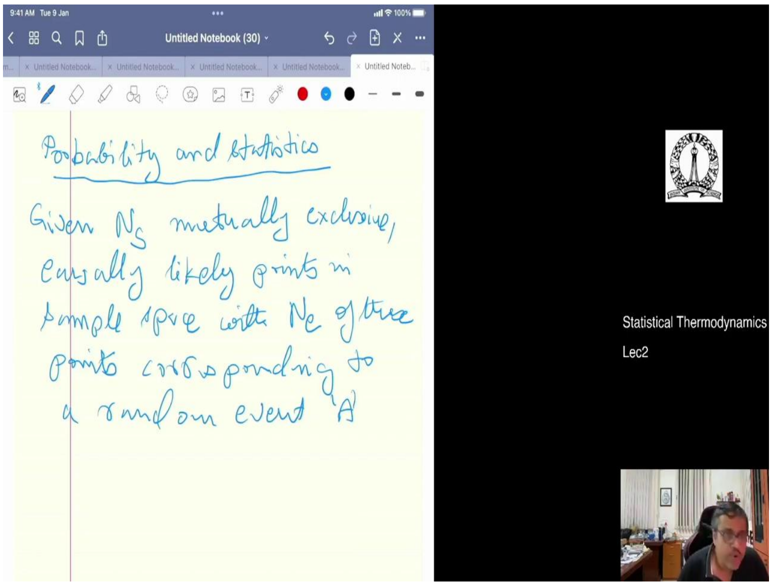
Statistical Thermodynamics for Engineers
Professor Saptarshi Basu
Indian Institute of Science, Bangalore
Lecture 02
Basic Probability Theory and Statics

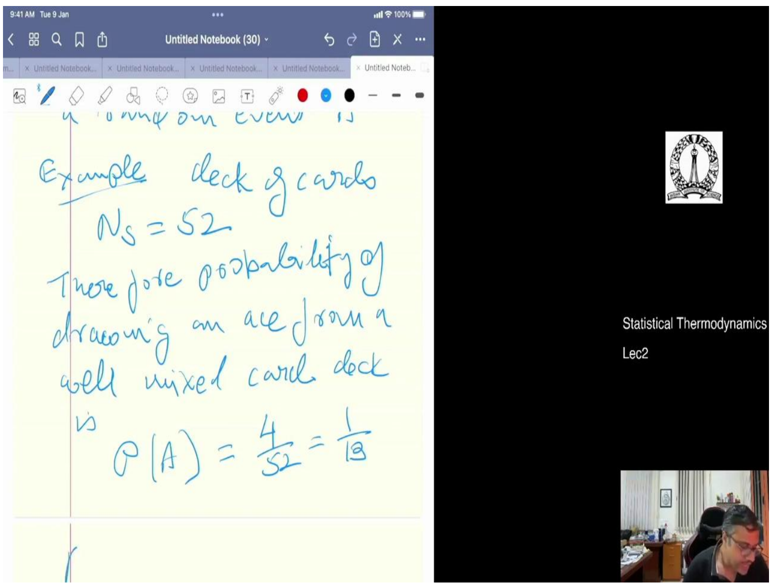
So, welcome to the second lecture of Statistical Thermodynamics for Engineers. So, in this particular lecture, we are going to cover the principles of probability and statistics.

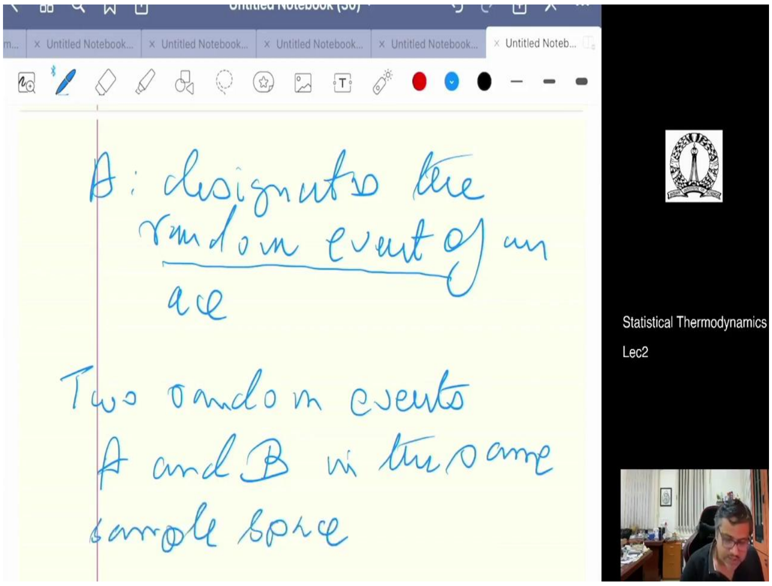
(Refer Slide Time: 00:17)



So, let us look at it things will be, probability and statistics. So, let us take a situation given say N_S mutually exclusive equally likely points in sample space with any of these points corresponding to a random event A. So, given that there is N_S mutually exclusive equally likely points in a sample space with any of these points corresponding to a random event A.

(Refer Slide Time: 01:38)

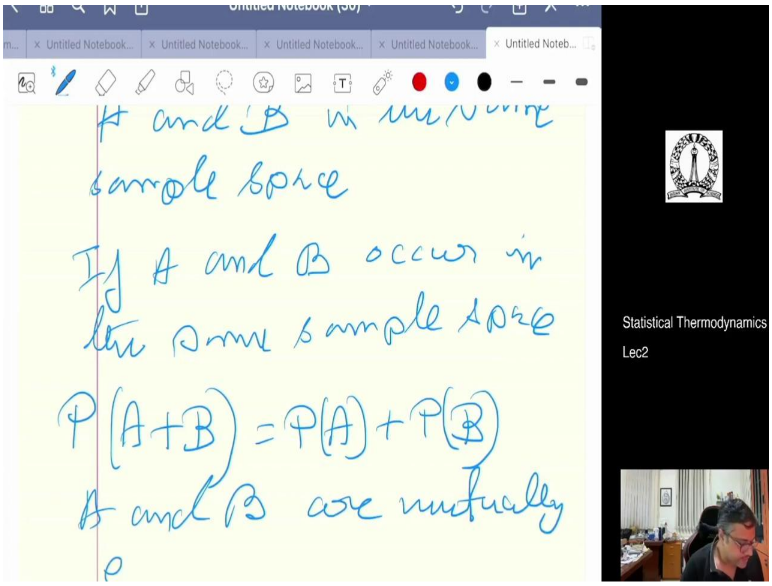


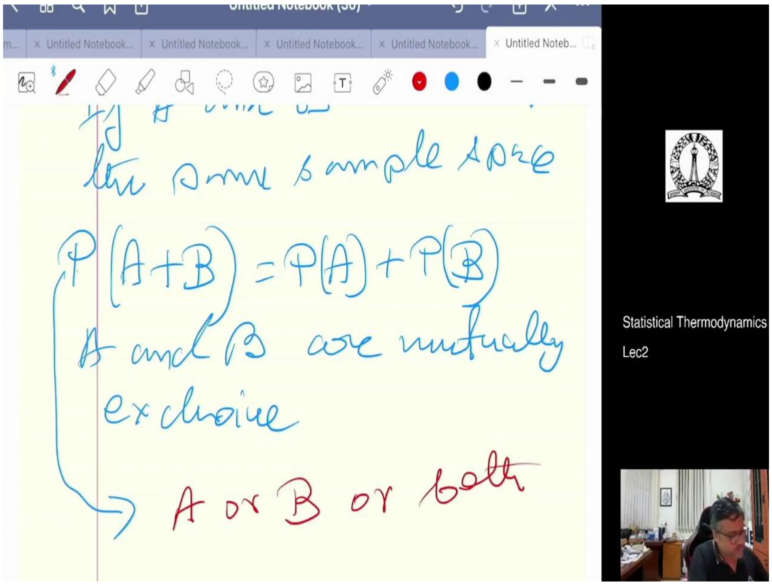


So, let us take an example. So, that you can understand the best let us take a deck of cards. So, deck of cards. So, where N_S is basically 52 and a deck of cards there are 52 cards as we know. Now, a probability therefore, if we want to know the probability of drawing an ace a well-mixed card deck is given as P(A), where P(A) is equal to 4 by 52 or 1 by 13.

