Optical Methods for Solid and Fluid Mechanics
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Module No # 08
Lecture No # 31

As we have seen in the last session we showed you some videos of typical photoelastic fringes. And the first thing that anybody would notice when you look at some of these fringes is the fact that they all look rainbow coloured right and so the word rainbow naturally raises the question of you know where this fringes are coming from? And it is not surprising to know that they have related to the, phenomenon that finally leads to a rainbow which is a reflection.

And so to really understand where fringes are coming from why some materials show fringes some materials do not show fringes in fact most materials do not show fringes. If you take a piece of metal or a piece of opaque plastic you do not see any fringes there. But if you look at glass various types of glass if you look at, transparent rubbers transparent plastics these do show fringes to varying degrees of course varying intensity degrees.

And so to understand why this is the case we need to delve a little bit deeper into the nature of light itself. And so that is what we do today I will discuss some very basic ideas from a slightly different point of view. I am sure many of you have seen this in you know 10, standard or 12 standard physics textbook before it is very elementary. But if you think a little bit a little bit more carefully the exact reasons behind why some of these phenomena actually occurs perhaps not so trivial and that is what we will discuss.

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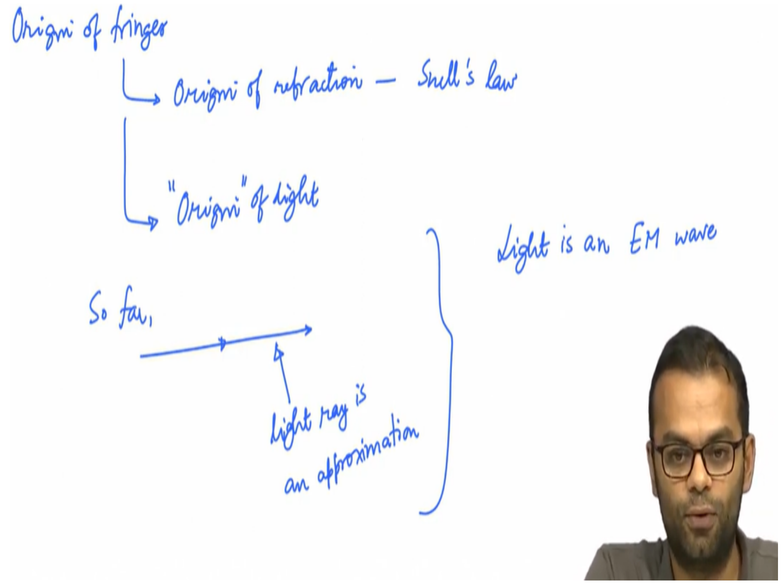

So in order to understand the origin of fringes we need to understand first the origin of refraction why does refraction occur we all know, refraction of course we know Snell's law you probably heard this multiple times before. We know how to calculate what happens when light is incident on a transparent object what is the path that it finally takes does it deviate away from the normal towards the normal sin theta in refractive index all that stuff is known right.

So we know operationally what it is but then do we understand where, refraction comes from a microscopic point of view why do; some materials refract light why do; some materials not refract light and so on. So that is something we have to consider and when we do this we will very quickly realize that in addition to looking at the origin of reflection we have to look at the origin of light itself.

And this I know this sounds melodramatic but to understand why, certain phenomena happen you need to look at the wave nature of light and if you go back to the beginning of the class and to what we have discussed so far. We have always been talking about light as being array right so far for us light is just array when you draw a lens diagram you know right thin lens equation and so on you always draw array right.

But light is not just array this is an, approximation so a light ray the notion of a light ray is an approximation and light is in fact as we know a wave and it is not any wave it is what is called an electromagnetic wave I am sure you have seen this phrase also before. And you need to go back and look at what electromagnetic wave is and why it does what it does when it interacts with materials right.

So it takes us into some fairly alien, territory we will try to cover that in as qualitative a manner as possible and hopefully this will give you some background to help you understand why refraction occurs in the first place. And then from there we will step back and try and understand why birefringence or rather the phenomenon that underlies photo velocity why bio reference turns up in many materials.

