Optical Methods for Solid and Fluid Mechanics
Prof. Aloke Kumar
Department of Mechanical Engineering
Indian Institute of Science – Bangalore

Lecture – 03
Tensors and Deformations

Welcome back everybody so, we were discussing tensors in the last class, vector spaces and tensors and where we stopped last time.

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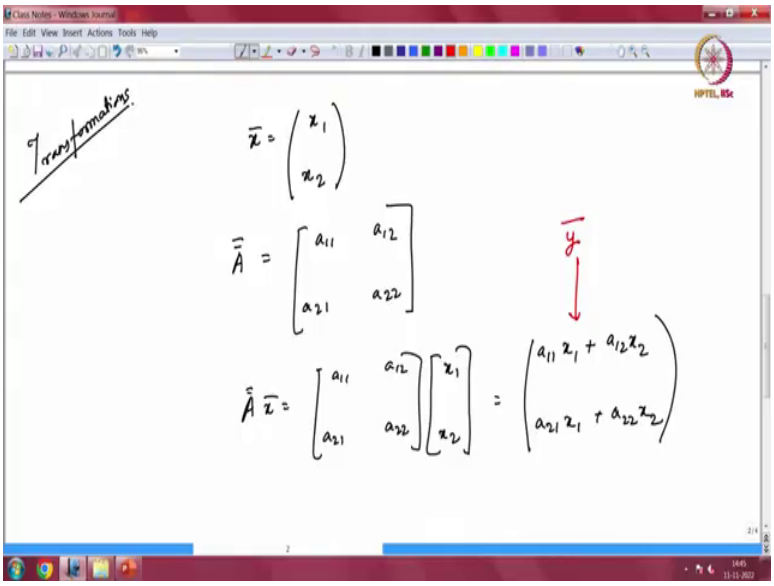

This example of transformations and we were discussing how, if you have a column vector. Then you can have this matrix two dimensional matrix, consisting of four terms as a tensor that can operate on this column vector and give rise to another vector. So now, these we had also discussed the definition of tensors and this falls very much in line with that because it is acting on a finite dimensional vector space.

And then you have the vector space is also a real Euclidean space. So, matrices such as these will represent tenses of some form. When we introduce transformations, we said that a transformation acts on a vectors to give you another vector in the same space. I am not going to repeat that part again I think from here onwards we will just take that as granted but the idea is that you have a transformation which acts on of one vector and gives rise to another vector.

Now, you can generalize this idea. Obviously, going even further and you can define something.

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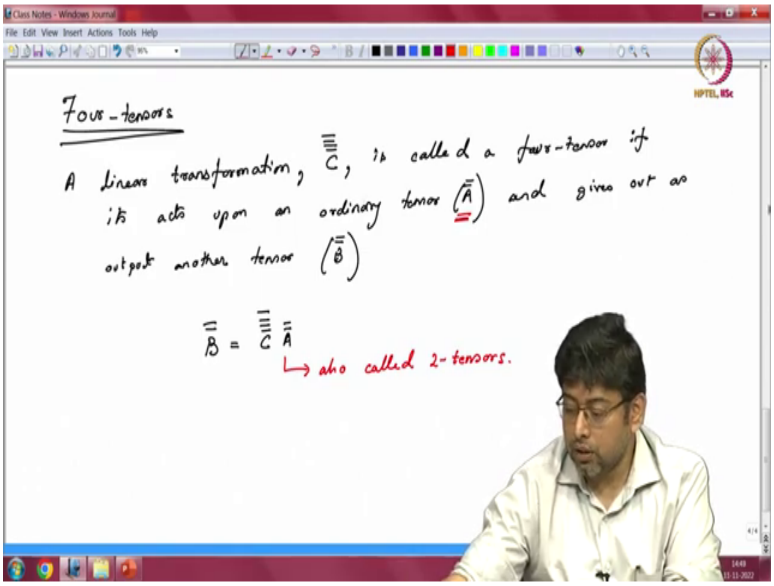

For example, as a four tensor and four tensors although we will not need it so much in this class but I think it is just appropriate to quickly discuss this. A four tensor is a linear transformation and we will call it let us say C now, we have already put two bars for a tensor. So, we are just for four tensor is going to become a bit cumbersome later I will put four bars. And it is called a four tensor if it acts upon an ordinary tensor.