So, here a basically designates, that designates the random event of an Ace. So, this should be now understandable that the probability of drawing an ace from a well-mixed deck of cards is given as that random event is called a. Now, if we consider two random events, if we consider two random events. So, now let us consider two random events, A and B, these are the two random events A and B in the same sample space, two random events in the same sample space.

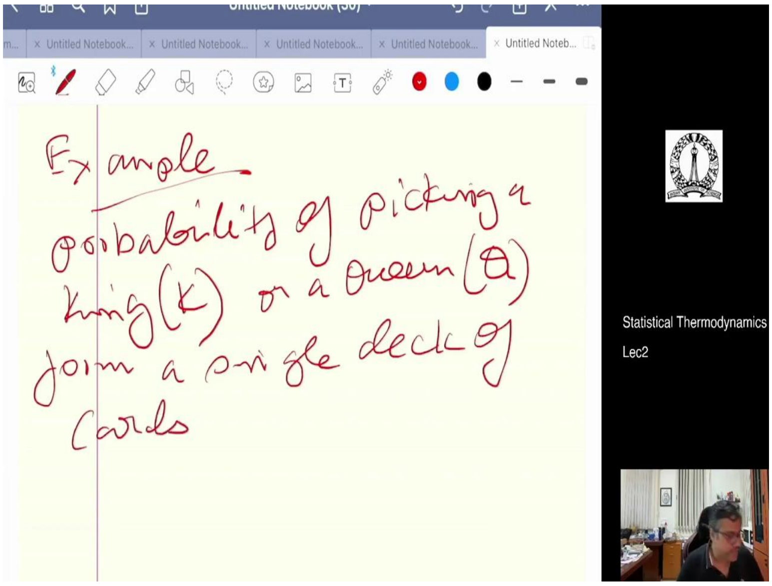
(Refer Slide Time: 04:00)

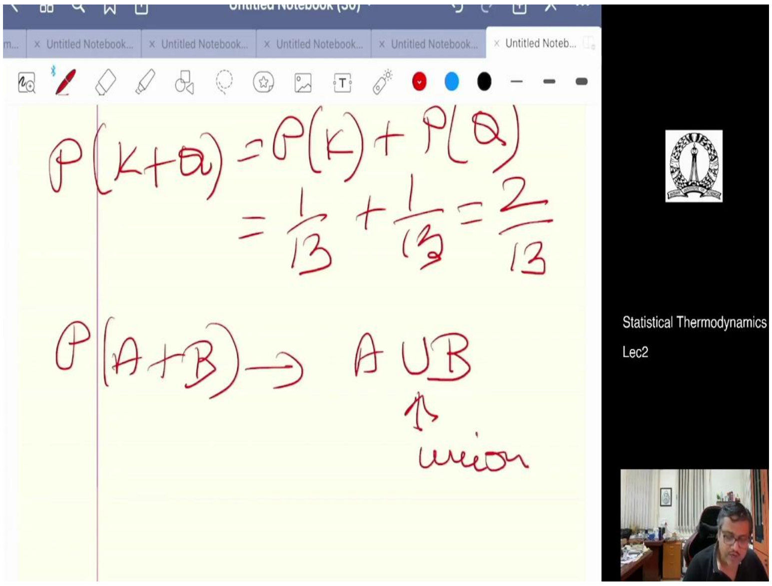




Let us look at that. Now. If that is the case, then if A and B occur in the same sample space then probability of A plus B, is probability of A plus probability of B, because A and B are mutually exclusive mutually exclusive. So, physically this means, this probability of A plus B means that A or B or both this is what it means. So, physically this is what means that A or B or both.

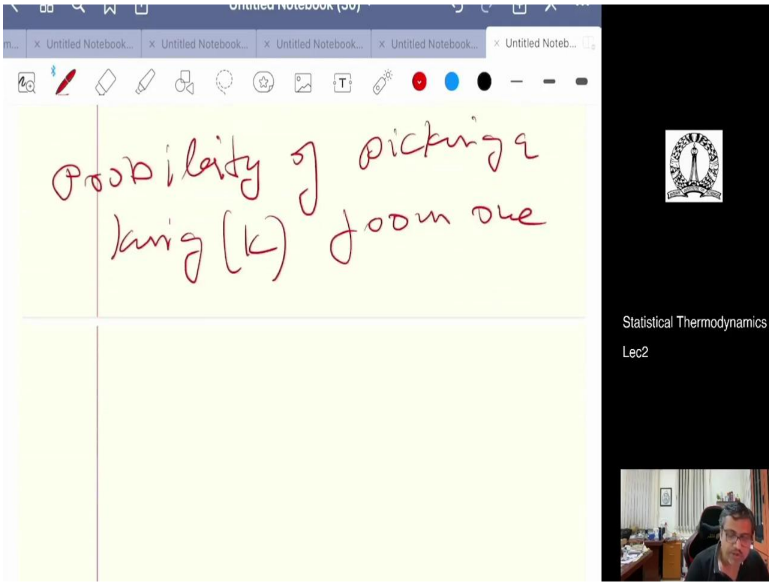
(Refer Slide Time: 05:11)

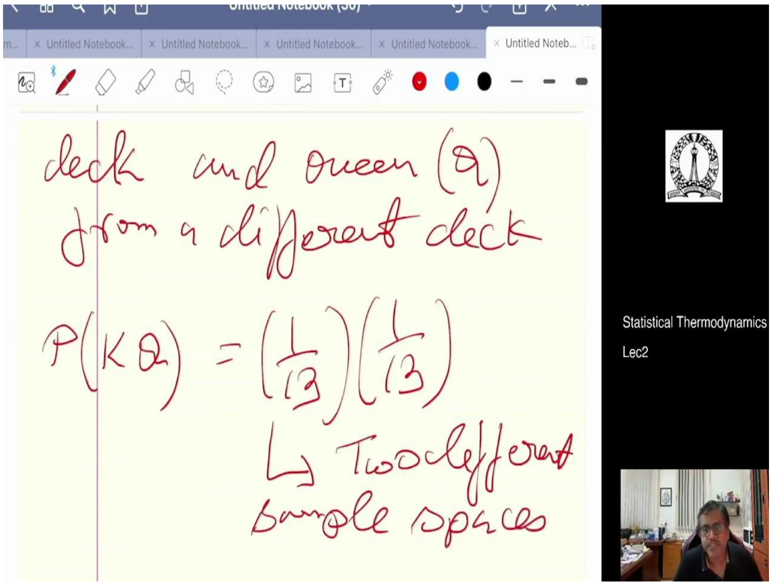




So, example, you can take an example right now. So, say for example So, let us say the probability of picking a king say that K or a queen Q, from single deck of cards, let us calculate this. Now, so this will be P (K plus Q) is probability of getting a king plus the probability of getting a queen which means , it is 1 by 13 plus 1 by 13 which is basically 2 by 13. So, probability of A plus B is more like A union B.

