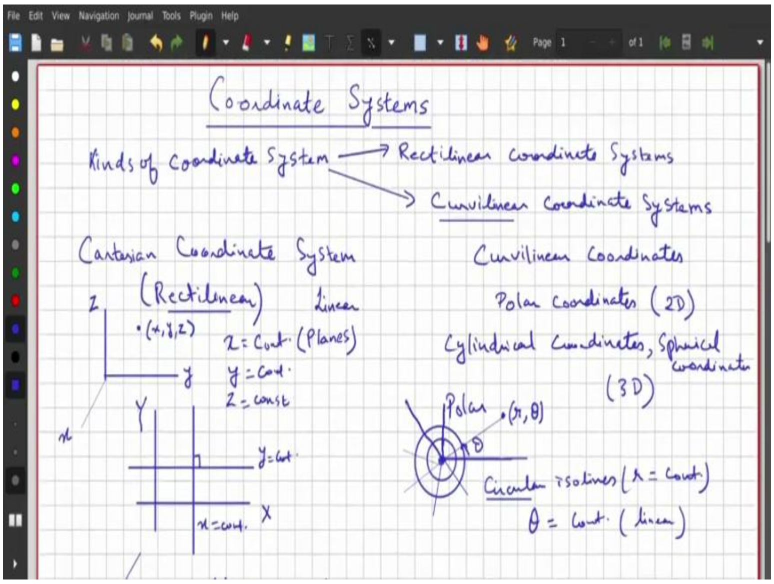

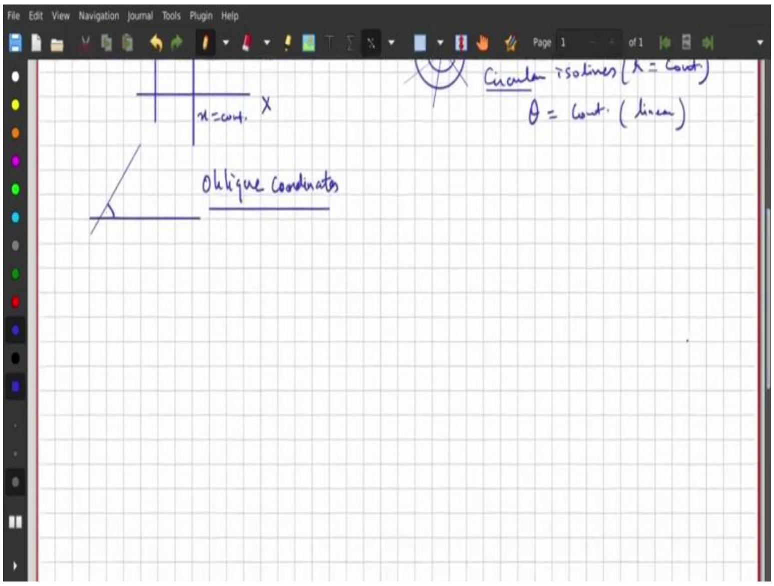

So, up to now you I think so by our from our undergraduate studies as well as our high school studies we have seen the use of coordinate systems quite a bit and we have different kinds of coordinate systems, kinds of coordinate systems. And the two generic way of characterizing is based on let us say, so we call one class of coordinate systems, rectilinear coordinate systems, and another one of this generalization basically is the curvilinear coordinate systems.

So, rectilinear; example of rectilinear coordinate system is Cartesian coordinates, for example. The Cartesian coordinate systems, Cartesian coordinate system. That is rectilinear. And here the word rectilinear means that the constant let us say we have let us draw the Cartesian coordinate system. We have the x. This is the y-direction and then this is the z-direction. So, x equals to constant, y equals to constant or z is equal to constant. These all define planes. Or in two-dimension you can think about them as lines. The define lines and they are linear. So, these are linear objects. So, that is, hence the thing rectilinear.

And on the other hand, examples of curvilinear coordinates, curvilinear coordinates are for example polar coordinates, and this is in two-dimension, and we have the cylindrical coordinate, cylindrical coordinates, and then we have spherical coordinate systems, spherical coordinate system and that is in two-dimension. So, these are examples of curvilinear coordinates because as you can see, let us take an example for, draw the polar coordinates.

