Optical Methods for Solid and Fluid Mechanics
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Module No # 08
Lecture No # 32

So we have been discussing some microscopic ideas of what happens when light interacts with materials? Specifically we looked at situation where an incoming light ray light wave traveling along a certain ray direction excites electrons in a material electrons that are bound to the atoms electrons that are free and so on does not matter. Those distinctions do not matter at this point it is just, a fairly simplified conceptual picture that we are interested in.

And based on that we realize that there are 2 contributions to the actual field that you look at when light passes through a material when any electromagnetic wave passes through a material. The first of course is the incident wave itself the one that is coming from the source and the second one is the total contribution from, the oscillating electrons or from the excited electrons that are moving around in response to the incident field.

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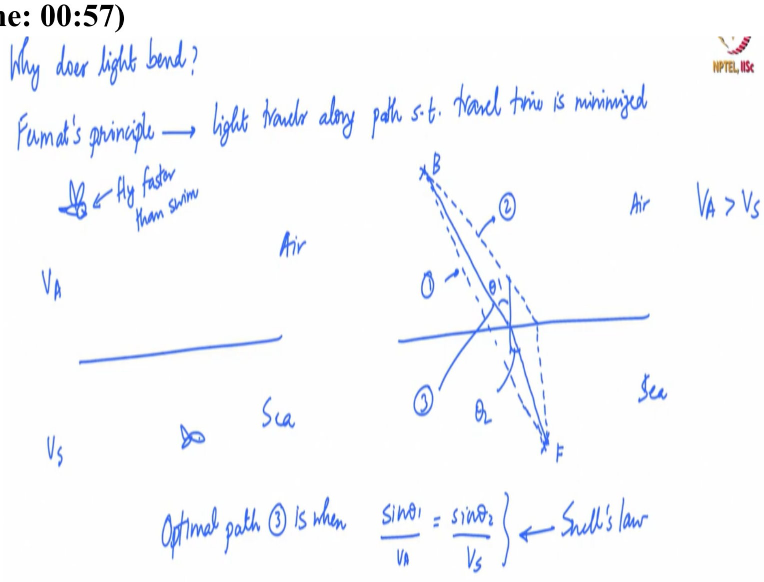

Now if you add them up and look at the final field we saw that there were 2 contributions we call this the E full which had these 2 contributions E bar that is the coming in incident original incident wave and the E bar emit which is the sum of all the emitted waves from each of the, electrons in the material. Now the final if E full we said had a phase difference with E if you remember and we call this delta.

And we said that the value of delta determines how the material responds to a particular light source a particular type of incoming electromagnetic wave. If the delta was pi by 2 then the wave moves slower the superimposed wave compressing of those 2 components is slower, than the incident wave. This is what happens typically in a transparent material like glass or transparent polymers and things like that and it leads to Snell's law as we will see very shortly and you get the usual definition of the refractive index and so on.

The second case was when delta was pi then you had complete absorption so the emitted field was exactly out of phase with the incident field. And so the total field was 0 which means that you would not see any effective field coming out from the piece of material so it is completely opaque and we also you know in passing saw that you can get amplification which what happens in the case of a laser when the delta is closer to pi.

And usually we also mentioned this at the very end the situation somewhere in between case 1 and, case 2. So you have one transmitting component and you have one absorbed component and for most practical cases for photo elasticity we will assume that the delta is close to pi by 2 we will not worry about absorption. It turns out for x-ray tomography will worry about both. So the same ideas will hold through when we discuss x-rays and remember x is also our electromagnetic waves it is just, that the frequency is very different from optical light which is in the visible spectrum.

So then we went on to this question of why light bends going back to our original question and I mentioned that there is something called formats principle you have probably again heard of this before. And it says that light travels along a path such that the travel time is minimized. So we look at some of, these consequences today and then we will try and explain from microscopic picture.

Where this bending comes from and why different wavelengths will bend differently when you have composite light like white light and consequently why you see fringes right that is something we will discuss and this will naturally lead us to the idea of what is called double refraction or birefringence which we, will discuss in the next session. So going back to Fermat's principle this is a best illustrated using a very simple example.

So let us say you have you have land and you have c and let us say you have a bird flying like this and it wants to catch a catch a fish that is/has swimming right or that is stationary let us say it is a stationary fish. Now let us assume that the bird is like a duck, and it can swim as well as fly so this bird can give it a beak and this bird can fly faster than it can swim typically that is how ducks are I suppose.