(Refer Slide Time: 06:38)

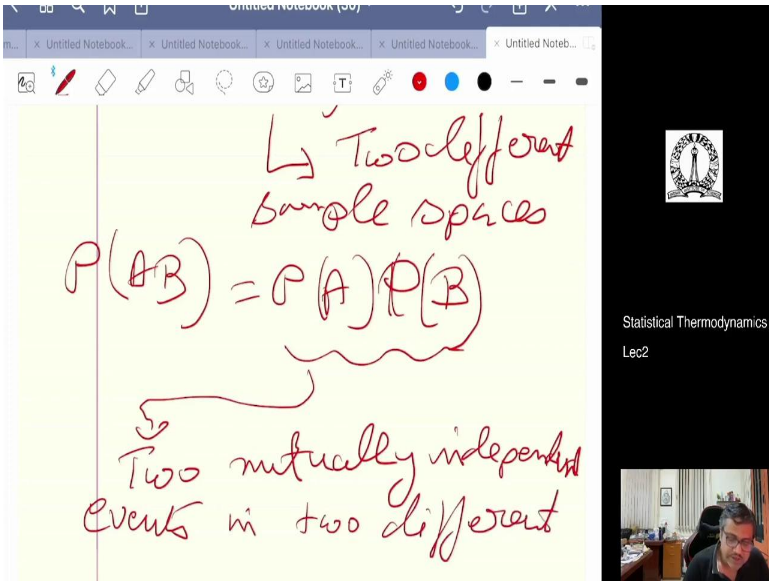


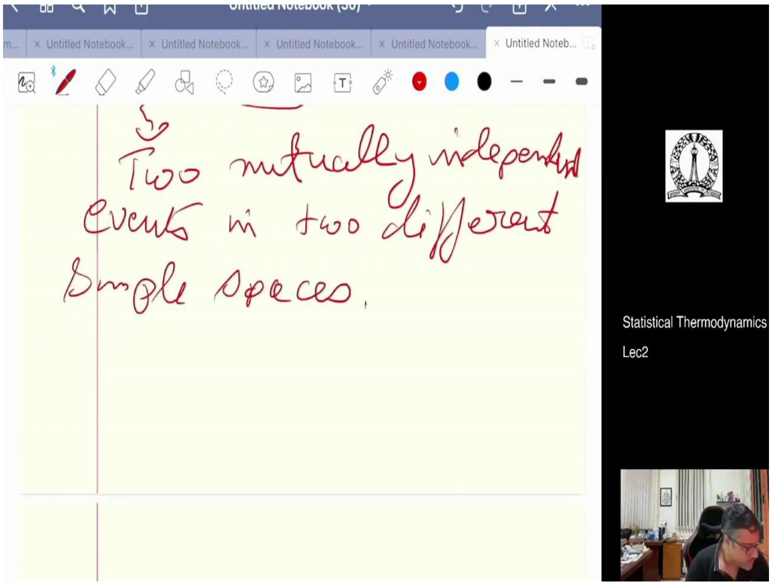


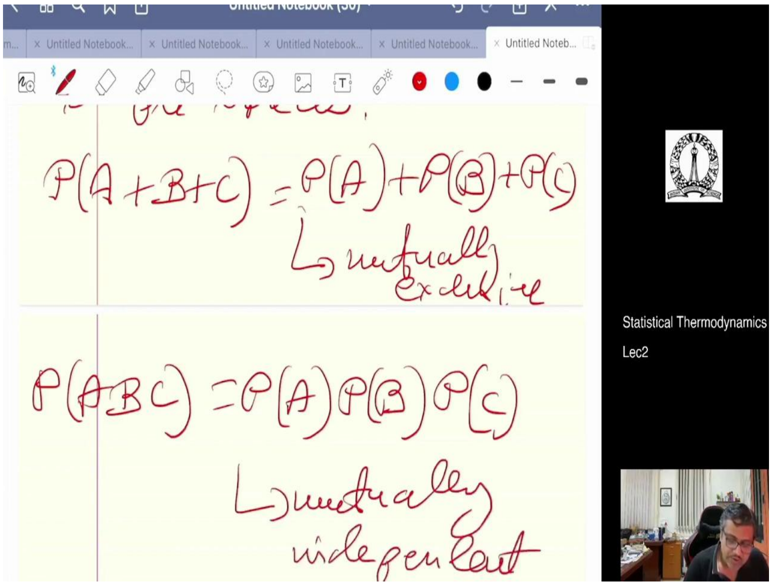
Now, let us take the same example and take it a little bit forward and take the same example and say the probability of picking a king which is K from one deck and a queen which is Q from different deck. So, these are now two different packs of cards that we are talking about, so, they are not from the same deck they are from two different decks.

So, in that case we write probability of KQ is more like multiplicative 1 into 13 by 1 into 13. Because in this case these are two different sample spaces. So, these are two different sample sizes. So, in this case your P(AB) is more like P(A) into P(B). So, this essentially implies that these are two mutually independent events in two different samples spaces.

(Refer Slide Time: 08:34)