Let us say sum tensor A and gives out as output another tensor. Obviously, it similar type of tensor it cannot be a different type. So, here what we will have is? We will write this as this, so, C let us say acts upon A and gives rise to another tensor B. Now, these are tensors we have already seen before and these are also this can also be called two tensors. Now, four tensors are very important in the field of solid mechanics, container mechanics in general.

We will not encounter them so much here but it is just important to see how you can just take this idea forward and more forward. But in the experimental techniques that we are going to discuss it is these tensors are going to be the important ones that we will encounter. And we will see why that is? So, till now what we have been doing is we have been giving a very dry set of definitions for tensors.

But it is important to understand why, what some of these tensors can do? And why they are needed? Now, I want to add a page this wait a second.

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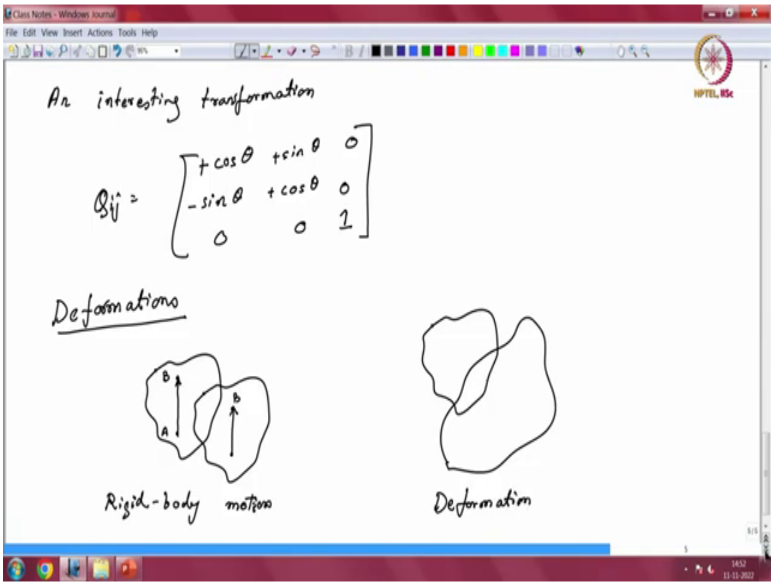

So, let us say I will give you an example of an interesting tensor and I want you to work out what this tensor actually does? I will give you hints also. So, sorry by definition, word is incorrect it is an interesting transformation. So, I will give you this particular matrix. So, this is a three dimensional matrix consists. The first entry is + cos theta then + sin theta and the next one is – sin theta and this is again + cos theta.

And we will populate almost everything else with a 0 except here and I am going to ask you to see what this does? Now, this is a rotation matrix. So, I have already given you a very good hint as to what it will do? But I want you to work this out for yourself, maybe take a radius vector and operate and so, theta can be any value you want to choose. So, to just put some value of theta and you will see that what this will end up doing is rotating that.

So, I just want you to work this out it is quite simple really but something which is nicely illustrative still. Now, this rotation the reason I discuss this is because you might have seen this before but in mechanics the kind of tenses we are going to be dealing with, they are going to be slightly different and there is a good reason why we deal with many of them. Now, in order to understand that we have to understand the idea of deformation in a little bit more detail.

So, we will look at deformations. So, we will have real bodies and we will have them deform. So, let us say we have a body like this and I am just going to create some copies of this. So, I am going to create one more and now in this case, what I will do is? I will draw something slightly different. So, what I am trying to show here is? You have some, body which is shown here? By this particular outline and this outline you are seeing all over.

Now this is a body let us say there were two points in it. Let us say some point A and some point B and let us say this is a vector connecting that. Now, this body has changed places and it is deformed, perhaps or perhaps not but it looks exactly the similar same in this case. So, I am actually, trying to show a rigid body movement here. So, this body has just gotten displaced in space without so, this is a rigid body motion.

Sorry, this is and this is a case where you, actually, the body is deforming. So, this is a case of deformation.

