Optical Methods for Solid and Fluid Mechanics
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Module No # 08
Lecture No # 33

So far we have discussed the origin of refraction it goes back to this wave nature of light and the fact that the frequency of or the velocity of the phase velocity of the light changes as it enters a medium. We define what the refractive index was we all have seen that in some form before and we delved a little deeper into trying to understand why refraction occurs? And why light should bend and, consequently why you see different changes in direction when you go from a denser medium to and error medium and so on.

I will just backtrack little bit so that we have some continuity and then we can take up a discussion of bi-refringence from here.

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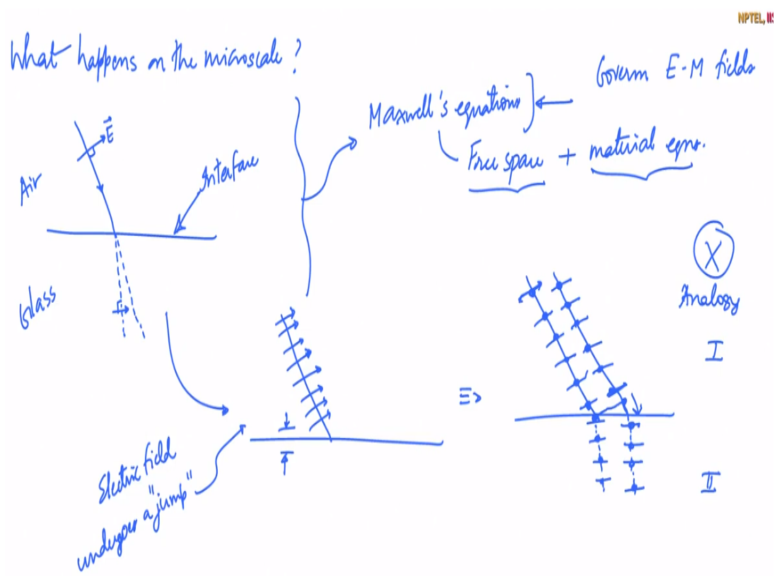

So here is what we were discussing on the micro scale remember we mentioned that there are typically 2 waves that are going on. One is the incident wave, of course which; is your incoming light wave and the moment it enters a medium typically has atoms which have electrons naively speaking and the electric field that accompanies a traveling light wave excites these electrons they oscillate.

And since they are oscillating charges they emit light of their own and the final light that actually is propagating is the sum of these 2. And so we said that, to understand why light should bend we have to look at the total interface wave or the total wave pass the interface pardon me.

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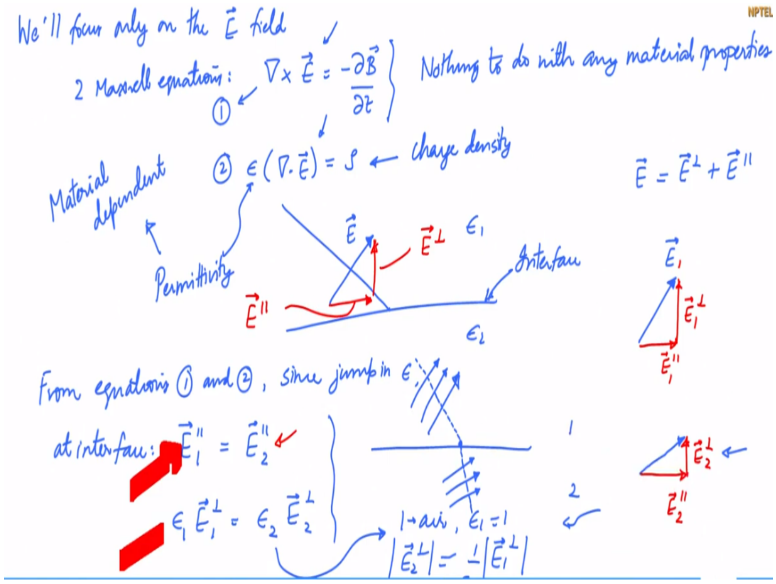

And that is what we did we went back to this Maxwell equation the equations themselves we do not need to remember these equations. But at least from the point of view of understanding where this light bending comes from we can loosely rationalize it using just 2 of these, Maxwell relations involving the electric field. I also told you that the magnetic field was not particularly important because it does analogous stuff and it is always perpendicular to the incoming electric field.

And based on this we did some simple calculations and we found that at the interface you have 2 conditions you have this which is that the component of the electric field parallel to, the interface remains unchanged. And the component of the interface a component of the field that is perpendicular is altered in the ratio of the epsilon being the permittivity of course which is also related to the refractive index.