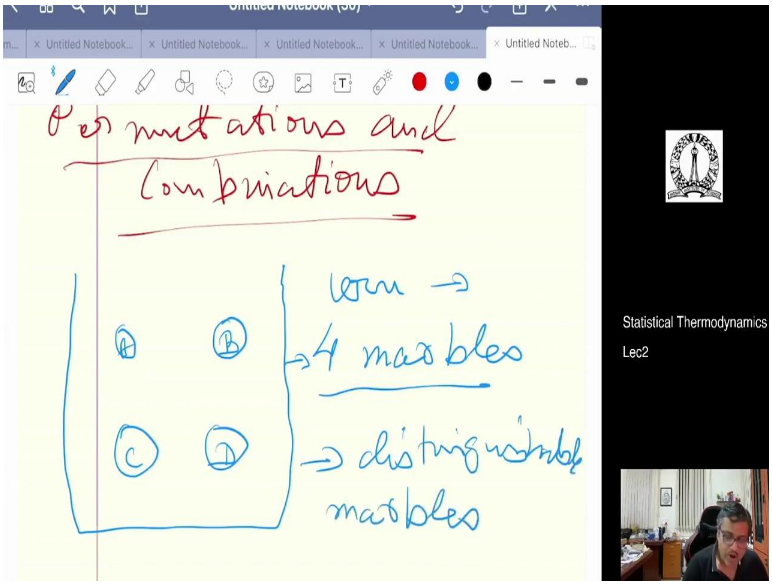


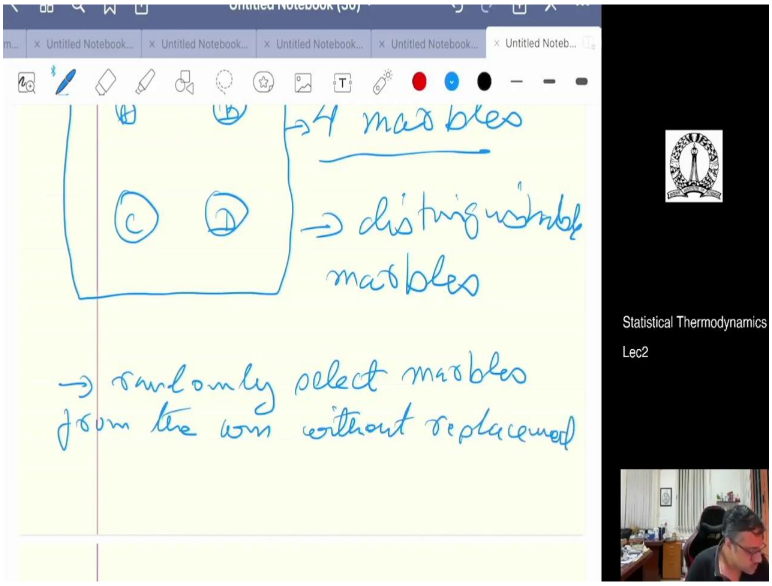


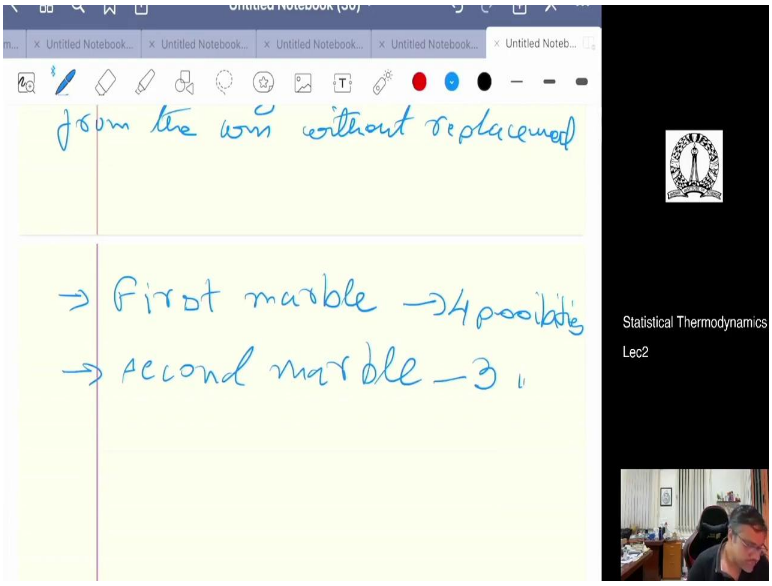


So, when we talk about the P(A plus B plus C) it means P(A) plus P(B) plus P(C) these are what we call mutually exclusive events these are mutually exclusive in nature and when we say P (ABC), P(A) P(B) and P(C) these are mutually independent. So, these are mutually exclusive and then they are mutually independent. So, now, let us look at some of the permutation and the combinations in this particular case.

(Refer Slide Time: 09:34)


Untitled Notebook...
Permutations and Combinations
urn →
A
B
C
D
→ 4 marbles
→ distinguishable marbles
Statistical Thermodynamics
Lec2

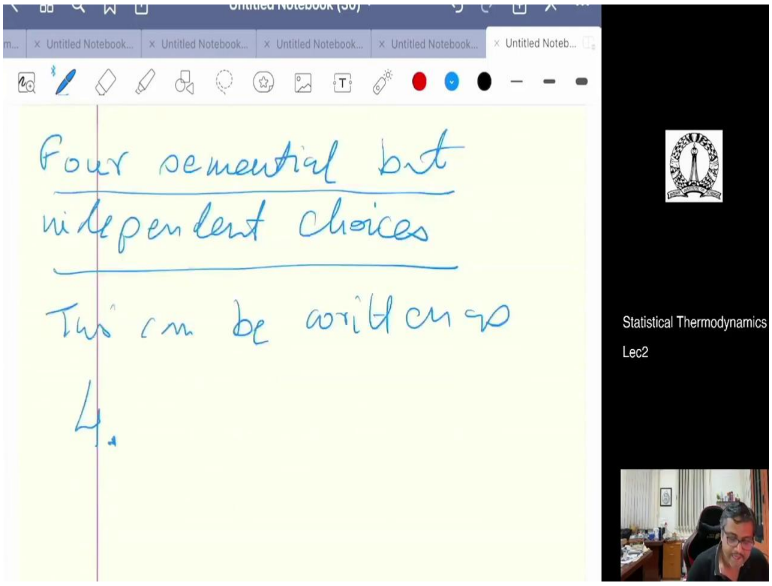

Untitled Notebook...
→ 4 marbles
C
D
→ distinguishable marbles
→ randomly select marbles from the urn without replacement
Statistical Thermodynamics
Lec2

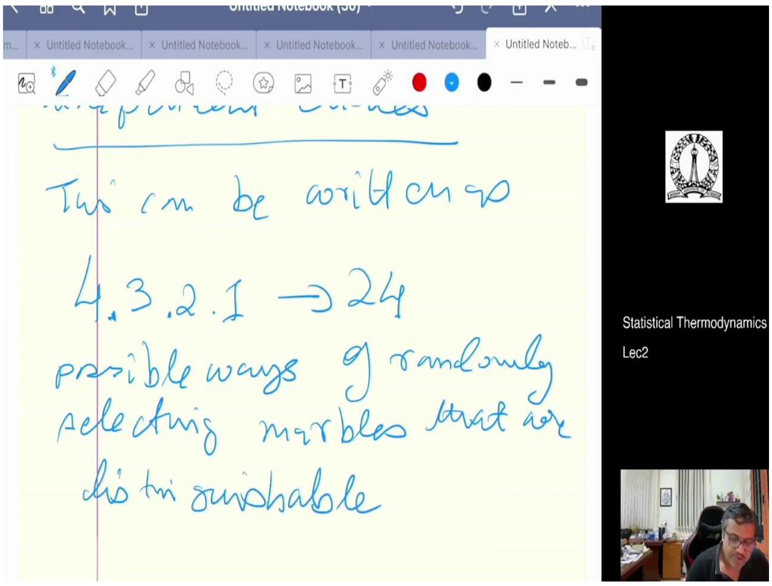

Untitled Notebook...
from the urn without replacement
→ First marble → 4 possibilities
→ second marble — 3 ,,
Statistical Thermodynamics
Lec2

So, this is like rehash of statistics that we are doing. So, permutations and combinations now. So, let us look at those now. Now, let us consider urn which have been say four marbles and which are marked as A, B, C and D. So, this is an urn which consists of four marbles. So, these are four marbles and they are distinguishable marbles. So, our aim is to randomly select marbles from the urn without replacement. So, what is our aim? Our aim is to is to randomly select marbles from urn without replacement.

So, naturally when you do something like that, what you see? You see that, so the first marble obviously you can have 4 possibilities, 4 possibilities, so, first marble we have 4 possibilities, the second marble on the (())(11:39) tab it has got three possibilities because, you know, we are not replacing the marbles and then the third one has 2 the fourth one has a new one.

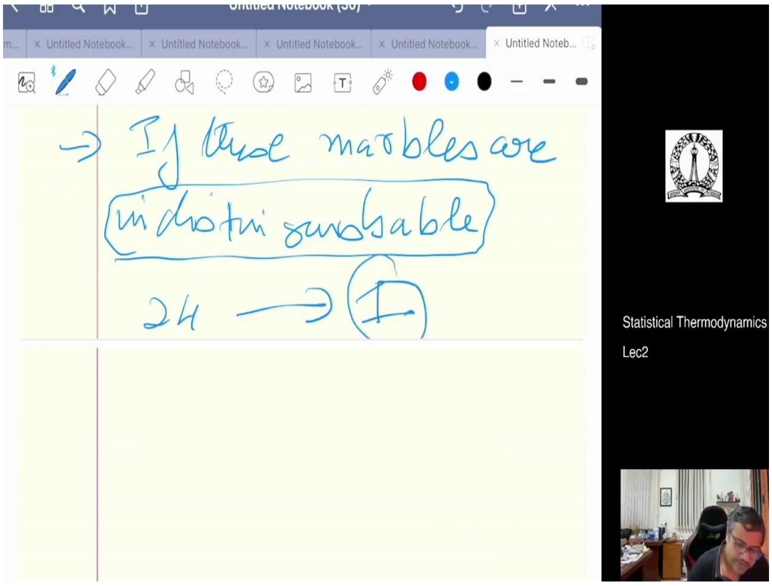
(Refer Slide Time: 11:51)