And so how can the bird get to the fish and presumably the duck wants to eat the fish right we will assume that; as well how can, it get to this in the shortest time. So it can travel at a speed v l in land or sorry in the air is not land the, apologies the bird is flying. But it is not running it can travel consequently let us call this v a in air and v s in water or c does not matter. Now there are a few strategies you can think of if the bird is extremely hungry and wants to eat the fish in the shortest amount of time wants to minimize its travel time it can do a few things.

The simplest perhaps the most naive thing the bird can do is, let us call it point b and point f b for bird and f for fish I suppose. The most obvious thing is it goes straight over here right now you might say this is a possible solution but the distance it travels in the c is actually larger than it can get away with because it is speed in the c is lesser. So this it turns out is not the most ideal route to take if it wants to minimize its travel, time.

Another option of course if you think about it is to travel as far as possible in the air because the velocity is highest in the air and then travel as less as possible in the sea. Because the velocity the c is lesser so we are assuming that v a is greater than v s so this is one option this option which we will call 2 is certainly better than option 1. Because the distance you cover in, the air is larger so the time you take equivalently is smaller.

But it is still not the optimal solution right and so the optimal solution is if it takes a path that is inclined at an angle theta 1 and an angle theta 2. And the optimal path which I am going to call 3 is when the sine theta 1 by v r is sine theta 2 by v c. If I multiplied and divided this by some constant let us say speed, of burden vacuum or something then I would get the expression that corresponds to Snell's law.

So it really tells you that the light that is traveling through these media is actually going in such a way that it minimizes its travel time right. So it is consistent with format principle so now the next question is why should this happen why should it minimize it is travel time and the answer to, that is fairly deep and it is not something that we can get to.

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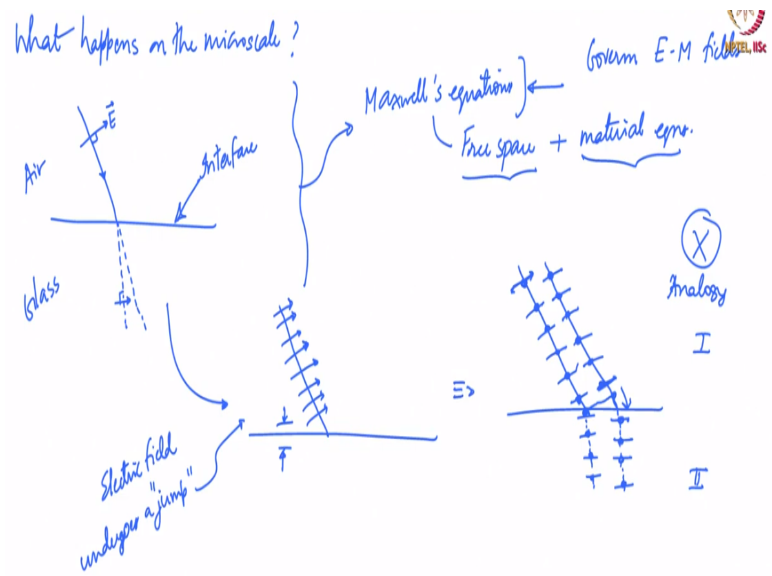

In fact I am pretty sure I do not know the entire answer myself but let us ask the equivalent question or somewhat equivalent question of what happens on the micro scale? Now remember the light that is coming in the incident ray is actually an electromagnetic wave. So there is an e bar that comes like this and there is a, corresponding e bar perpendicular to the direction in which it travels in the glass. So this is again perpendicular to this is again perpendicular to this.

So the E bar remember is always perpendicular to the direction of propagation so you cannot have the same E bar in both cases they cannot be parallel to each other right. So that is something you want to keep in mind now what is, actually happening when the light comes and interacts so this is the interface of course. And for simplicity again we will just take air or vacuum analogous and let us say glass.

The full description of this wave is obtained from a set of partial differential equations called Maxwell's equations and in order to explain why this bending occurs you need to look at Maxwell's equations we, will do that in a second. Before that I just mentioned one common explanation is used certainly an explanation I have seen before but it is not correct and just to sort of myth bust we look at that explanation which is commonly given.

So the usual thing that people will usually say is you have light coming like this these are the electric field vectors along the light travel along the ray direction, all the more perpendicular parallel to each other let us say it is polarized. So you know everything is the same direction for now we will assume that. And the usual argument that is given is that this is analog is going to be thought of as analogous as soldiers marching right.