Now consequently the arrow direction changes as you see this figure here and so this is really the origin of this bending of light as you go partial interface.

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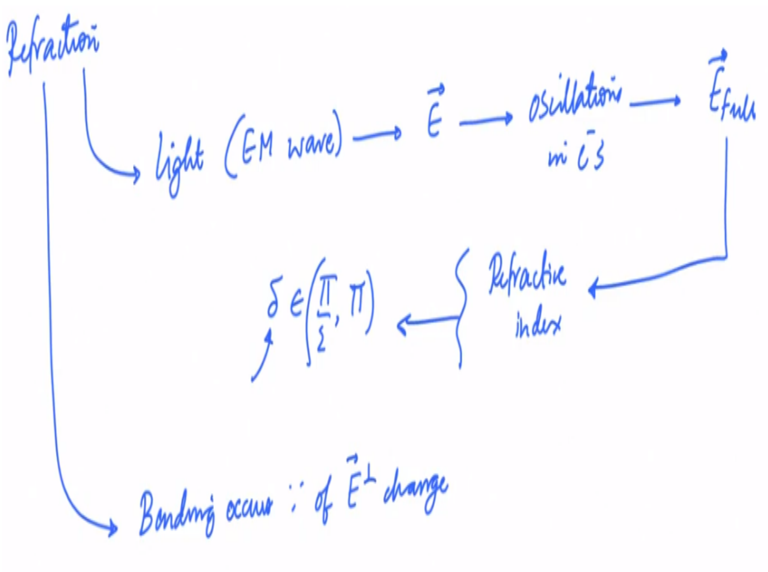

And I think we also briefly mentioned about the fact that you can have bending towards the normal away from the normal these are things that we have seen in a class in a gray optics or a geometric optics class before perhaps in high school. And all of this can be explained of course by just taking the ratio epsilon 2 is to epsilon 1 and then that tells us which way the light ray is going to bend.

So, now we understand you know sort of summarizing we understand why refraction should occur and why bending should occur. And incidentally in the process of discussing this we also figured out that this change can give you absorption it can give you different degrees of absorption. You know you can make materials opaque and so on these are some of the stuff we also discussed in the context of this, bending business all right.

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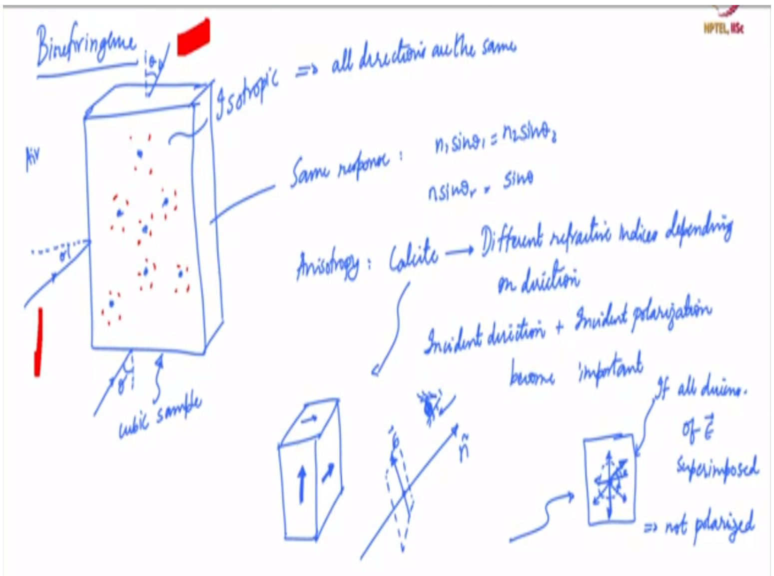

So then we came out to bi-refringence and this is what we will start discussing today this is wearable sort of continue. And I told you that in a typical material in a typical let us say a glass or some polymer of some sort the material is usually isotropic right which means it is the same in all directions. So if light is incident in one of these directions that is shown here this or this or this is going to basically react in the same way right it is going to see the same structure locally.

So it is going to see the same types of atoms and there is going to see the same types of atoms are going to see the same type of electrons near the atoms. And so it is going to result in the same types of bending or deflection. Now so that means that the refractive index of, course is independent of the direction its independent of what direction the electric field is in what direction the propagation is in and so on right so this isotropy that you see in most materials.