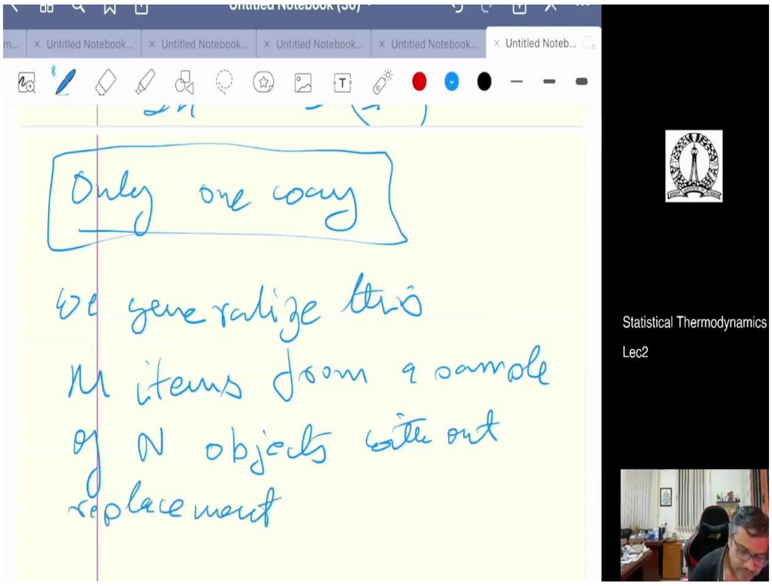




So, these are what we call four or sequential but independent choices, 4 sequential but independent choices they are sequentially... So, this can be written as, this can be written as 4 the first one can be drawn in the case of 4, the next one is 3, third one is 2 and then 1 which gives rise to 24. So, there are 24 possible ways of randomly selecting marbles that are distinguishable.

(Refer Slide Time: 13:22)





If these marbles are indistinguishable... Guess how many ways they can be ranged or they can taken out so, there is only 24 just goes to 1 and there is only one way, it is because these models are now indistinguishable in nature. So, we generalize, now what we can do is that, we can generalize this, our hope is to generalize we generalize. So, it is like to choose M items from sample of N objects without replacement.

(Refer Slide Time: 15:00)

of N objects without replacement

$$P(N, M) = N(N-1)\cdots(N-M+1)$$

permutations $= \dfrac{N!}{(N-M)!}$

$M < N$

Statistical Thermodynamics

Lec2

$P(N, M) = N(N-1)\cdots(N-M+1)$

permutations $= \dfrac{N!}{(N-M)!}$

$M < N$

If $M = N$

$$P(N, M) = N! \qquad 0! = 1$$

Statistical Thermodynamics

Lec2

four sequential but independent choices

This can be written as

$$4.3.2.1 \rightarrow 24$$

possible ways of randomly selecting marbles that are distinguishable

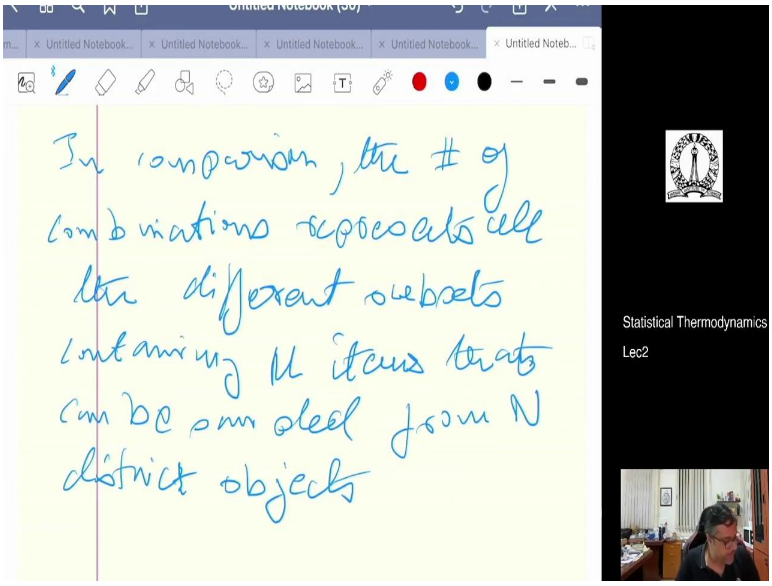
Statistical Thermodynamics

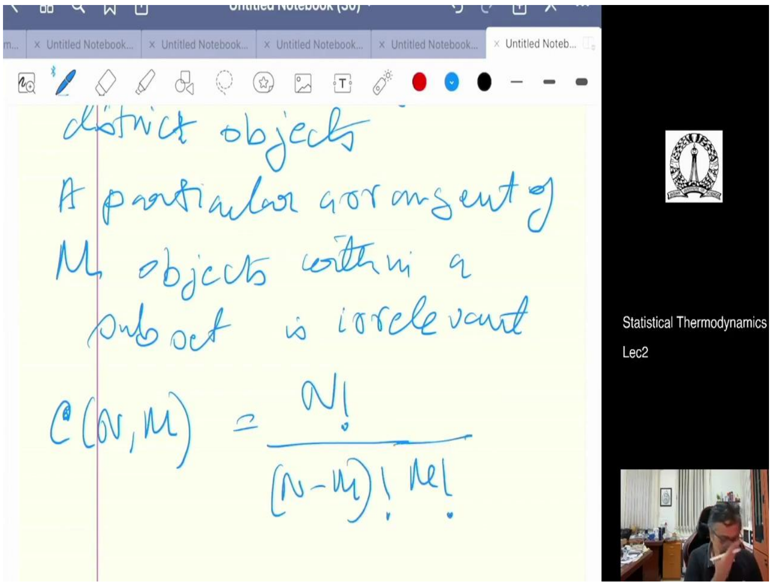
Lec2

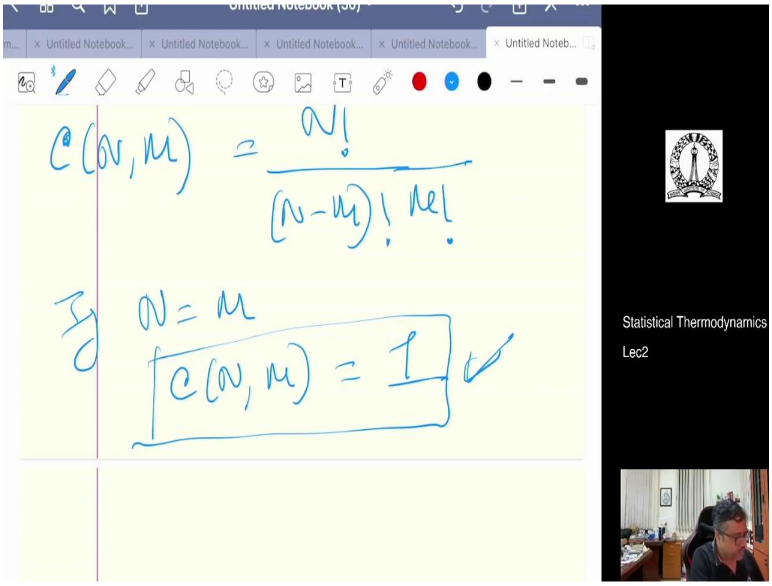
So, our rotations we will write P as a permutation of N objects and M items taken out, so these are the permutations, this is given as N N first you taken out N and then minus 1,... N minus M plus 1. Now, this can be written in a formal way of N factorial divided by N minus M factorial. Whereas, of course, your M is less than N, so, this is a criteria.