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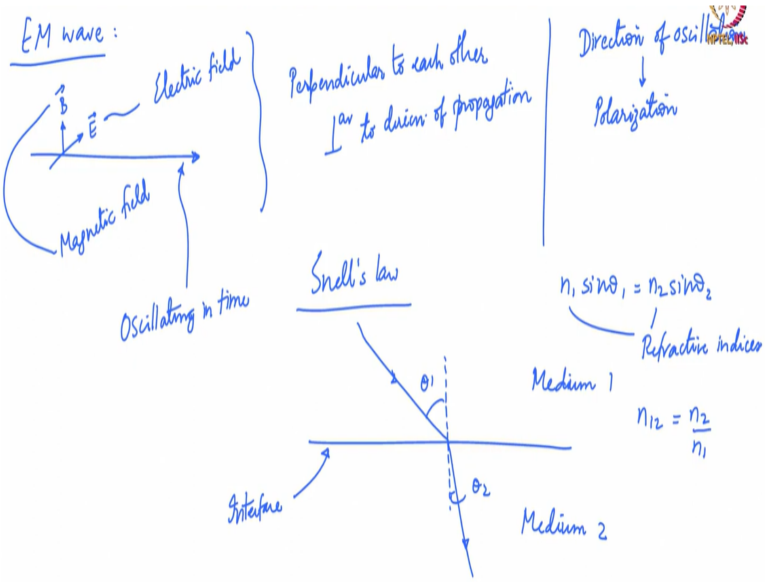

So let us start with basic idea, of an EM wave so a light is going this way let us say this is the direction of propagation and this is well known right. I mean if you have a light bulb you know that is the origin of the light right because there is some filament presumably if it is a tungsten filament light bulb then the tungsten is getting heated up it is emitting light. So I am using photons and they are all coming in, a certain direction towards you.

So the direction if you approximate the light bulb as a point so the direction of propagation is clear right it is coming from the source to you. But along each point of the propagation direction there are actually 2 vectors right there is what is called an E vector and a B vector t E is the electric field vector and the B is a magnetic field. So the electric fields, in the magnetic field are both always perpendicular to each other and then also always perpendicular to the direction of propagation.

What this means is that at every point along the ray there is a field electric field there is a magnetic field that is perpendicular to it. And these electric and magnetic fields are basically changing in time. So they are oscillating in time and they are oscillating if you have a light wave in vacuum. Let us say without interacting with any material then they are/have oscillating as they go along the electric field is oscillating like this along the light along the ray and the magnetic field is oscillating let us say like this along the way.

Now the direction of oscillation of one of them let us say e bar just to keep things, concrete b balls perpendicular always the direction of oscillation determines what is called the polarization offline. And if light is plane polarized that means that the direction of oscillation is always in one direction along one axis if it circularly polarized the axis rotates in a circle it is elliptically polarized and so on.

So it will give some details of that later on but basically, polarization is related to this direction of oscillation. Now if you recall Snell's law for refraction it tells you basically that if you have an interface. So this is an interface and you have a medium 1 and medium 2 then lights coming in like this is going to get refracted like this and if you draw the normal to the interface you call this theta 1 and you call this theta 2.

This is the, direction of propagation then Snell's law says that n 1 sine theta 1 is n2 sine theta 2. And we all know that this n 1 and n 2 this is these are called the refractive indices of medium one and medium two with respect to air right. You could take the ratio and then write n 1, n 2 then 2 by n1 is the relative refractive index of medium 2 with respect to median mode right. So this is standard Snell's law again from 12 standard physics you have probably seen this many times.

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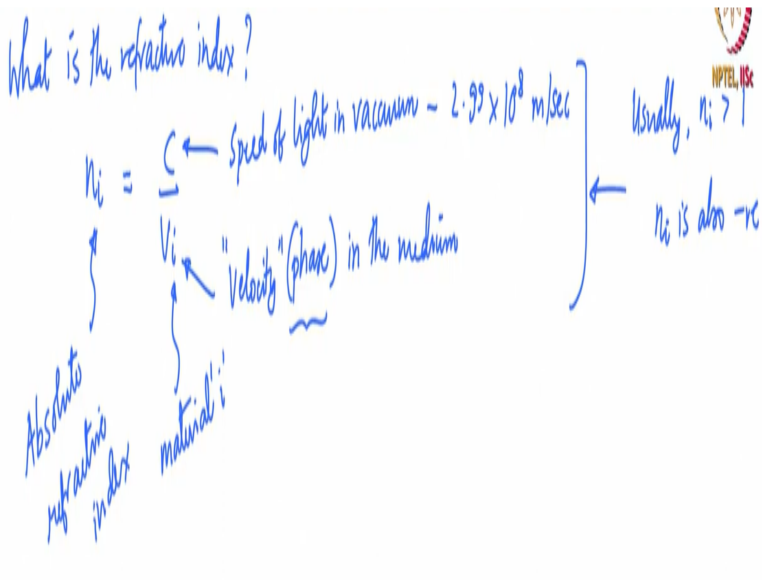