So if you take 2 such rays and then the soldiers are going like this so this is the analogy they can march faster in the first medium compared to how far they can march in the second medium right. So then the usual explanation is that they come like this and then this guy slows down so his speed reduces. So he does this and then this guy also slows down and eventually this bends right this is the explanation I have seen before.

And if you think a little bit it does not really make sense because even if someone, were to slow down there is no reason for them to bend unless this person is going to bend the entire line with them as they slow down right and this is not something that usually happens. So this analogy is not correct what actually happens is that the electric field undergoes a jump. And remember the electric field again because the electrons in the material are all oscillating and they are generating their own field and so on.

You suddenly see a jump from this side of the interface to this side of the interface and this precisely this jump that is responsible for the bending. Now in order to quantify this jump you have to look at Maxwell's equations and Maxwell's equations basically govern electromagnetic fields. There are 2 variants of Maxwell's equations that are called, the free space equations and then they are what are called the material equations.

We will deal with this not with this applies only to free space with isolated charges with materials Maxwell's equations are usually averaged over some small volume. And then you generate the material equations it does not matter for now it does not matter which you know what the distinction is between them but it, helps to know that there are 2 versions.

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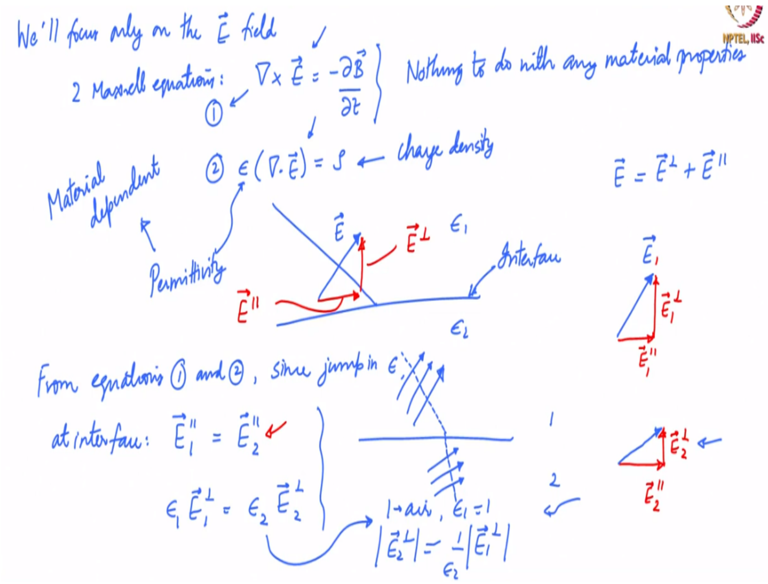

So we will focus only on the E bar field remember I said the b bar is always perpendicular so whatever the E does the B will sort of mimic it in a perpendicular direction so that is all we have to worry about. And if you do this there are only 2 Maxwell equations that are of interest. The first says that the curl of the electric field is, proportional to the time derivative of the magnetic field.

There are some factors of c and so on and the units are you know depends on what system you use Gaussian units or SI units and so on I want to talk about all that just qualitative or semi quantitatively we try to understand what is going on. So this is the first one notice this is this has nothing to do with any material properties here, so this applies equally to all materials. The second Maxwell equation which you have probably seen before in the form of what is called Gauss's law is this is the charge density on the right hand side it does not matter what that is for now for our particular application and this is called the permittivity.

And the permittivity of course is a material parameter so it depends on, which material you are talking about and this charge density is like a free charge density for our cases zero because we do not have any free charges floating around. So these are the only 2 Maxwell equations we have to worry about now if you take the interface and you look at the incoming wave like this and you take the electric field vector.

I am going to draw it bigger just to illustrate, what I am talking about you can resolve this vector into 2 components you can resolve it parallel to the interface. So this I am going to call E bar parallel and you can dissolve it perpendicular to the interface this is called E bar perpendicular. So you have this is the original labor that is coming along at every point in the light ray and you can split this as E bar parallel and E bar, perpendicular parallel perpendicular.

Now from this let me call this equation 1 and 2 so from equations 1 and 2 you can show that if the magnetic field is continuous there is a since there is a jump in epsilon. Remember there is epsilon 1 and epsilon 2 for r will be 1 and epsilon for the other material. The electric field which obeys 1 and 2 and also has this jump in epsilon is subject to, these 2 conditions at the interface.