Now, if M is equal to N then the P N comma M becomes nothing but N factorial which is exactly what we saw here to look at it. So, this is basically 4 factorial. So, this is N factorial, where if you recall 0 factorial is 1 plus 2 this is... So, you can understand so, this is how this is very simple so, instead of four objects now, we have N objects and you know you are taking out M items of them you can take all the N objects in that case M and N are the same.

(Refer Slide Time: 16:30)





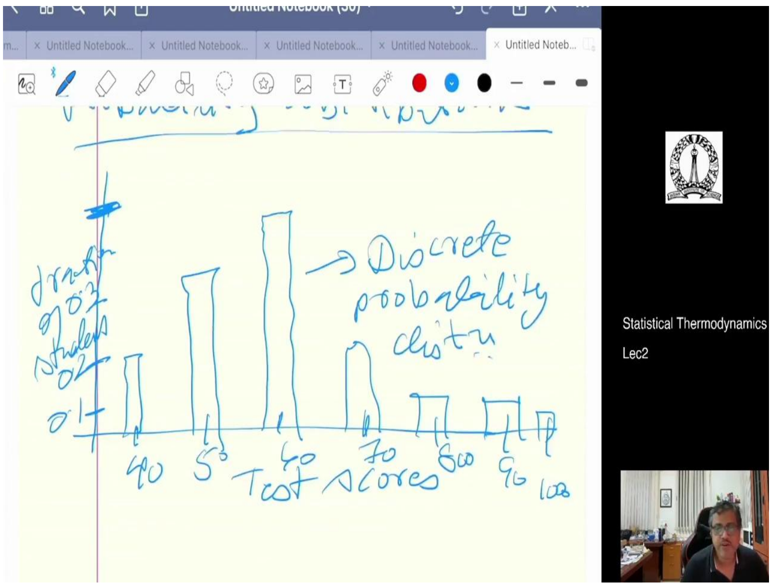


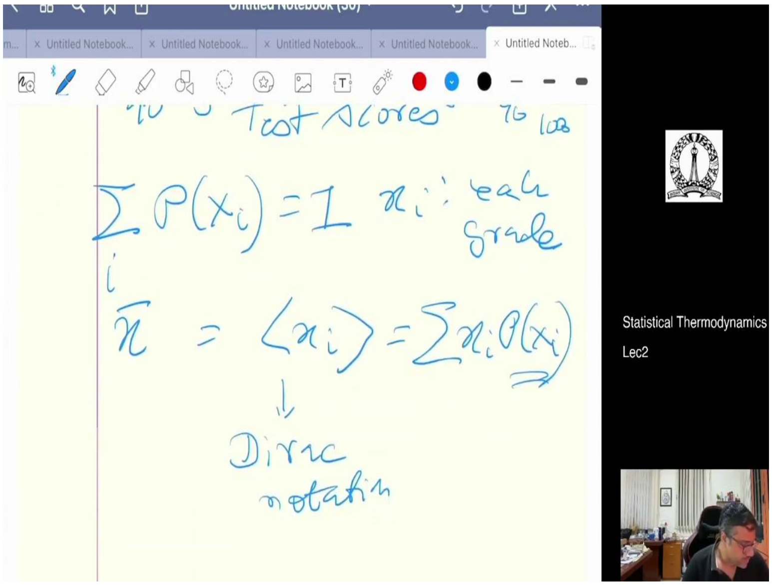
Now, in comparison the number of combinations represents all the different subsets containing M items that can be sampled from N distinct objects. So, either on the particular arrangements of M but it should be remembered here look at it carefully, a particular arrangement of M objects within a subset is irrelevant.

So, basically if you write it as C (N comma M), this is N factorial divided by (N minus M) factorial divided by M factorial, same arrangement you are not taking into consideration here and if N is equal to M then C N comma M becomes equal to 1, which is exactly what we saw in the last one, that we may have four objects which are basically indistinguishable, and you are taking all four of them out one at a time. There is only one way that you can do it. And so, therefore, this is what it is.

So, this is a more generalized representation of what will happen when you actually have this. So, this brings to that you have now can see what a permutation and a combination actually as looks like in this particular case. So, now let us take a step back and let us look at some other few other things, which is called the probability distributions now.

(Refer Slide Time: 19:33)

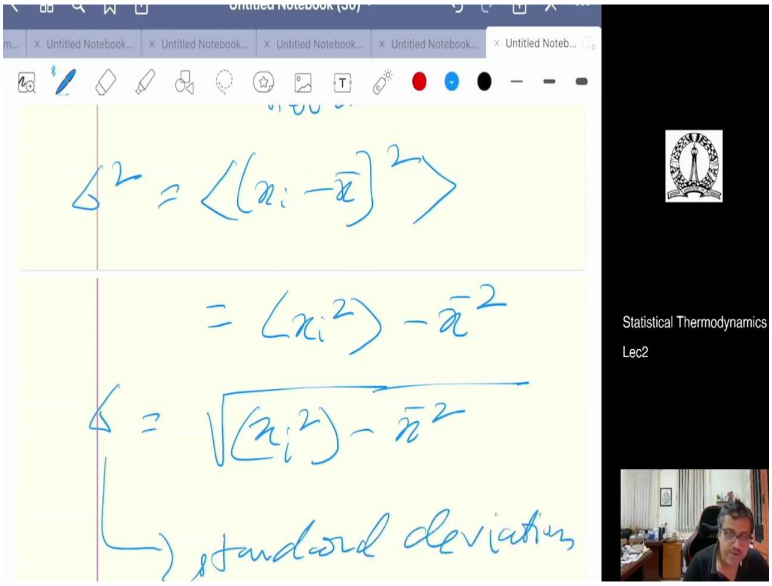


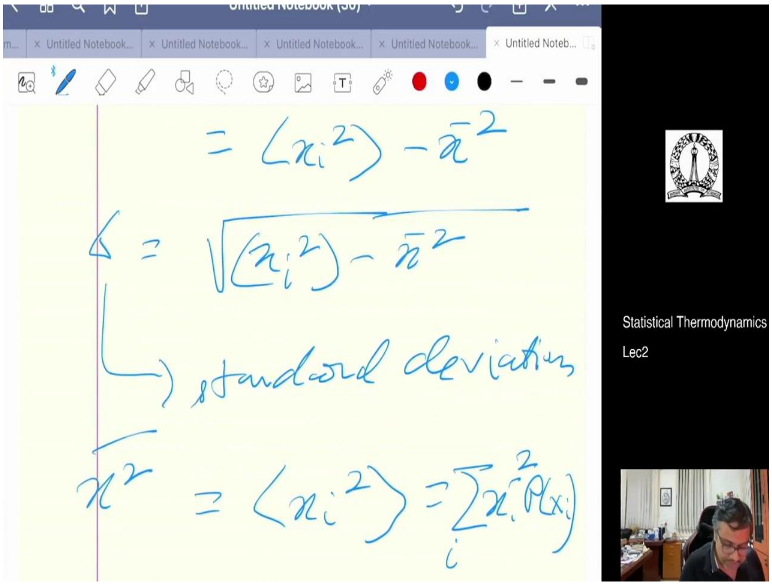


So, probability distributions the rest of the things you can self-study from the book. So, probability distributions is like basically if you have this kind of a chart. So, for example let us look at it… so this is like for example, the test course in a particular exam. So, this is 40, 50, 60, 70, 80, 90 and 100.