So, if you see the polar coordinates is defined by some origin and then that so here the iso you can think about let us say in a two-dimensional rectilinear coordinate, we draw the two-dimensional rectilinear that is Cartesian coordinate let us say x and y. So, here the const x is equal to constant line and y is equal to, this is y is equal to constant line; y is equal to constant. Similarly, in x is equal to constant line is again a straight line, x is equal to constant.

So, as you can see, these are straight lines, linear and also they are orthogonal. So, these all are orthogonal coordinates which we are discussing whether it is Cartesian. In a fashion a coordinate system can be arbitrary. So, we can have coordinate systems which are let us say like this. So, where the angle between axis is not 90. Those are oblique coordinates. These are oblique coordinates. So, those are also useful in certain types of problems but for this material, this segment specifically what we will be dealing with is orthogonal coordinates.

And similarly, for example, here in terms of the curvilinear space we have the polar coordinates. So, if you; so you have a pole and from which you have different radial directions. So, basically, and these directions are categorized by an angle theta and then you have concentric circles. So, that represents polar coordinates.

So, here any point is represented; so for example, in a Cartesian coordinate any point is represented by 3 numbers, a set of 3 numbers, three tuples x, y, z. In polar coordinates, this is a 2D representation so any is represented by an r and theta where theta is the angle from some reference axis and r is the distance of the point from origin which is also known as the pole. So, that is; so this is an example as you can see, this is a curvilinear coordinate because one of the isolines as you can see are spherical like here these are circularized isolines, circular isolines which are the radial lines, which represents r equals to constant.

On the other hand, the other lines, other isolines theta is equal to constant, those are still linear objects. But here since one of the coordinates is as you can see it is nonlinear hence this belongs, the polar coordinate belongs to curvilinear coordinate systems. And the extension of the polar coordinate in a higher dimension is a cylindrical coordinate and we also have the spherical coordinates. So, these are some of the basic examples of coordinate systems that we use in all of physics.

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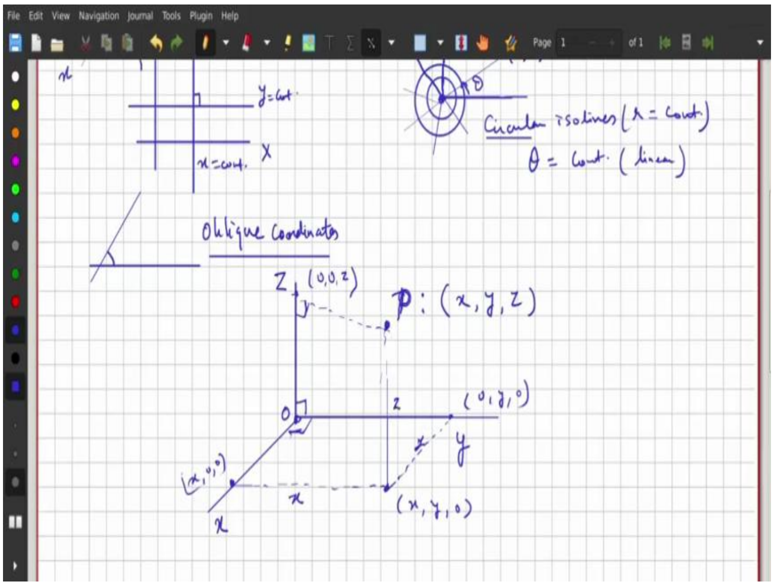

So, let us say, so let us say we have a Cartesian orthogonal coordinates. So, this is the origin. We have the x-axis, y-axis and the z-axis. And let us; this is a Cartesian coordinate as you can see because it is linear, it is a rectilinear coordinate system and it is orthogonal because these all angles as you can see are 90 degrees. It is a 90 degree. So, any point in this coordinate space P, let us call that point P that is represented by 3 numbers, a set of three numbers: x y, z. That is like that. So, these coordinates are x, 0, 0. This is 0, y, 0. You see here, this is an x, y, 0.