The first is that the parallel component on the one side is equal to the parallel component on the 2 side. And the second is that an epsilon time the perpendicular component of the one side is epsilon 2 times the perpendicular component of the 2 side. I am not going to derive these because you can apply the divergence theorem from here to this equation, take a small pill box and then you can derive this you can apply the stokes theorem to this and get this and so on.

So we will leave out the derivation but nonetheless this is what it boils down to so what this means is that the as the light ray enters the medium like this the component that is parallel so this component remains unchanged. So if I wanted to find out the vector here into the and let us say my incident vector is like this is the incident E bar this is E bar perpendicular and this is a bar parallel because E bar parallel remains unchanged.

So this will be in one the E bar parallel 2 is the same because of this equation and the E bar perpendicular into becomes smaller. The reason is that the equation that we have written down here if one is let us say add then epsilon 1, is 1 then the E bar 2 perpendicular is 1 by epsilon 2 E bar perpendicular 1. And if I take a magnitude on both sides then epsilon 2 is greater than one and so f E bar 2 is less than E bar 1 perpendicular so the vector in the perpendicular direction becomes shorter.

So the resultant is actually something that looks like this so the incident is all like this let me just redraw this so that it is the same orientation as the vector I have drawn here. So this vector is like this is the incident the transmitted guy is parallel to this so it is going to look like this. And not just that every vector along the light ray is going to look like this and every vector along the light ray here is going to look like this.

So the vector the light ray which is perpendicular to the electric field is, going to be like this so this is the net result of the 2 equations that you see written down over here.

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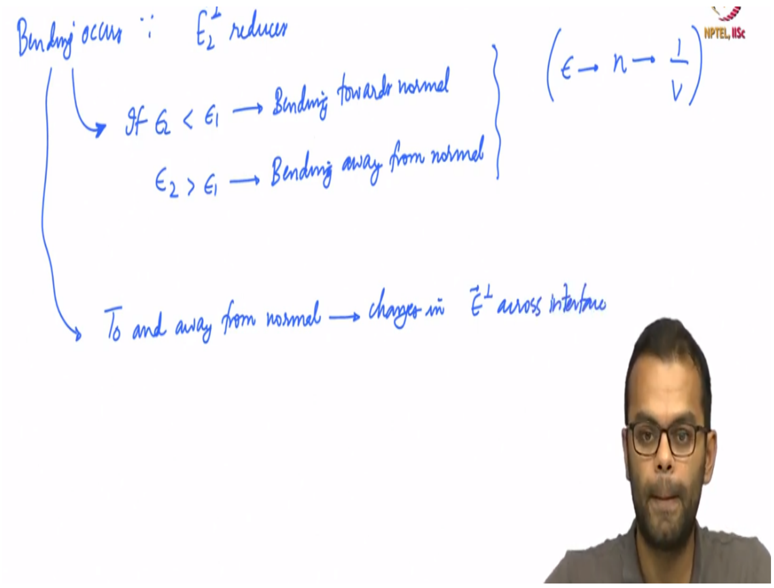

So the bending of this light is only because of this change in E 2 bar now notice also from here you can also derive the fact that if epsilon 2 is less than epsilon 1. Then you have bending towards the normal and if epsilon 1 is greater than or sorry if epsilon 2 is greater than, epsilon 1 which is the case we just discussed then you have bending away from the normal. So all of this is a consequence of the same thing right and the other important thing of course is epsilon is related to the n the refractive index.

So larger n means larger epsilon and vice versa and this is related to 1 by v so larger v smaller epsilon smaller v larger epsilon and the same discussion we had, before comes through. So the bending 2 and away from the normal occurs because of changes in E bar pub across the interface ok that is just something you want to remember on a microscopic scale this is exactly what is happening.

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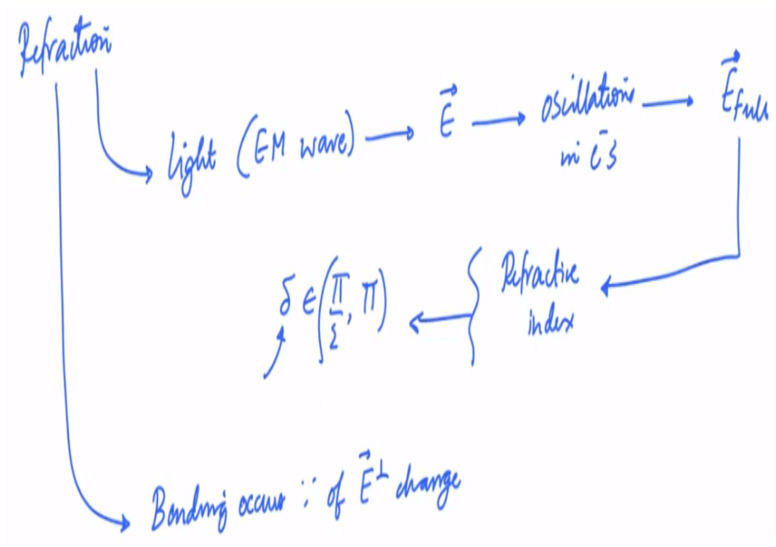