Now anisotropy of course comes up when you have some specific orientation of these atoms in a crystal. For example give you the example of calcite and now depending on which direction the light is, coming in with respect to the orientation of the atoms in the structure you are going to have different types of responses right. And so now the direction of the electric field with respect to the specific direction of the specific crystal direction becomes important.

And as we will see today that is really where some of these effects come from and that is where we start our quantification of, bi-refregations in the first place.

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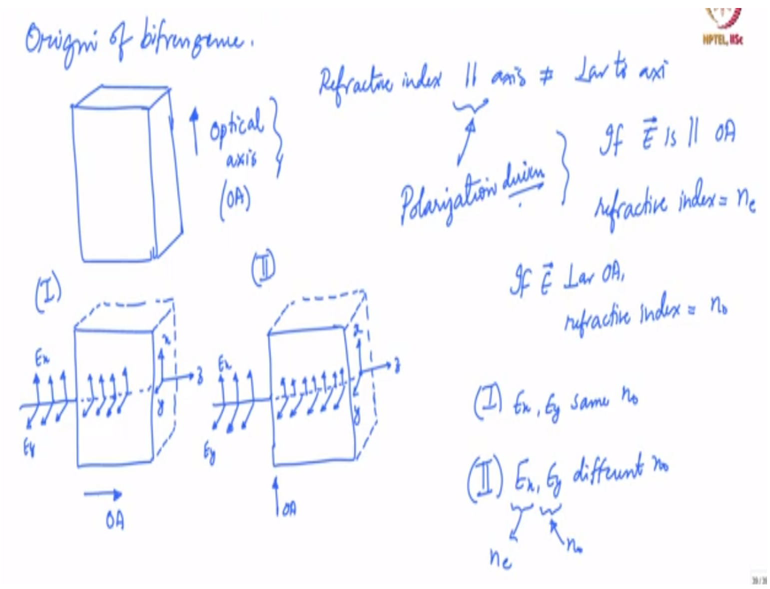

All right so let us take off from here and sort of see and look at the origin of birefringence. Now typically there is a property of many of these anisotropic crystals that distinguishes one particular direction and this particular direction is often called the optical axis or the optical or the optic axis. And usually what will happen is that the refractive index the axis is different from that perpendicular to the axis.

And when I say parallel and perpendicular I am referring to the polarization direction. So if the light polarization is parallel to the optic axis you will see a different refractive index which means that component of the light will travel at a different speed compared to the component that is perpendicular to the, axis to the optical axis. So just to be a little bit more precise let us take 2 different situations so here we have our optical axis like this I am going to call the optical axis as OA.

And in this case in a different case we have the optical axis like this so the materials optical axis is pointing upwards and here it is pointing to the right. The material of course is 3 dimensional so, you can imagine it has a third dimension has thickness it looks like a cube for instance and will ignore that for now. And let us assume that we have light coming in like this so this is our direction of propagation this is the direction of propagation and we will also make access.

So this is a z-axis this is let us say x y comes out like this same axis here x z and y all right. Now I am, going to denote the polarization along the so remember the propagation directions along the x or the polarization the vector electric field vector can either be along x or along y or in some intermediate directions so it has to be the x y plane. So I will draw 2 components this is the e x component this is the e y component.

So everywhere along this way you have these 2 guys in there, going like this and blah and the same thing is coming here. You have x e y and is going like this at every point there is an oscillating electric field it could be this is the component along x the component why in general it is along some direction in the x y plane. So you can resolve it along these 2 axis and those are the values you get.

In the first case the optical axis is in, the direction of propagation so if the polarization is parallel to OA or of the electric field vector is what is reference to as polarization a very revealed vector is parallel to OA. Then the refractive index is let us see n e just a nomenclature subscript e refers to extraordinary but does not matter for now it is just a different value.

And if E bar is perpendicular to o a then the refractive, index is n, o stands for ordinary but again we do not have to worry about the nomenclature right now. So if you look at case one this is case 2 in case 1 the e x component in the e y component have the same refractive index which is n, o because they are both perpendicular to a. So as the material as the light goes through the material the 2 components will still continue to be, together right and they will move along they will move along at a slower speed.

Because the refractive index is not one right in fact it is larger than one so the speed inside the material is lower than the speed in vacuum we have discussed for this refractive index means but they will go together that is the most important thing this is in case 1. Now in case 2 things become a, little bit more interesting now you have an e x which is along the optical axis direction and you have an e y which is perpendicular to the optical axis reaction.