So what is the refractive index that is the next question? So we are going layer by layer we are starting from what we know and then we are going into something that is reasonably unknown at least to us these phenomena are you know fairly well known they've been known for several decades at least. But let us try and understand this in a sequential manner so what is the refractive index?

Now refractive index obviously is a dimensionless number right because sine theta has no dimensions and so refractive index also is a dimensionless number. And it really is the ratio of the speed of light in one medium with respect to the other so this is what is called the absolute refractive index. So, this could be refractive index of glass in air for example c is the speed of light in vacuum.

When I say add I mean vacuum iron vacuum are practically the same for our practical purposes and this is 2. 99 into 10 power 8 meters per second. And this is the velocity or what is called the phase velocity I am going to put this in quotes I will explain what phase velocity is in a minute this is the, phase velocity in the medium. So the refractive index is the ratio of these 2 and its dimension is because both have dimensions of meters per second.

So usually for most materials there are some exceptions we will talk about them again little bit later usually for materials notice also the index I is for material I. So if you have 2 materials and material one will have effective index n1, corresponding phase velocity v 1 and so on. So usually n a is greater than one and this makes sense because speed of light and vacuum as you have probably heard is the fastest you can ever go you can go faster than that at least information cannot propagate faster than that and usually ni is greater than one.

But there are cases where n i can be less than one it does not violate any principle, of physics because you are talking about what is called the phase velocity and I will explain this in a minute. There are cases where n i is also negative these are called negative refractive index materials we will not talk about that. But there are active areas of research interest and there are also situations where the refractive index is complex which has a certain implication, which will get to in due course.

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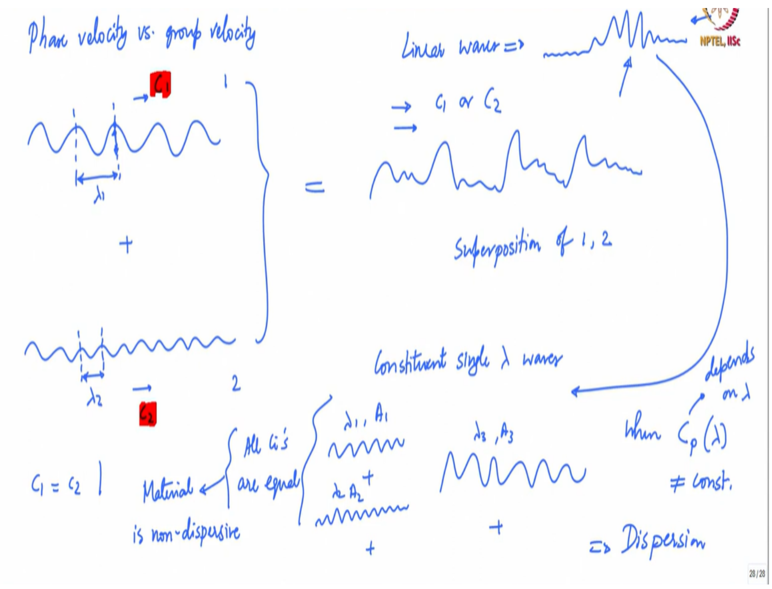

But this basically defines what the refractive indexes right so before we go further we have to understand this idea of a phase velocity as opposed to what is called a group velocity first. Now let us say I have a wave like this has a certain wavelength lambda 1 and then I have another wave which is like this and it has a wavelength lambda 2. Lambda 1 is, obviously larger than lambda 2 as I have drawn it here you can combine these waves and get a third wave which perhaps looks something like this.