Similarly, if you join like this, this is a 90 degrees. This point is 0, 0, z. So, you see this length is x. This length is y and this vertical height length that is z. So, that is that is a general representation of a point in cylindrical coordinate. Sorry for that for. This is following Cartesian coordinates. The same point can be represented in a cylindrical coordinate system in a different way. So, let us say if this is I will say Cartesian coordinate, Cartesian coordinate.

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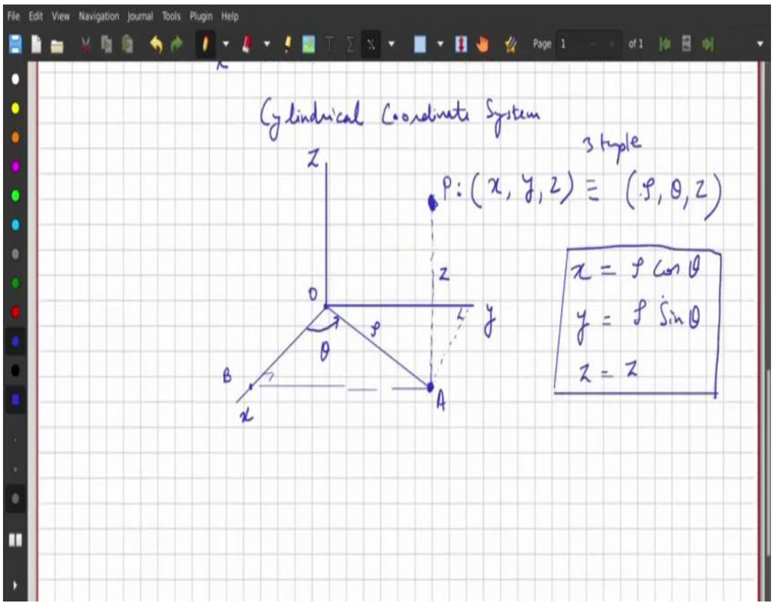

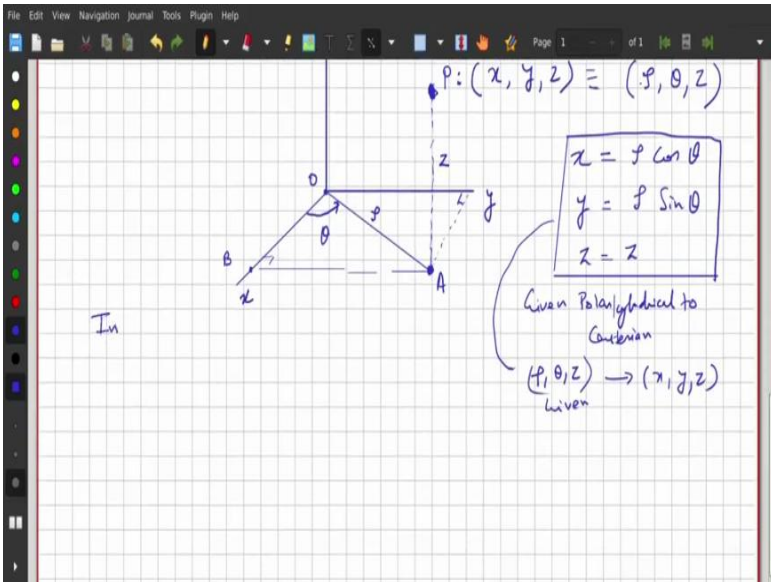

So, let us do cylindrical coordinate system. So, let us again draw the Cartesian coordinate just for reference purposes. Because that is a system in which it is always easier to write things down. x, y, z: that is the general our Cartesian coordinates, and let us say we have our point P whose Cartesian components we know that are given by x, y and z. And we know what those are now.

As we did before so, same point P, since it is in 3 dimensions it needs 3, 3 tuples to represent the point but we can use different points. This same point x, y, z is equivalent to in cylindrical coordinate systems to which is known as let us call that rho, theta, and z where rho is the distance from origin to this point. Let us call that point A. So, this distance from the origin to point A, that is rho. The angle that OA makes the x-axis that is called, that is the theta. That is the azimuth angle and this is the height, which is z itself. So, that is a cylindrical coordinate system.