So the e y will go at a certain speed and the e x will go at a different speed so if the e x any initially were together the e x could start going faster. For example than the e y so 1 has refractive index and e and the other experiences, are refractive index n, o so they move at different speeds. Now which one moves faster which will move slower whether any is larger than a no or the other way around does not matter for now it is just the fact that the 2 of them moves separately it move at different speeds.

So what this does is if I look at the let us say I look from this direction now I am going to look from the y direction just, to make my drawing a little bit easier.

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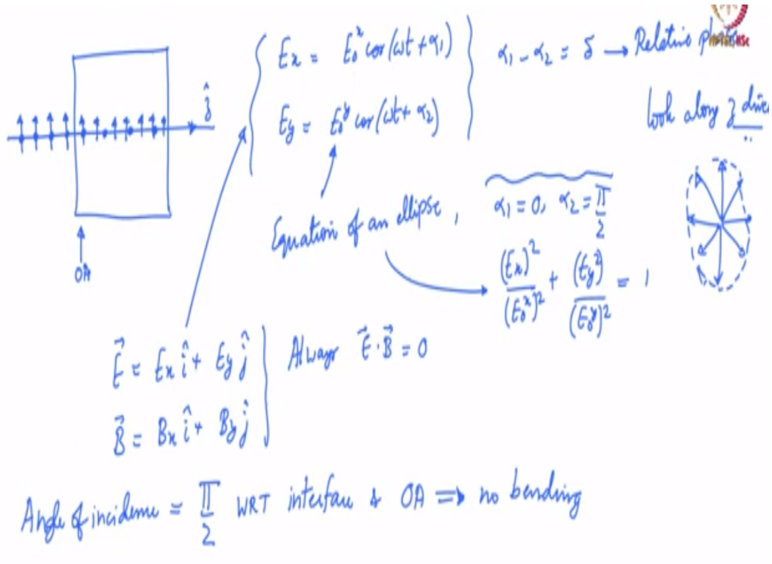

So then the y polarization is going to be an arrow coming out I will denote it by a dot by polarization the exploration will be up and down I will denote it by a the exploration will be up and down I will denote it by a little line. So this is my y polarization this is my x polarization so the optical axis is in this direction then one, of them will lag behind the other. And so you would expect that when the 2 waves emerge from the material they will have a different phase right.

So if for example your e x was some e 0 x Cos of omega t + alpha 1 and e y is some e 0 by Cos of omega t + alpha 2. And then there is a relative phase of alpha 1 - alpha 2 between the 2 waves as they emerge right. Now in general if you, look at these for example e 0 x and e 0 y are you can write this down as the equation of an ellipse right depending on alpha 1 and alpha 2 the relative values the ellipse will be aligned either along the x direction the major axis will be along x or along some direction in the x y plane right.

If let us say for example alpha 1 was 0 and alpha 2 was pi by 2 then you can write down e, x square by e 0 x whole square + e y square will be 1 because this guy will be Cos omega t this will be sine omega t and then u you divide and square and add you get one right. So this is the equation of an ellipse of course for this special combination of alpha 1 alpha 2 the ellipse major axis lies along the x axis or the y axis does not matter depending on the relative values of e 0 x and, e 0 y.

And the minor axis will lie along the other one another of x or y in general for any arbitrary alpha 1 alpha 2 the major axis minor axis will be rotated and will be in some direction in the x y plane but nonetheless this is the equation of an ellipse right. So in general if you look along the ray so if you look along the z direction the electric field is going to be oscillating, like this in an ellipse because that is the equation of the ellipse just something to keep in mind.

And of course remember we are only talking about the electric field there is also a corresponding magnetic field. So your actual electric field is your ex I hat + e by j hat and the magnetic field is correspondingly b x i hat + b y j hat and of course you always have e bar and b bar, perpendicular. So you know you will have a corresponding b x and p y right but just keep that in mind.

Now notice that the angle of incidence here is what is normal right because the direction z is perpendicular to the interface and it is also per particular to OA which is the optical axis so this is pi by 2 with respect to interface and the optical axis. You can see that because it is along, the z direction now the moment this happens there is no bending right. We have seen that already with Snell's law with figured out why that happens and so on and the same logic applies here as well.

The only difference being you have different refractive indices for different polarizations but the direction of incidence is still perpendicular and so you will not see any bending. So for all the cases we will discuss now pending is not really going to happen there is no refraction per se but there is double reflection in that there are 2 refractive indices and we have to keep track of them.