I am making this up it is not the exact sum obviously because I am drawing it freehanded now this is obviously called the superposition of way 1 and wave 2 this being wave 1 this being wave 2. And it turns out that for linear what are called linear waves any waveform right, even if you have a wave that looks like this you can break this into constituent single wavelength waves. So for example this fellow here you can write this as this plus so these are all single wavelength wave.

So this is lambda 1 lambda 2, lambda 3 and so on different amplitudes a 3, a 2, a 1 and so on. So you can break up any of these breaks up any such wave like this or any signal of, this form into constituent waves. You might have a lot of constituent waves but nonetheless it can be done in some very definite way. Now if this wave had a speed c 1 this wave had a speed c 2 direction of propagation which means that the time taken for this point.

Remember this as the wave propagates a point here is going up and down up and down and the time taken for that to do one full up, and one full down and come back to its starting point is then. That of course divided the wavelength and divided by the time is the wave speed c 1 and likewise over here. Now if c 1 = c 2 or indeed if in this case all c i's are equal that means that the material or the wave or rather the material in which the wave is propagating is non-dispersive.

So very simple thing that you can, probably imagine is if you just take this example the one we have drawn here. If you have c 1 and c 2 equal then the speed at which this wave moves forward will also be the same as c 1 or c 2 right. Because there is no change everything is moving at the same speed the wave will retain its shape and it will keep propagating at the same speed.

Now interesting things happen when this c which I am going to, call this which I am going to give a subscript of p for face when this c p of lambda is not constant which means it depends on lambda when this happens then you have dispersion. So when the wave speed for the individual sinusoidal waves depends on the wavelength of that wave then you has something called dispersion.

Now to explain what dispersion is I will show you a very short animation so, that the idea is clear and we will annotate on top of the animation.
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Here are 3 panels showing you the difference between group velocity and a phase velocity. So this is borrowed the source is provided over here you can easily make an animation like this in Mat Lab or python it is very simple to do you only need a suitable way to visualize final time dependent waveforms but this is just one, to illustrate the point. Now let me show you what these individual panels represent.

But before that there are 2 dots that are marked in each panel there is the red dot which represents the phase velocity and there is a blue dot that represents the group velocity. So for example if you take this panel the red and the blue are always coincident because the phase velocity in the group velocity, are the same. You can see that they are going together the phase velocity and the group velocity not being the same there are 2 simple cases that I will that are illustrated here.

So let us take the case with the group velocity is 0 the phase velocity tells you this for example if you see this red that is going here that is following this trajectory it tells you how that, point at the same phase is moving right. But if the group velocity which is the added sum of everything is unchanged then the total envelope of the wave does not propagate right. So if you look at this the outline I have drawn here is stationary even though stuff is wiggling inside there is some change inside the overall envelope is still in the same location and that is because the group, velocity is 0.

On the other hand if you do have a finite group velocity and the group velocity is lesser than the phase velocity then this case will happen. So you see that the red spot moves faster than the blue spot and the blue spot represents in some sense the speed at which the entire wave drain is propagating. So the red is point on the crest in this case on one crest does not, matter which one you could take anything and then remove the same speed as the one shown here and the blue is what are called the wave drain motion velocity.

So the first panel is a non-dispersive case so if I took this waveform and I split it up into various components with various constituents each of those constituents will move the same speed. The second case is dispersive the, third case is also dispersive it is an extreme case a standing wave the third one because it is not propagating. But let us look at the second one second one is also dispersive which means some frequencies or some wavelengths I should say will move faster than other wavelengths.

Now if you go back to our discussion of what the refractive index was remember refractive index is c by v i in the, case where the medium is dispersive some wavelengths moving faster than other wavelengths implies that v i is a function of lambda. So the refractive index will be a function of lambda ok and this is precisely why you see dispersion in a prism in the word dispersion comes from here of course.

Because it is all dispersive if you take a prism and you put white light going through remember white light, is there is no color called white of course. We have learned that you know in school it is comprised of all possible wavelengths so going from red all the way to blue and beyond and so on. And each of them since this prism is dispersed the medium is dispersive the refractive index the velocity with which each phase or each wavelength propagates is dependent on the wavelength itself and so the, refractive index is a function of the wavelength.