And you can see from this triangle like OAB, we can write a representation of or the relationship between the Cartesian and the cylindrical coordinates and the representation is if you see properly the x coordinate is rho cosine of theta; y coordinate is rho sine of theta and the z coordinate stay as it is okay because that in that direction, it is the same as the Cartesian. That is the transformation as you can see from Cartesian to polar coordinates.

And so that means given rho, theta and, z we know what is x, y, z. We can write the reverse transformation. So, the inverse, write the inverse transformation. So, this is for this is given polar or cylinder to Cartesian. So, rho, theta, z are given, and, using these set of equations we can find what is x, y, z.

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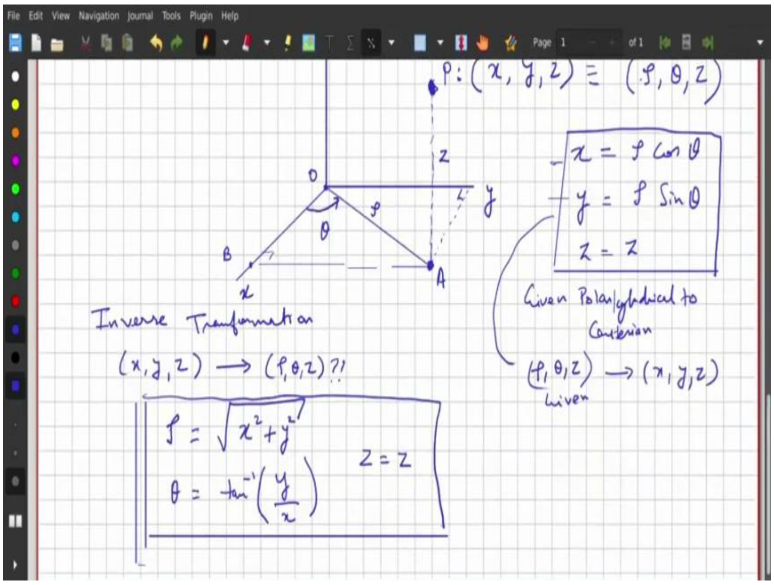

Let us write the inverse transformation. The inverse transformation means that means we are now given x, y, z and then we have to write rho, theta, z we have to find out. This is, find out. This is unknown. So, if you see that that is quite easy to write just from these two equations, rho as square root of, it is a distance. This is basically rho that is x index y plane x squared plus y squared. That is rho.

What is theta? Theta is as you can see is a tangent. If you divide this equation by this, the second one this one by this one will get the tan theta which is y over x and z stay as that it is. So, these are the inverse transformations. Then these are the inverse transformations. So, we understood about cylindrical coordinate systems and this is Cartesian coordinate systems. So, let us go to spherical coordinate system.

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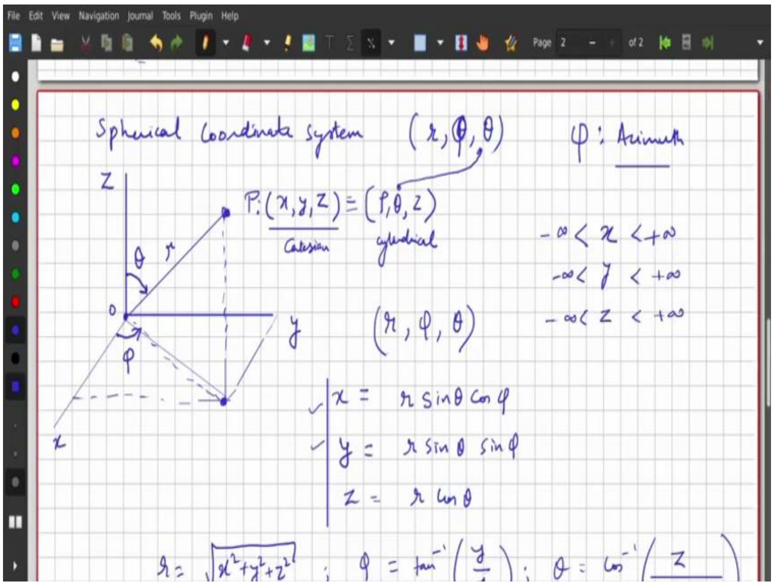