So now this tells us if we summarize what we have seen so far so refraction basically comes because you have light which is an EM wave of course. So it has, an electric field the electric field causes oscillations in electrons. Again there are other sources other means by which an incident light can interact with atoms there is inelastic scattering and so on which we will not discuss about all that.

But just for our practical purposes we will keep this fairly simple and perhaps a little bit oversimplified but nonetheless. It results in oscillations in, the electrons so you get an epsilon full which is the superimposition of the incident one and the resulting field because of these oscillating electrons. And the E bar full gives you the origin of the refractive index and that come if you remember through delta which usually is between pi by 2 and pi this is the phase difference between the 2 components.

And it being between these 2 values, tells you how much is transmitted how this is affected and so on. The other aspect we also saw is that this bending occurs because of this change in the perpendicular component of E bar and this explained why something went towards the normal awareness on so. Now we understand where refraction comes from so let us now look at double diffraction we know single refraction has proven double, reflection or what is called birefringence.

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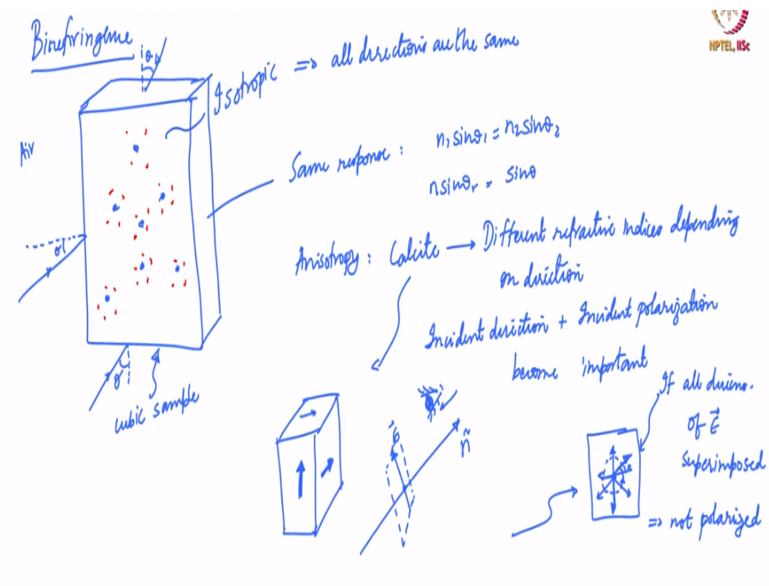

So far when we talked about this material piece or sample I said that you have these if you remember a discussion from last the last session you have these electrons around these atoms and so on and then they are oscillating etc. But we inherently made the assumption that this material is isotropic. And isotropic of course means that all directions are, the same which means if the light were incident on this phase let us say you had a cubic sample.

If the light were incident on this face not normally maybe I will change the light will incident on this face like this at some angle theta. Or if the light were incident on this face at the same angle theta or if an incident on the front face or the back face or any of the side faces you would get the, same response which means you will get the same n 1 sin theta 1 = n 2 sin theta 2. If you assume that this is add then you get n sine theta r is sine theta i which is theta here right so I will just call it theta nothing matters right the direction does not matter.

But this will change if you have anisotropy in the material so very good example of this is this crystal called calcite. So if you ever, manage to get your hands on a calcite crystal you will see that it shows different reflective indices depending on the direction. So for example the crystal of calcite will have one particular direction it could does not have to coincide with this could be this direction.

For example in which light coming in or you know polarization in this direction or electric field this direction will have a, different retardation versus electric field in this direction already field in this direction. The moments you have this now suddenly the incident direction and remember the incident polarization become important. Direction we can understand because you know it is Snail's law has sine theta and so on so the theta matters but the incident polarization also matters.

Because so far we have been, talking about the electric field remembers this is the e this is the direction of propagation. The electric field does not have to be in a fixed direction the only requirement is that the electric will be perpendicular to the direction of propagation if I call this n hat let us say direction of propagation. The electric field can be in any plane in any direction on the plane perpendicular to n, hat.