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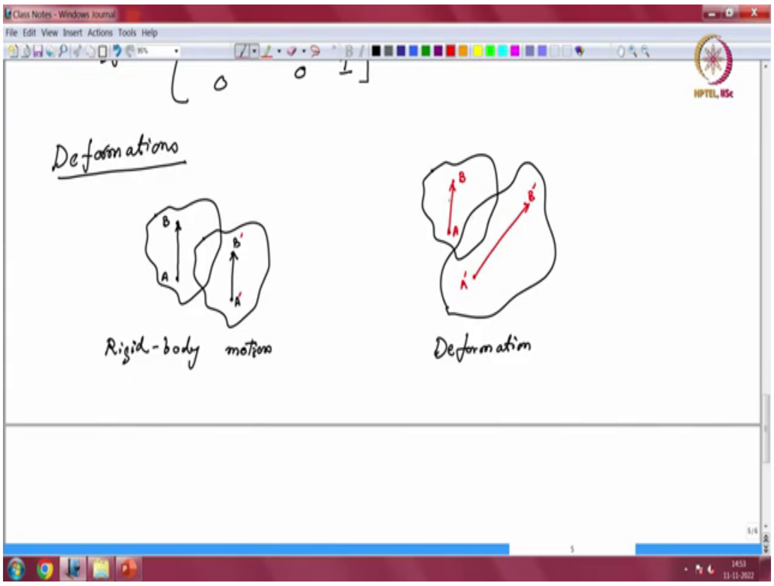

So, here now these two points. So, in this particular case, in the former case, the vector A B is simply translated in space or rotated but does not change as much the relative positions do not change. You take any two points and they will not but in this particular case where the body has deformed you had so, this is A dash B dash. So, you had these two points but now, let us say the body has deformed.

And this new vector is now looking different which is still connecting the same two points in the system but the vectors are now very different. So, this body has undergone deformation and now, we have to the one of the ways in which we can understand deformation is by keeping track of these vectors that we had. So, what is a body? It is just a collection of many different points.

So, we can draw all these vectors from let us say one point to other points in the body. And then we see how these radius vectors change and that will give us some idea of the deformation. So, this is how we keep track of deformation. So, we are going to look at deformation of a body. So, now the whole point is that what you have in the field of solid or fluid mechanics is you have deformation of some sort. And the whole idea of the experimental technique is to keep track of this deformation.

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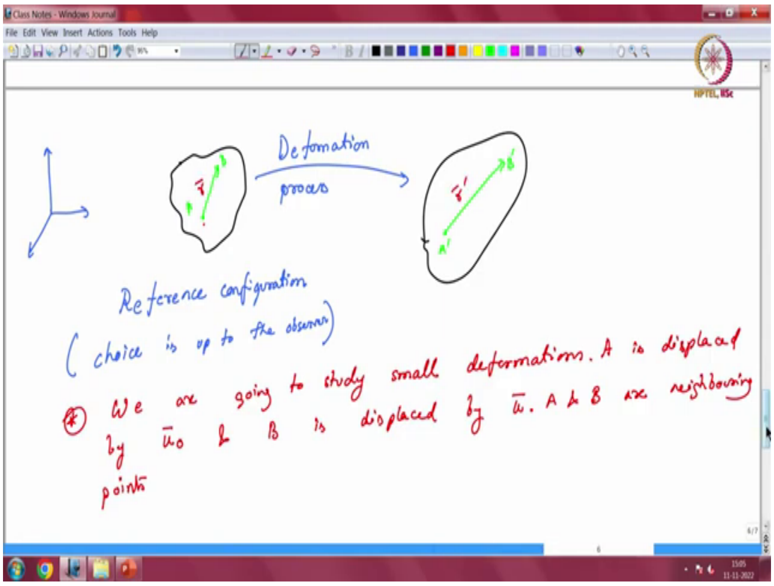

Now, just like we do for dynamics, what we will do is? We will say that we have some coordinate system that is already set up. Now, you have this body that you had I just copy paste the photo from previous time and this body is now undergoing some process and is now going to deform into a slightly different shape. So, this body has undergone some process. It is deformed, so, some deformation process is going on.