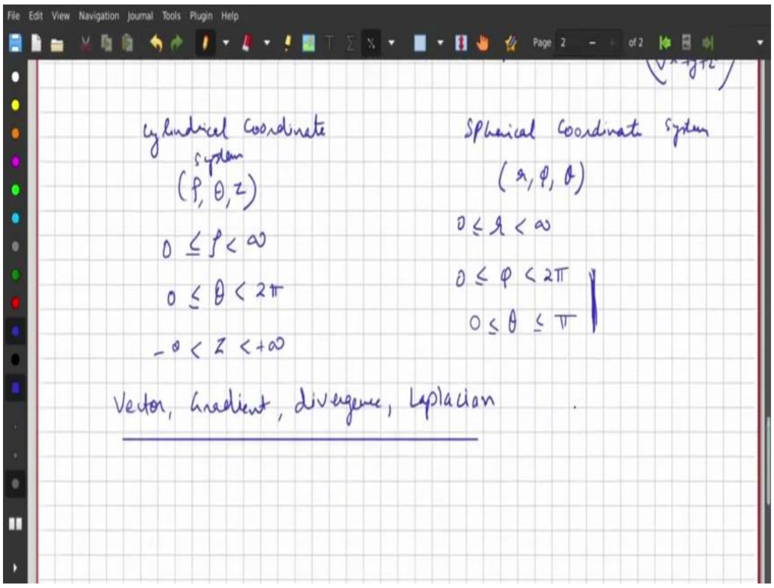

Spherical coordinate system. So, let us again draw the Cartesian coordinates for reference. This is our x, cartesian y, z and this is the point P whose x was given by x, y, z. In Cartesian that point was given by rho, theta not rho. I think it will be called it, so it will be called at theta, z. So, this was in Cartesian, Cartesian. This was in cylindrical. And now we represent the same point in spherical and then that spherical coordinate system is given by a set of numbers which is r, theta or let us write it r, phi, and theta; r, phi, and theta. And note by the way, in the cylindrical coordinate system, we use this theta and here also we have theta. These two are different. We will see.

In a cylindrical coordinate system, theta is the azimuth angle and in spherical polar coordinates phi is the azimuth angle, azimuth. So, this point, the distance of that one from the origin that is r, and similarly, let us drop the projection of that point onto the x-y plane and this angle is the azimuth angle known as phi and the angle between OP and z-axis is theta and that is known as the zenith angle. So, these are the three r, phi, and theta are the three set of numbers that represents the same point P in spherical coordinate systems.

So, let us write a similar relationship between the Cartesian and the spherical as we did it for cylindrical. So, you see, let us draw the projections and see x equals this is x and you see that is our sine theta which is this length. This is r and this is r sine theta and, the projection of that on the x-axis is r sine theta cos phi. So, that is x. Similarly, y will be r sine theta phi and you see the z will be the r cosine theta.

So similarly, let us write the inverse transformations as we did for, so we see the inverse transformation. r is the square root distance from the origin, x squared plus y square plus z square. And phi is, it can be represented by dividing let us say this equation by this equation which is you will see tan inverse y over x and theta is cos inverse that is you can see from here. It is r, r, z over r. So, that is z over r is the square root of x squared plus y squared plus z square. Those are the inverse transformation in the spherical coordinate system.

So, one very important point that we need to keep in mind is the range of the variables. So, in the Cartesian coordinates, x range is minus infinity to plus infinity, same thing for y and same thing for z. So, it is unbounded in all the 3 directions. It is infinite. For the cylindrical and the spherical coordinate systems, let us write that down. So, for the cylindrical coordinate system, we have it is given by rho, theta, and z.

If you see, cylindrical coordinates is given by rho, theta, and z. and rho goes from 0 to infinity. Theta is the azimuth, okay, this is the azimuth and that goes from 0 to 2 pi. So, you see in that direction it is bounded and z is in unbounded again. It is from minus infinity to plus infinity.