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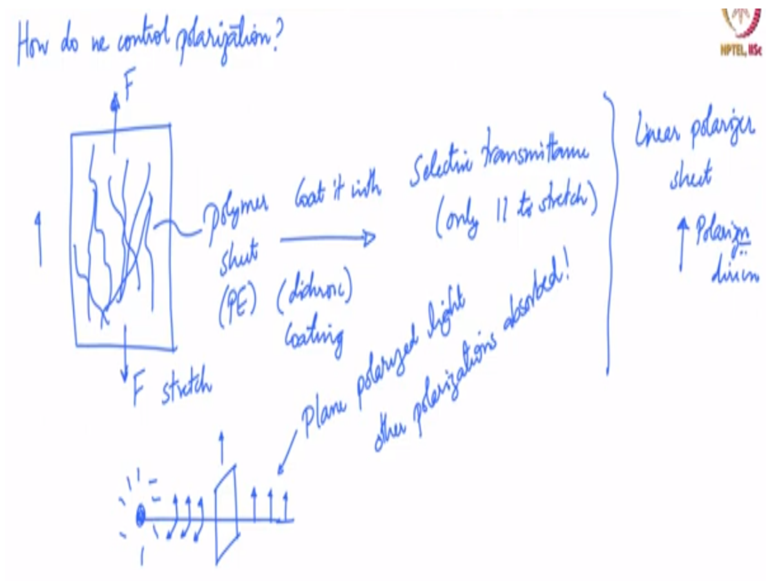

So again something to keep in mind now the next question is perhaps a little bit more natural which is how do we control the polarization? This is important because I mentioned at the, start that there are polarizer sheets that you can use that are circular polarizer linear polarizers and so on. And so we have to just do a small detour to understand how polarization can be controlled.

Now it is quite simple to do in principle although in practice it is a little bit non-trivial you can take a polymer sheet polyethylene typically and then you can stretch it. Usually this is p e, polyethylene and when you stretch a polymer sheet if you have stretched you know plastic bags you know you have probably seen this before. The polymer chains will all tend to align in the stretching direction so this is load stretch.

Once you have stretched it you can coat it with the suitable coating there are several coatings they are called dichroic coatings and what they will do is that, they will allow the sheet to pass light without much absorbance in the polymer chain stretch direction. But they will absorb light in all the other directions so they will allow selective transmittance so only bilateral stretch right.

This is what you call a linear polarizer sheet and basically linear polarizer sheet has one direction called the polarization direction. And for this, case its upward right so the moment you have a light source typically light source Emits light with all types of polarizations and you pass it through a polarizer sheet that has a polarization direction like this. Initially you are going to have e x and d y in various ratios right and you have an ellipse the electric field follows an ellipse.

The moment it passes through a polarizer sheet like this, it is only going to have this polarization the other polarization will get absorbed. So this light with this single polarization is called plane polarized light the other polarizations are absorbed. So this is how you generate plane polarize light so here is what we are going to do so how do we use this?

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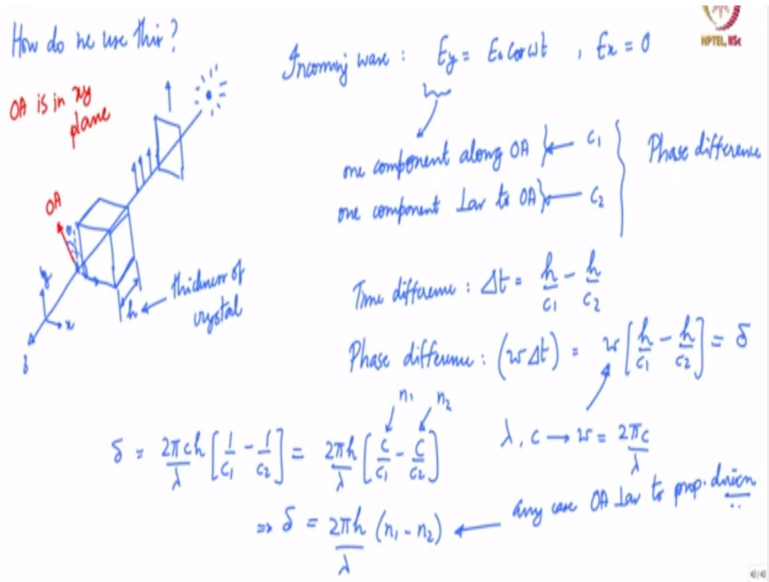

So far we have sort of delved into some of these more fundamental microscopic aspects. But, it is time to put this stuff to some use right and this will be our typical configuration. So you have this is a 3 dimensional drawing so this is our z direction x is up. And well, let us make x here and y here this is easier right so it is going at an oblique angle and I have a polarizer sheet looks like this polarization direction is like this.