And as a result, the body is changing its size and shape. What is the causality for this is something that you might have to work out in your very specific case. So, we are not here to specify what is why this body is deforming, like that. The idea of the most experimental techniques is to keep track of this deformation and give you that quantitative information. And once you have that then you can go back and address your query as to why that might be happening.

But the idea of most experimental techniques stops at keeping doing a good bookkeeping of this deformation process. So, how do you track of the keep track? So, we are just showing you last time that we have this radius vector that we had drawn between a point A and point B. And these are obviously arbitrary points, so, you can choose whichever point. So, we do that again here, let us just say we have point A and point B and we have a vector here.

And then you have a new this is your A dash and B dash so, these are the same points but in the deformed body. So, now, just like we do in dynamics everything is relative, so, motion of a body is relative. So, you have to define the coordinate system. The inertial coordinate system with which you are going to measure let us say the velocity of a body of a moving body.

Similarly, in this particular case you have to define the reference configuration of the body. Because you know just the inertial reference frame is not enough. You also have to define the configuration of the body with which you want to compare the deformation. So, you have to define something called as the reference configuration. So, let us say this is the reference configuration.

Now, I would be very careful here is that the choice is arbitrary. This choice of the exact reference configuration is up to the observer. So, the choice is up to the observer as to what you denote as the as the point with which you are going to compare the rest of the process for many different experiments. For example, if you were doing a tensile experiment, where you had, let us say a metal.

And then you are going to pull it at two ends or you are then the usually what people do is? The before they apply the force and before they start pulling they will define that state as the reference state. But it is not necessary for you to do that but here we will assume that what the reference state is. And then what both the points A and B are moving with respect to each other.

Now we are also going to do another thing we are going to put a star mark and say we are going to study small deformations. What do I mean by small? Well, we will see what I mean by small will be able to apply some Taylor series expansions here. So, let us say, the point u point A is displaced by so, A is displaced by u not as a vector and B is displaced by another vector u.

And while I have drawn A and B here deliberately in an exaggerated form, actually, A and B are varying are neighbouring points. The reason they are neighbouring points is because I want to apply my tools of calculus which are we are all very familiar with and good at. And we will use that to understand the deformation. So, there is one going to be one more problem in this course which is going to be with the letters we use to name our variables.

When we see in fluid mechanics the variable u is often reserved for velocity. But in solid mechanics u is often used for displacement. So, we obviously have if you look into textbooks, you will see that the variable u in fluid mechanics, is almost always for velocity and the variable u in solid mechanics textbooks is always for displacement. Here we are discussing both, so, we will we have to be very careful in how we name our variables.

So, I will just go ahead and use u here, as a variable for displacement and then when it comes to velocities, I will try to have ux, uy, uz to delineate that. But u, v, w will be variables to this to denote displacement. So, my vector u is basically composed of three components u, v, w in the three coordinates in the three independent directions.

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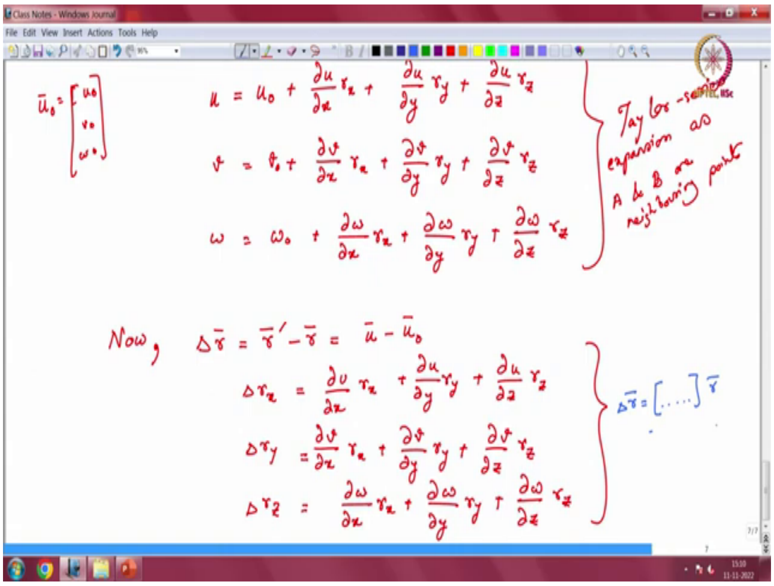

And so, I can relate my u to u not by using the Taylor series expansion and I have already said that the two points are neighbouring points. So, I can relate u and u naught through my Taylor series expansion and so, my vector u naught is nothing but the column vector u v w. So, now I can go ahead and I can one more thing was let us say this radius vector is r and this radius vector is r dash.