So, you have like (())(20:17) thing… something like this. So, and this is the fraction of the students. So, this is like you know you can have 0.1, 0.2, 0.3 etc. So, this is what you call a discrete probability distribution, this is called the district discrete probabilities or discrete probability distribution, this discrete probability distribution means that when you actually know you know sum the probability of individual scores equals to I is equal to 1 that xi is basically each grade. So, when you write x bar which is basically nothing but you know this is how I write this is called direct notation, so, this is not in summation xi and P xi which is basically each grade multiplied by its corresponding frequency in this case the probability.

(Refer Slide Time: 22:07)

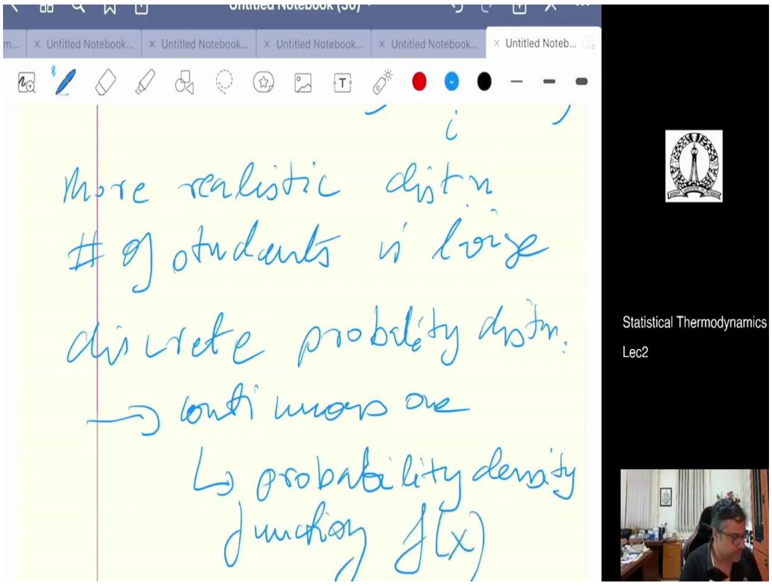


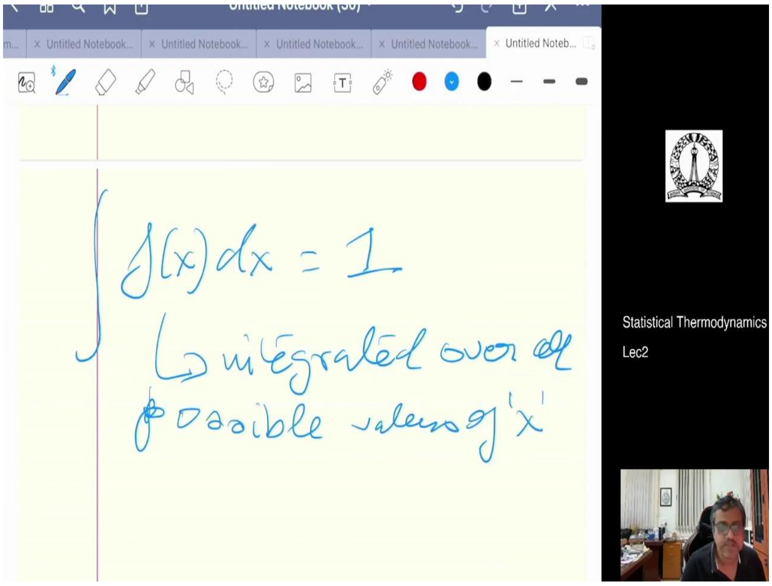


So, similarly, you can get something what we call the variance which is sigma square some of these are known to you so, it is like xi minus x bar square and then the average of that. So, this can be also written as xi square bar minus x bar square and a subset of that which is called the standard deviation essentially is x square and so, this is basically now, this entire thing psi square bar minus x bar square and then you take the whole root of that. So, this is basically what we call a standard deviation and that was the variance...

So, if you have to consider a variance that is basically xi square bar is just the individual terms so, xi square which corresponding to xi which is nothing but a way which, is like if you have done you know weighted so, you in each score has got a particular distribution. So, this is exactly how you compute it. So, this is a percentage so, in this case that is the probability.

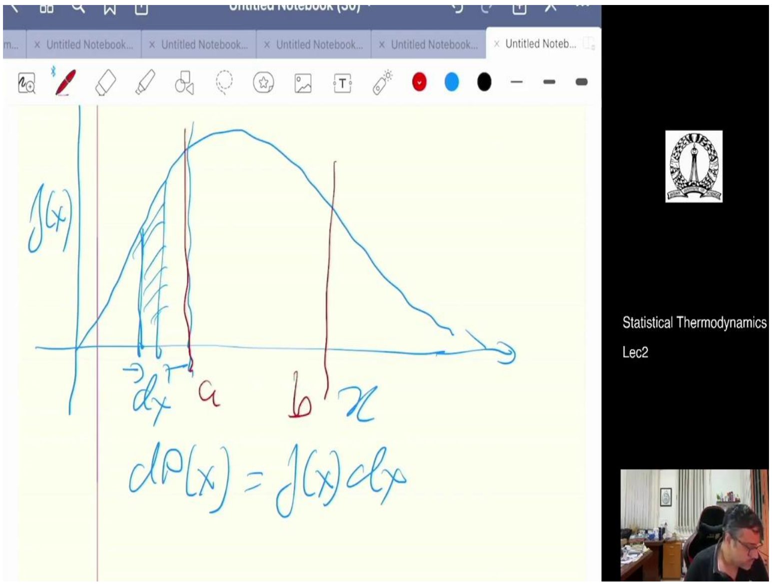
(Refer Slide Time: 24:09)

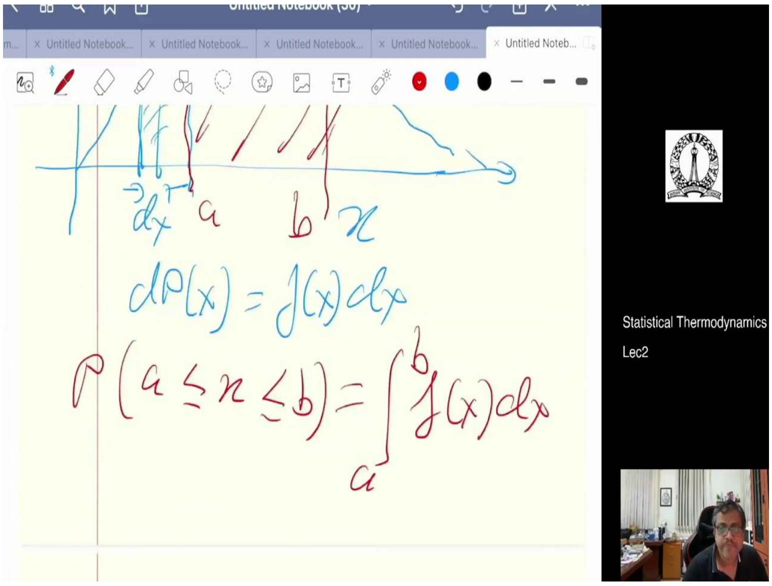




So, if you consider the case that now, if you consider a case in which a more realistic distribution in this realistic distribution, so in this realistic distribution, where the number of students in this case it was rather small on distribution of students is large very large. So, therefore, that is the discrete probability distribution in that case discrete probability distribution in that case approaches, what we call a continuous one, which is also known as the probability density function fx. So, of course, the probability density function has to obey the fact that fx and dx is equal to 1. So, this is integrated over all possible values of X.