Let us write the spherical coordinate system that is given by r, phi where is the azimuth now and theta is a zenith. So, r, 0 to infinity. Phi is 0 to 2 pi and theta is 0 to pi. So, it is bounded in 2 directions now because this represents the surface for; a constant r will represent a surface of a sphere in the spherical coordinate system.

So that is the generic understanding of all the 3 types of coordinate systems, and we will generalize all these ideas so that what we are basically what we want to do is basically vector calculus in all this coordinate cord-; in all these coordinate systems. That is the general our main goal.

That means we need to know how to write quantities, like a vector, the gradient of a vector, divergences, divergence, the Laplacian. So, that is such in general the aim that we have in our mind. So, we have to represent all these quantities in different coordinate systems. So, that is a goal as you can think about.

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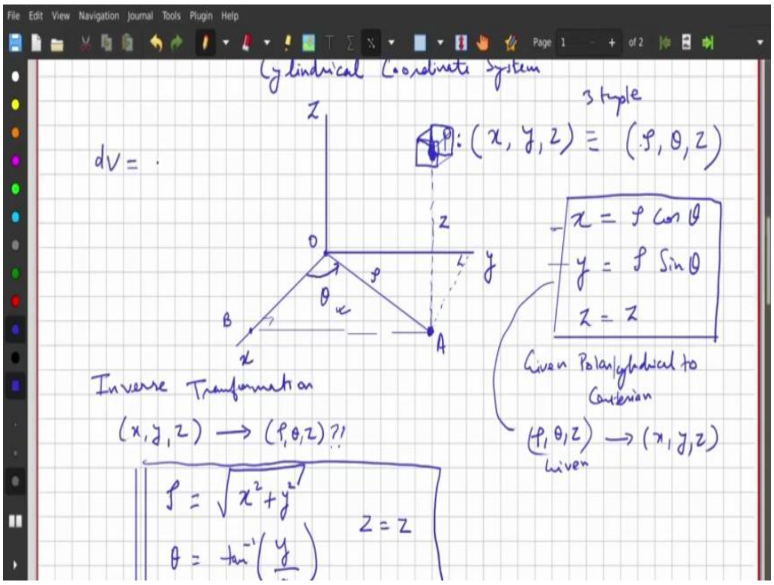

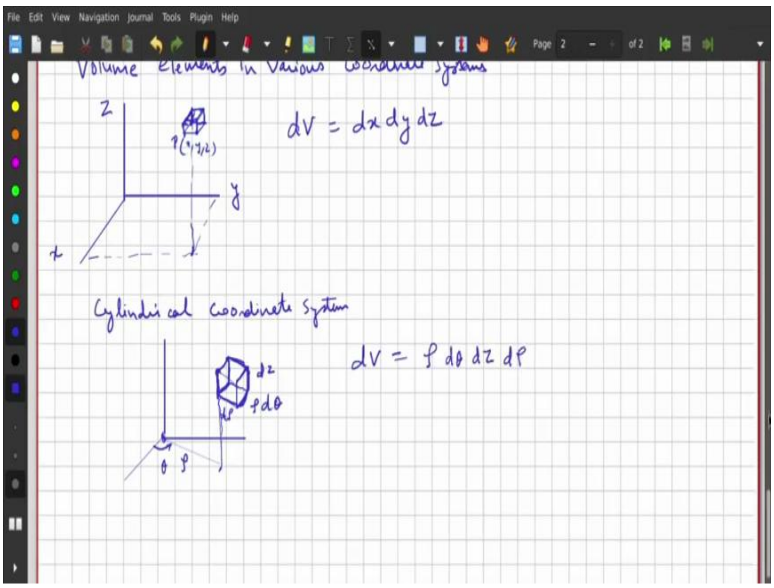

So, before moving on to the writing or let us say discussing the vector calculus in various coordinate systems, an important point to note down is knowing the volume elements. So, for example, the volume element that means let us draw an infinitesimal volume near to point at point P, basically centered at point P, and you see that it is a cuboid something. Something like this.