And so remember this is not norm this is incident at some angle theta and so the corresponding theta that you get will change depending on what wave we are looking at? So this is your red blue green etcetera will have different values of theta 2 let us call this theta 1 will obviously not be equal to theta 2. Because the incidence is not, normal to this to this edge and of course the exact value of theta 2 will depend on lambda because n depends on lambda or n 2 rather in this case and that is why you see breakup of colors when light passes through a prism right.

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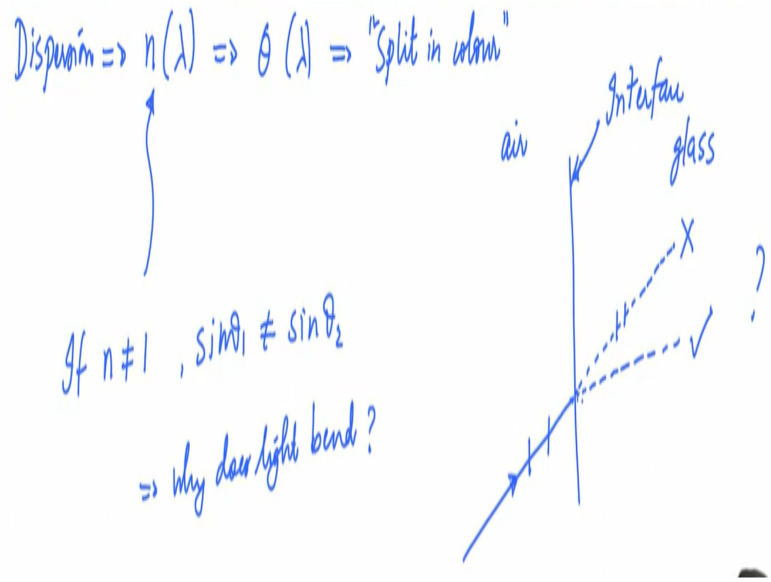

So coming back to where we were we know now that dispersion implies that the wave the sorry infective index is a function of the wavelength. And so this means that the angle of refraction is a function of the wavelength which means you will see a split in colour. Now this is fine this makes sense but the more fundamental question we want to ask first is the refractive index is a function of wavelength.

But if refractive index is different from one right if n not equal to one then the sine theta 1 or consequently theta 1 is not equal to theta 2 or, sine theta 2. So the next question we want to ask is y does light bend why should, it bend in the first place refractive index is different velocity is different but why cannot it just move slower right. So you know you could as well think of a situation where let us say I have my interface like this and you have incidence like this.

And I am telling you that the light goes faster let us say, this is air light goes faster here and then this is glass let us say and light glows grows or goes presumably slower in glass why cannot it just continue going like this but just go slower right. So when delta t if light has moved this much here why cannot it just move this much in glass velocity is lesser why should it bend right?

So that is the next question we have to answer why does it, have to do this and not this and the reason we are asking this question is because it leads to this discussion of an ordinary or extraordinary array which comes up in by refrigerants and if you have this background it will make understanding that a little bit more easier.

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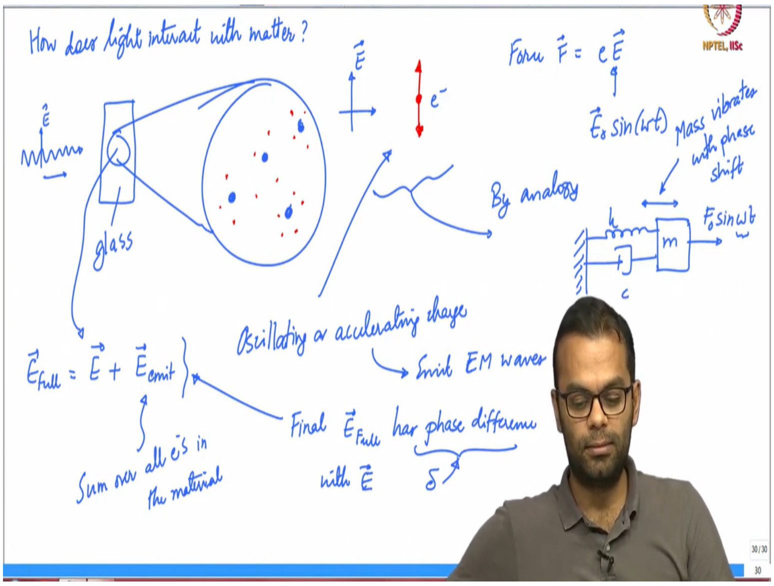

So why does light bend to appreciate why light bends we have to first go a little bit more deeper and ask the question how does light interact with matter? You have a slight interact with matter now at the level which we have been discussing so far it just you know only thing that matters is n and anything that you see as a consequence is theta. But if you look at it a little bit more microscopically so you have your piece of glass and you zoom in and zoom in what happens?