So the light that is coming out effectively is, pole wise like this right there is no x component let us say. And now I pass this light through a bi-refrigent crystal again we are not talking about stress optic by refrigerant stress optic law and so on I have not discussed any of that stuff yet. So this stress is not really coming to the picture the elastic part of photo elasticity has not had been touched upon but once you understand what, goes on with the bi-refrigeration crystal it is a simple enough thing to extend it to photographic materials.

So the bi-refrigent crystal let us say the optical axis is like we discussed it is in a plane perpendicular to z so it is in the x y plane. So I will make the optical axis or denoted in red it is in this direction so optical axis is in the x y plane and it is in this direction at some angle. So if I take this like this it makes an angle let us say theta with the y axis and it makes an angle pi by 2 + theta with the x axis.

Now what will happen is our incoming wave or in or incoming wave has e y let us say the c 0 Cos omega t that is our incoming way it is only polarized along the y direction e x is 0 of course. This guy will now have one component along the optical axis, and one component perpendicular 2 optical axis. This will go at a speed let us say c 1 and this guy will move at a speed c 2 because they have 2 different refractive indices.

And depending on how far they propagate which means depending on the thickness of the crystal. So this is the thickness of the crystal depending on this you will see a phase difference and we can actually quantify, that phase difference right and how do we do that? Well the time difference which I am going to call delta t is h by c1 - h by c 2 right. The first wave takes h by c 1 seconds to move through the crystal and the second fellow takes h by c 2 right.

So the time difference between the 2 of them is h by c 1 - h by c2 the phase difference is simply omega times delta t right. Remember right passing, through the cursor does not change its frequency right it is going at the same frequency because it is not losing any energy we are assuming that absorbance is minimal here. And so the frequency remains unchanged so the phase difference between the 2 waves is given by omega times delta t.

So you can write this as omega times h by c 1 - h by c 2 and I will call this phase difference, delta. And depending on the wavelength of the light as it was coming through the medium before it went into the crystal the wavelength lambda and this c the freak the velocity of the light or speed of light in the vacuum or in the air can be related to omega by omega is equal to 2 pi c by lambda right that is a very straightforward relationship.

So if you put this together you will get delta is, 2 pi c by lambda times h 1 by c 1 - 1 by c 2 which is nothing but 2 pi h by lambda c by c 1 - c by c 2. And if you recall our discussion about the refractive index this is refractive index one and this is refractive index 2 because the ratio of the speeds right absolute refractive index. So this is 2 pi by lambda times n 1 – n 2 that is the phase difference.

And this is applicable to any case where the optical axis is perpendicular to the propagation direction any crystal of thickness h you will always see this phase difference of the optical axis is perpendicular to the propagation direction.

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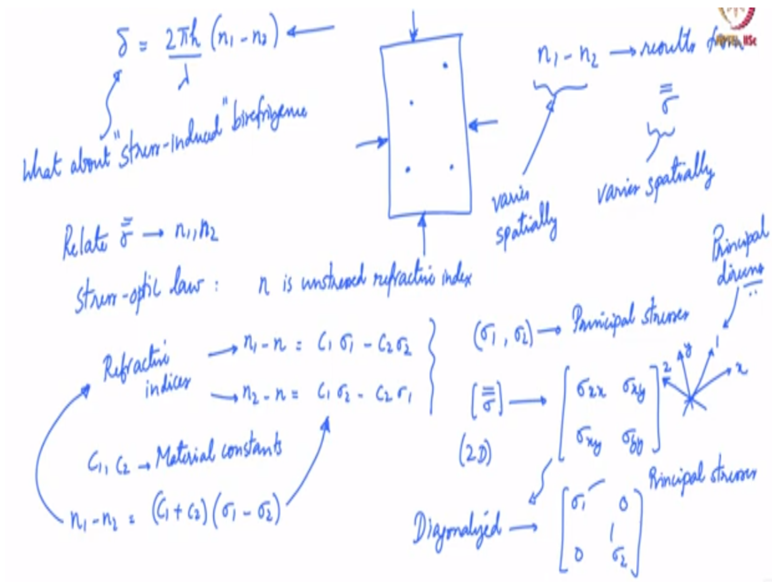

So let me write that down again so this is 2 pi h by lambda into n 1 - n 2. So now the question is what about stress induced by refrigerant so far we have, sort of touched upon this the origin of this birefringence as being structural. So you know this is arrangement of atoms or whatever in the material has this sort of property that the orientation is such that the response of the electrons and so on is different in one direction was the other direction and so on that is a structural origin for this delta right that is a structural version.