And that volume of that small elementary volume and cylindrical and let us call it in Cartesian coordinates, let us do it for Cartesian coordinate. So, if we write down the volume elements, volume elements in various coordinate systems, various coordinate systems. So, we have this x, y, z and this is the point let us say P given by x, y, and, z. So, a small elementary volume at that point P will look something like this. It is a cuboid. And that elementary volume dv has magnitude dx, dy, dz.

And let us do it in cylindrical. So, just by the nature of the cylindrical coordinate system, you will see that let us say we have again now coordinate system and this point P now but that point, let us see. This is rho, that is theta. So, this is point P. So, a small volume element there will look something like this.

As you can see, so, if you see the sizes of this small differential volume and we saw the sides parallel to the z-axis is dz. This which is parallel to let us say the x-y plane, this is basically this length as you can see from this. So, this is rho and this is theta. This elementary part will be this is rho d theta. And this is, this is measuring in the rho direction, so this is d rho. So, the dv, the elementary volume becomes rho d theta dz d rho.

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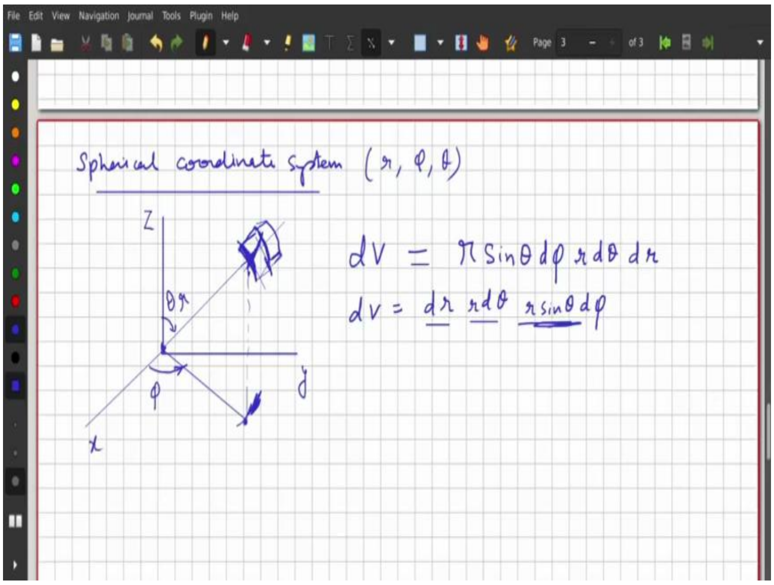

And let us do the same for spherical and spherical coordinate system, spherical coordinate system. So, let us write the volume element for spherical coordinate system. So, this is x, y, z, and any arbitrary point P in spherical polar coordinates. This is given. This is r. Projection of that on the x-y plane. This is the azimuth phi. So, it is in this system, we have r, phi and theta, the zenith. And this is theta.

So, when if you draw the coordinate system, let us say the elementary volume in this coordinate system, this will look like and this gets- here we have the 2 sides of it and the third direction is in the radial direction. So, this has to be projected in the radial direction. So, that is d. let me redraw it. This is point P. This is the direction of dr. This is one direction and then we have another direction which is parallel to this, which is something of this.

So, this will represent an element which looks something of like this and something like this. So, bear with me. My drawing for here. It is difficult to draw digitally. So, that let us write dv. This is the volume element and if you see the 3 dimensions of the cube, r; so it will be r sine theta d phi. So, that is in this direction. That is r sine theta d phi. That is parallel to that direction times r d theta, which is this direction times dr. So, that is the volume element in the spherical coordinate system. That is the generic volume element.

So, it is, let me write it in a better way. So, the dv is dr r d theta and r sine theta d phi. So, these are the directionalities as you watch radially. This is the zenith and this is the azimuthal direction. That is the volume element in the spherical coordinate system. So, now we will go to figure out exactly how to write the various vector like the gradient, divergence, Laplacian for any general arbitrary vector field in these various coordinate systems, and that we will do in the next segment. So, that is it for this segment. Thank you for watching. And see you in the next segment.