Now you will notice the following the light wave that is coming in of course has its electric field in this direction and it is propagating in this direction the magnetic field is into the screen of course that is I am going to assume that we are not going to talk about the magnetic field. Now because whatever we say of the electric field will apply analogously to the magnetic field as well. When the light passes through this, material what does the material actually have?

So these are let us say atoms my idea of atoms are fairly cartoonish but the atoms themselves have electrons around them right. For glass you know you know solar line or you know inorganic glasses typically amorphous and so it does not have a regular arrangement of atoms are in a you know disordered state so to speak the collection of atoms is not on a, lattice. But the presence of these electrons basically is responsible primarily responsible I should not say only responsible for interacting with light so here is what happens?

Your light source comes like your light ray wave comes like this the electric field is like this and remember that the electron is a charge particle. So the moment the electron sees electric field it experiences a force, which is charge times field we know that. And so if the field is oscillating up and down if the field is doing something like this then the charge also starts doing this.

Now this is you can think of this by analogy with a system that you are probably more familiar with which is you take a spring and a damper and you attach a mass. If you apply a force to this mass and you will get it to, oscillate the mass will start vibrating of course but it will vibrate with a phase change right so or a phase shift. And if you go back and try to work out this problem it is not very difficult to do you have probably done it before the phase shift will depend on the ratio of the applied frequency and the natural frequency of the material right.

And that of course depends on the spring constant it, is altered by the damping coefficient and things like that this is purely biology the actual equations are much more complicated as you can imagine. But for a conceptual idea this serves fairly well now the same type of thing you could imagine happens to the electron right it goes up and down up and down with some phase lag compared to the wave that is coming in.

Now the moment the electrons are, going up and down it is an accelerating charge okay oscillating or accelerating charge and if you have an accelerating charge you know equation of electron electrodynamics tell us that the charge will emit radiation. So this will also emit its own electromagnetic waves. So what you actually see in the material in this region what you actually see is the wave that was originally coming in or the, field that was originally coming in plus the sum of all the fields that are generated by all the electrons are there and all the atoms that are sitting in the material.

Loosely again this is a fairly simplified picture but it is a reasonably accurate picture I would think so the actual full field is not just the incoming wave which is passing through by itself. But also the emitted waves that are, emitted wave sum of all the emitted EM waves that are coming from all the electrons that are in the material. So naturally when you add these 2 the final vector or the final E bar full as I have called it here has a phase difference with the original incoming E bar we will call this phase difference delta.

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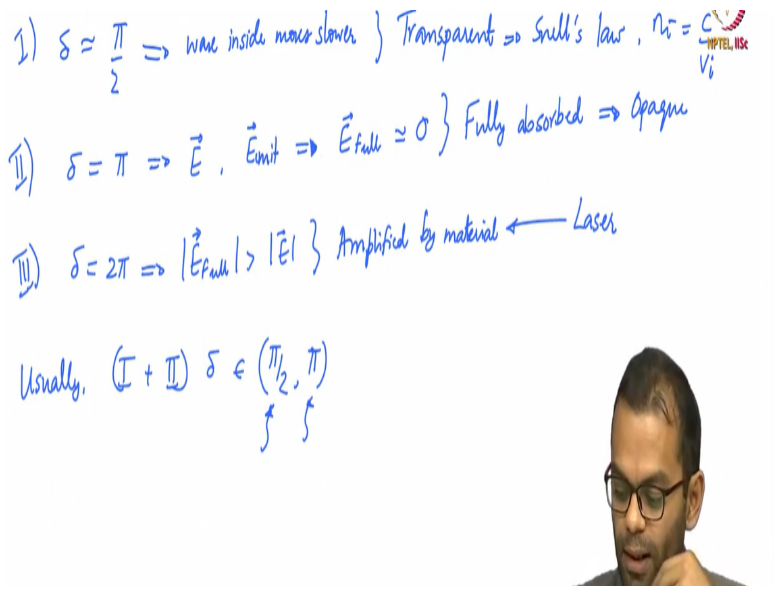

Now this phenomenon is the basis for many optical phenomena that you will see in nature for example if the delta is let us say around pi by 2 the phase difference between the incoming and the sum total that is coming out that is in the material. Then the wave inside moves slower if the delta is pi the phase difference is pi then when the E bar that is coming in is it a maximum then the e bar what do I call it E bar emit when the incoming a bar is maximum then if the, phase difference is pi the e bar emit is minimum.