But there is this other thing called stress induced by reference and loosely speaking what happens in this case is? If you have polymer if you have a polymer or even with glass particularly pronounced with the inorganic glass. If you apply a load then load of some sort then the local atomic rearrangements or in the case of polymers chain rearrangements are such that the n 1 - n 2 results, from an applied stress so the existence of these 2 refractive indices comes from the applied stress.

Now naturally since you know in general if you have a body and you load it to you know set of forces on its boundary the stress field very specially and so the refractive index will also vary specially and this is a really important thing to sort of recognize. The moment the origin for the, change in refractive index comes from a stress automatically accounting for the possibility that the refractive index changes from one point here to one point here to one point here and so on both n 1 and n 2 will change from point to point.

So the general way to relate the stress to these 2 refractive indices is by something called the stress optic law. And that basically says that if n is the, unstressed refractive index of the material then n 1 - n is c 1 sigma 1 - c 2 sigma 2 and n 2 - n is c 1 sigma 2 - c 2 sigma. I will explain what these terms are in a second n 1 and n 2 of course are the 2 reflective indices c 1 and c 2 are material constants.

And sigma 1 and sigma 2 are called the principal stresses you have probably seen you have probably, had this idea of a principles as discussed before. But just so that we are on the same page if you take any arbitrary stress tensor like this you can let us say it has this is a 2 d problem so we are talking only about 2 d for now we will talk about 3 d a little later. It has in general 3 independent components with respect to some coordinates or some.

So let us say x is like this and y is like, this the stress tensor can be diagonalized of course because it is symmetric and any symmetric tensor can be diagonalized. And if it is diagnosed is brought into this form and these sigma 1 and sigma 2 are the principal stresses. And the direction in which this diagonalization happens so basically the orientation along which the stress sensor becomes like this these are called the principal, directions right.

So the values of the principal stresses go into the stress optical now it is a very simple matter to obtain the difference n 1 - n 2 from here. You just subtract these equations you will get c 1+ c 2 times sigma 1 - sigma 2 from this.

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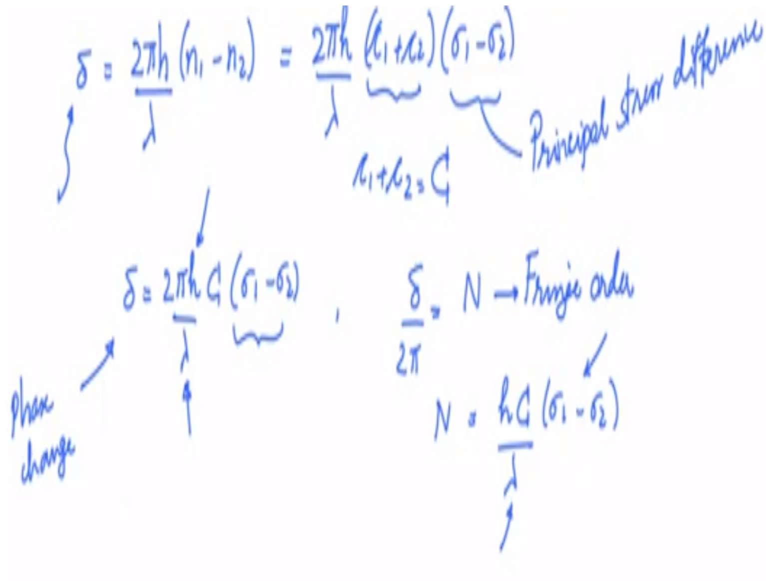

So now we can go back and write our expression for the phase difference and that becomes proportional to the difference, between the principal stresses. So this is called the principal stress difference it is not particularly deep terminology but whatever. And so our phase difference between the 2 waves is related to the difference in the principal stresses. We will combine this c 1 + 8 remember c 1 and c 2 are constants they are not the same c 1 c 2 we use for the wave speed so I will, probably make it curly like this.

And we will call this c 1 + c 2 as this c right this has nothing to do with the wave speed it is just a material constant. So this is really the first main relationship that we have in photo-electricity which relates the phase change of the 2 waves the interaction between them to the sample thickness the wavelength and the principle stresses the, difference in the principles. There is some other nomenclature you will see sometimes delta by 2 pi is usually denoted as n is called the fringe order.