So if I look from here at this plane the electric field in principle could go in any direction with respect to let us say some horizontal direction right it can go this way it can be this way it can be this way and it can change these directions as a function of time. So it could be a superimposition of one field like this and so on the, moment that happens if you have all directions of E bar superimposed.

Then that light is not polarized or un-polarized so typically light coming from the sun or from some light source is typically not polarized it is in order the field oscillates in all directions. Now remember what we said about the refractive index if the reflective index depends on the direction of the electric field then, depending on what light you are passing in you will see various types of effects in the light that is coming out.

So for instance if; the reflective index or if the retardation for an electric field in this direction is more compared to this direction. Then suddenly you will start seeing that these different polarizations are different e bar directions start splitting away from each other, and that really is what is happening in a by references phenomena. So the origin of the birefringence we will discuss this in more detail more quantitative detail in the next session.

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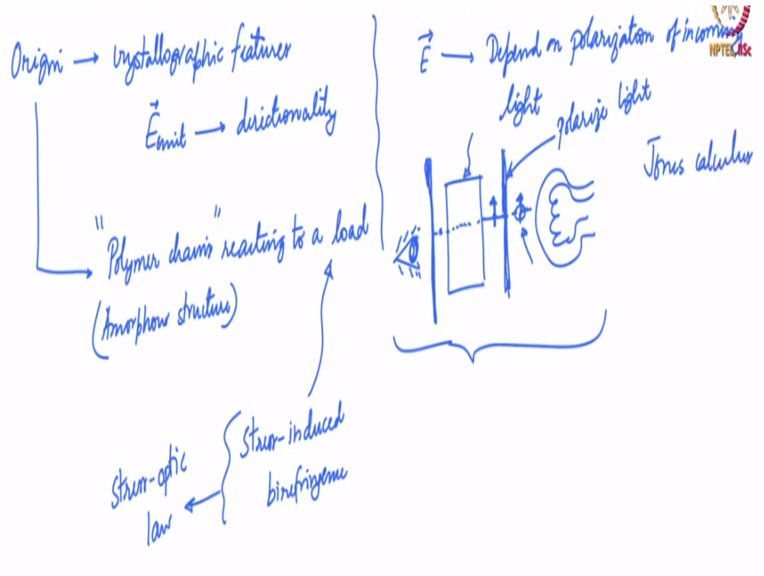

But the origin of the bi-refrigerants could be some crystallographic features for example in calcite. The arrangement of the atoms is such that the E bar Emit right has some directionality and that could come, because the way in which the atoms are arranged in the lattice and so that depends on the crystallographic structure. Or it could come because of polymer chains reacting to a load I am going to put polymer chains in court it could also be because of some more for generally some amorphous structure reacting to a load.

Polymer chains are a special class of course but it could be any mr structure and, this is what is called stress induced by refrigerants and is governed by something called the stress optic law which we will discuss in more detail. I am just giving you a sort of bird's eye view of where we are heading with this. And remember that since the E bar is affected these phenomena will depend both phenomena and specially stress engaged by differentian's will depend on the polarization of, the incoming light.

So if you remember the demo that we did a while ago at the very beginning of this section on photo-elasticity you had a glass sample and we put it between these 2 sheets and then we had a light source and we were looking at from the side you could see fringes right. And remember at that time I mentioned that you have polarizer sheets and the basic function of a polarizer sheet, is to polarize the light.

So a light coming from a light source is typically not polarized it has oscillations in all directions we will denote that like this we will discuss this notation in the next session but basically the light is polarized in all directions. When it passes through the sheet some directions are absorbed more than other direction so light coming in here is polarized. And, depending on how it interacts with the material depending on the bi-refrigerant properties of the material.

You pick up you filter out certain other polarizations with the second sheet and that is what you see in the form of fringes at the end. So what we will do in the next session is basically set up a framework for analyzing this entire process. There is a somewhat archaic term for this called, Jones calculus I will refrain from using this term because it is basically complex manipulation using complex numbers.

The name probably came up in the early nineteen hundreds before people started applying complex numbers many of these things. But there is a very systematic way to manipulate these polarizations and then obtain the final result of what you would expect to see when light passes through, the second polarization. So all our discussion of photo elasticity will revolve around how you break down this process.

And how; you systematically analyse incoming light interaction of the material and the interaction with the poltergeist so we will continue with this from the next session.