So when you add them up they will cancel each other likewise when the e bar is minimum the E bar m is maximum they cancelled each other and so it perfectly cancel out. And so the net E bar the E bar full is very close to zero this happens when let me let me come back to when this happens after I discuss the third case. So if you have delta, equal to 2 pi then the e bar full is greater than E bar the magnitude is greater than ever the incoming a bar.

So the incoming wave is amplified by the material right amplifies that and generates a larger amplitude wave. So this is exactly what happens in a laser by the way light amplification so amplification comes from here right the case 2 so this case 1, case 2 and case 3. In case 2 since the final field is 0 basically the light that you are sending in is completely getting absorbed by the material so it is fully absorbed.

And what happens when the light is fully observed you cannot see the light that you have sent in which means that the material is opaque. Whatever your sentence is gone it is not coming out from the other side and when this happens and the first case happens, then you have a transparent material and you have Snell's law. So wave coming in becomes slower so you have a refractive index and you have n i = c by v i n all of the stuff all of the good stuff we discussed before.

So in general usually you will see a combination of 1 and 2 so the angle typically delta is between pi by 2 and pi. Which means some part of it is transmitted, through which means it is transparent the material is transparent to that particular light and some part of it is absorbed right. So the ratio of how close it is to pi by 2 or how close it is to pi tells you the relative absorb absorption versus transmittance of the particular light in the particular medium good.

So we know that light gets absorbed or gets transmitted because the stuff that is, being oscillated is in phase or out of phase with the original latest incident on the material right but this still does not explain why light should bend.

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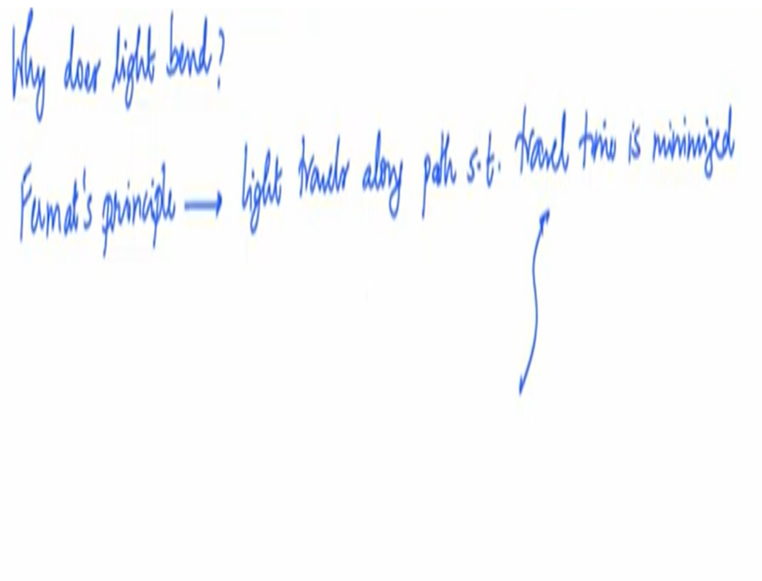

So we go back to our original question so when delta is between pi by 2 and pi let us say it is close to pi by 2 we forget about absorption for now when delta is pi by 2 why does light have to bend? Now there is a simple argument for well, not for explaining this but certainly to rationalize it and that is based on Fermat's principle. So Fermat's principle it tells us that light travels along a path such that the travel time is minimized.

If you are given a sequence of materials or different refractive indices and different consequently different velocities you can use Fermat's principle to determine what the direction of, propagation will be? So what we will do in the next session as we attempt to finally answer this question why light bends and why different wavelengths bend differently is we will apply this principle very quickly to light passing through an interface. And then we will see you know what the consequent picture is on the microscopic scale before we discuss birefringence.