So if you see that term this is what it means we will discuss what this means you know subsequently but right now this is delta by 2 pi. So n is h c by lambda sigma 1 - sigma 2 and you can imagine n changes with hatch engines with c, of course for a given material c is constant for a given thickness h is constant. So n really changes with this and with this really that is what we are interested in all right.

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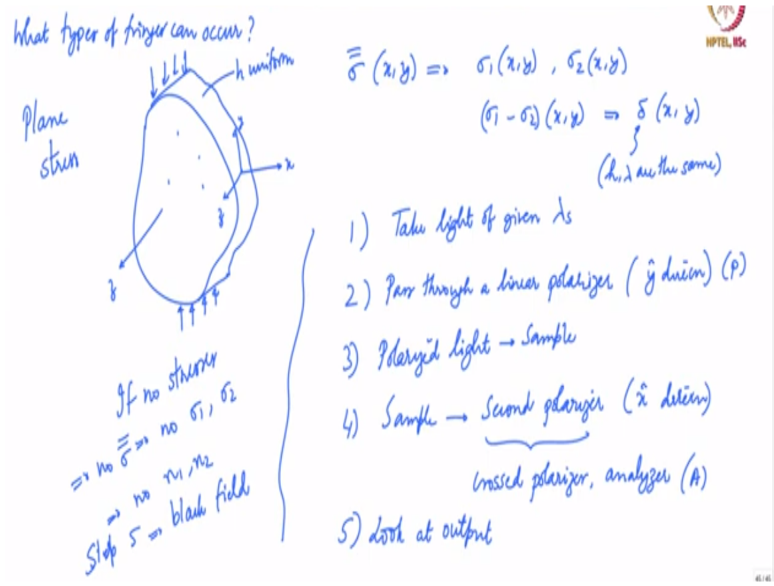

So the next question that we want to get to is what types of fringes can occur. So let us say you have a body like this it is 2 dimensional for now we will make our life simple and make it 2 dimensional. So it, looks like this and then it is loaded 2 dimensionally right so it is a plane stress problem for now. So we will assume that it is plane stress and as a result of this at every point so let us give some directions here.

So this is x this is y so z is out of you know plane of the body right the body is prismatic like I have shown here. So at every point the stress tensor is a function of x, y it is not a function of z because again plane stress. And this means that the principal stresses at every point are varying right which means sigma 1 -sigma 2 is also varying from point to point which means delta is also varying from point to point.

So everywhere in the material depending on the loading you will see a different value for the face for the phase change despite the fact, that h and lambda are the same remember. We are not changing that right that is a property of the body it is thick assuming that it is uniform thickness h and we are assuming that the light wavelength does not change of course. So lambda is the same and despite that you have this field of delta of x y and that is really what we are measuring in some sense in photo-elasticity.

So here is, a sequence of things we will do right and I will start off with this in the next session the first thing we will do is we will take light of a given lambda or a given set of lambdas. So it could be a spectrum like it could be white light it could be yellow light, orange light, blue light, green light does not matter we will pass the light through a linear polarizer.

Then the polarized light goes, through the sample the stressed sample remember the sample is under loading.

If sample is not loaded then the whole question of n1 and 2 does not turn up because there are no stresses inside. So we pass the light through the sample and then we use so let us say if this polarizer is along the y hat direction that is the polarization direction. Once it comes out of a sample so sample we pass it through, another polarizer second polarizer which is crossed with the first one which is called a crossed polarizer or a this is sometimes called as a crossed polarizer.

Or you know the photos literature is sometimes called an analyzer will usually call it a, this flow will usually call p and then we look at it. If there were no stresses if there was no birefringence even though the material is, optically transparent the first step will give you some white light it will pass through the material. I t will pass through the polarizer the second step will give you linearly polarized right.

The third step will pass that polarized light without any change through the sample and the fourth step will completely cut it out right because it is perpendicular to the incident light. So if no stresses, implies no sigma in place no sigma 1 sigma 2 implies no n 1, n 2 then step 5 will give you a black field or dark field you will not see anything because a and p are perpendicular to each other.

So what we will do in the next session is we will start with this sequence of steps five steps and then we will go one by one through each of them. And then we will see if stress is represent, what they should manifest themselves at and we will see that there are 2 types of fringes that result from the interaction of the light with this sample that contains sigma 1 sigma 2 is called isochromatics and iso-clinics. And we will try to understand how to break down their behavior properties and so on so we will continue with that from the next session.