(Refer Slide Time: 25:15)

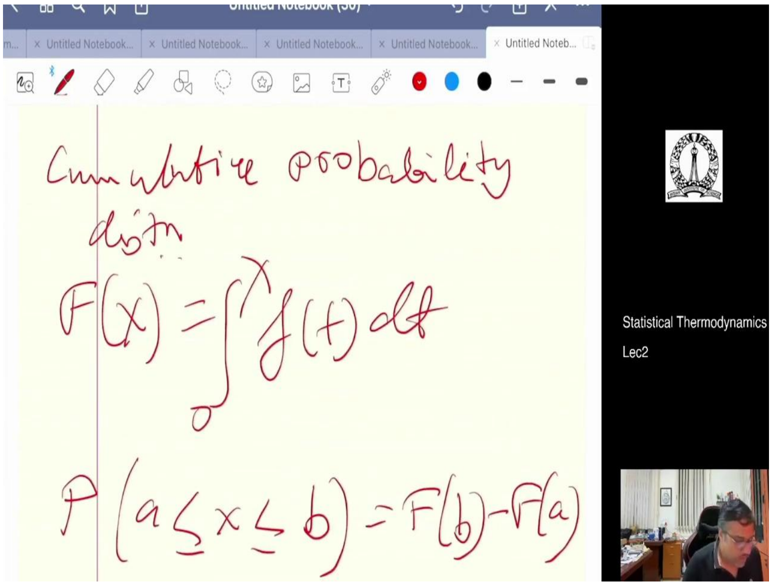


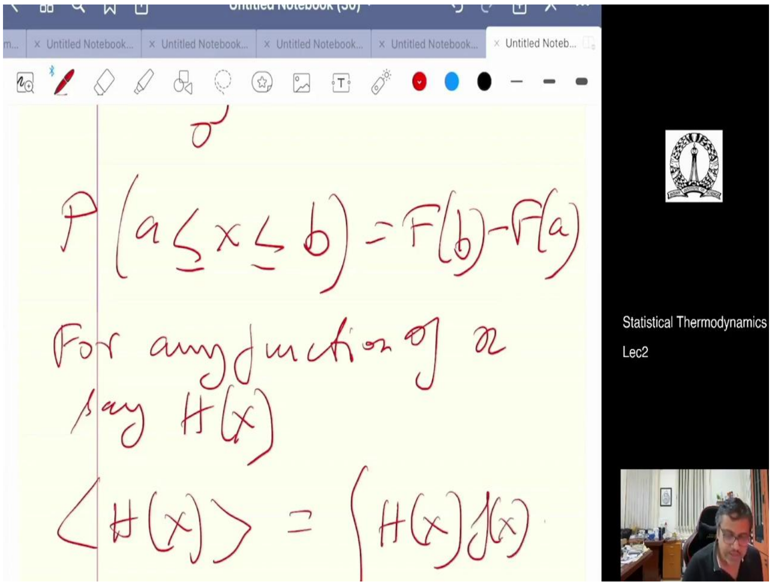


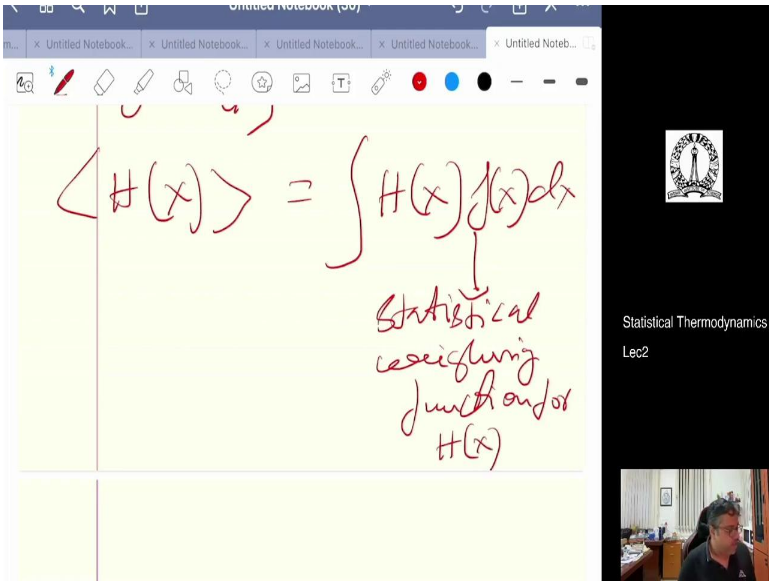
So, if we write it, we just plot it, you will find this fx So, this is something like a graph like this, for example so this is x. So, this is the graph fx and smart (())(25:33) for the convenience. So, this is basically a small segment which is dx, this particular graph. So, if I write dP x that will be fx into dx that is what it is, fx into dx and probable and if I have these markers as you know there is a point a, let us mark it point a and then mark a point b in this graph.

So, the probability of x greater than equal to a less than equal to b is given as integration from a to b fx dx. So, this is a probability that x lies between a and b which is basically this part of the graph. So, that is what it is. So, this is the probability of finding (())(26:41).

(Refer Slide Time: 26:46)

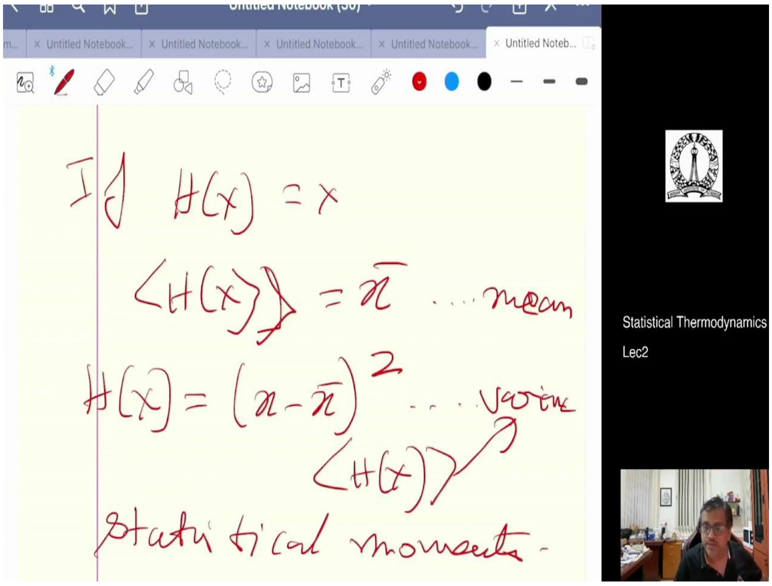






So, there is also something what we call the cumulative probability distribution. So, let us call that FX, FX is nothing but from 0 to X you integrate it is the dummy variable. So, and once you have P a greater than equal to x less than or equal to b you can write it as Fb minus Fa. So, that is that cumulative probability distribution. Now, for this couple of things more to note for any function of x say HX the function of X average of HX is basically HX into fx into dx. So, this is like as we told earlier it is a statistical weighing function for HX.

(Refer Slide Time: 28:22)



Now, if HX equal to X then average of H(X) is equal to x bar which is simple things the first moment which is the mean if HX is x minus x bar square and this gives you the variance. So, these are basically what we call statistical moments. So, in the next class we are going to look at some of the more distributions of such statistical moments and we are going to look at some distributions like the binomial distributions, the Gaussian distribution and a Poisson distribution before we move on to the, to how these distributions are linked with statistical thermodynamics in the first class. So, thank you. Thank you so much.