So, r was the radius vector in the beginning and then it became r dash. So that makes things consistent and now, the component of r in this case is rx + del u by del y r y + del u by delta z r z. These come from Taylor series expansion. And similarly, I do for all three cases sorry, not + not = + sign. This is again del v by del x into r x + del v by del y r y + del sorry del v. I am sorry del v del z r z and then W is this is not I have to put a note.

So, this is the relationship using Taylor series expansion as A and B are neighbouring points. Neighbouring points means they are separated by a very small distance, dx dy dz. So, that is why we are able to apply these. So, my vector u naught, is basically u naught v naught w naught so, I miss that not here. So now, if I want to understand how much has there been a change in my radius vector in the two cases.

Then I have to take these two radius vectors and 2 so, I have to take r dash – r. And that essentially biased down to my u minus So, now, if I had to write the three components of this z then this is nothing but del u del x r x and you can you know how to you can just copy paste. Actually, let me just do that I can just copy paste myself here. I do not have to write it, so, I am just going to copy paste this portion.

And then this class is normally got required. So, what we now is so, what we have basically been able to do with all this here is we have been able to relate this change in the radius vector and there is some matrix that we have to populate here. And if you multiply it with the original radius vector, we are able to find the change. So, some matrix which is basically these terms that we have just written.

So, the derivative terms and this now transformation allows me to connect the old radius vector to the radius vector in the reference configuration. I can write this down in a little bit more I am going to use in integral notation here to simplify things. You can just see that if you do not use integral notation things just become very tedious here. So, just to simplify life for myself, I am just going to use the integral notation.

And say that my r i is nothing but u i, j to r j where this comma here denotes differentiation, as is with index notation. So, this index notation just helps us write this down. In a more overall simplistic manner.

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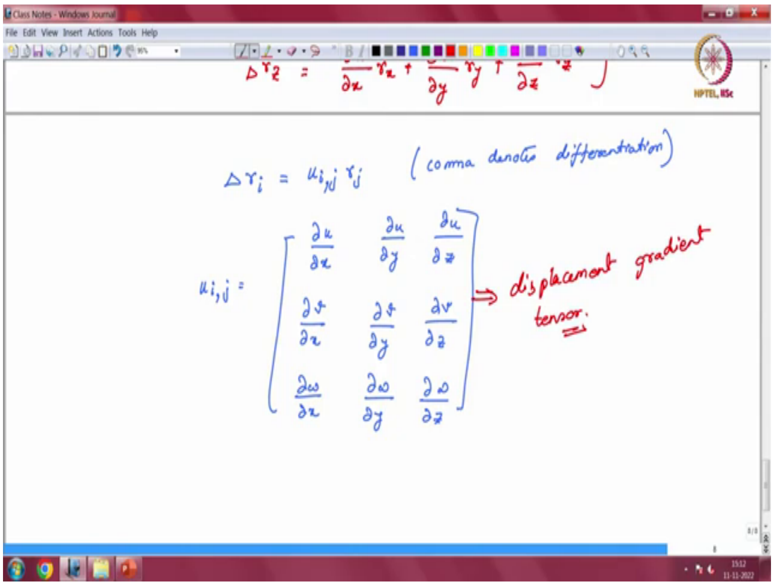

And now so, this u ij we already know what this is because we already worked it out. It is nothing but this particular matrix. So, if I had to this matrix which is I think you all understand, is a tensor is also called the displacement gradient tensor and you can see why it is a tensor. It is acting on one radius vector to give you another vector which allows you to keep track of the changes that are happening other deformation.

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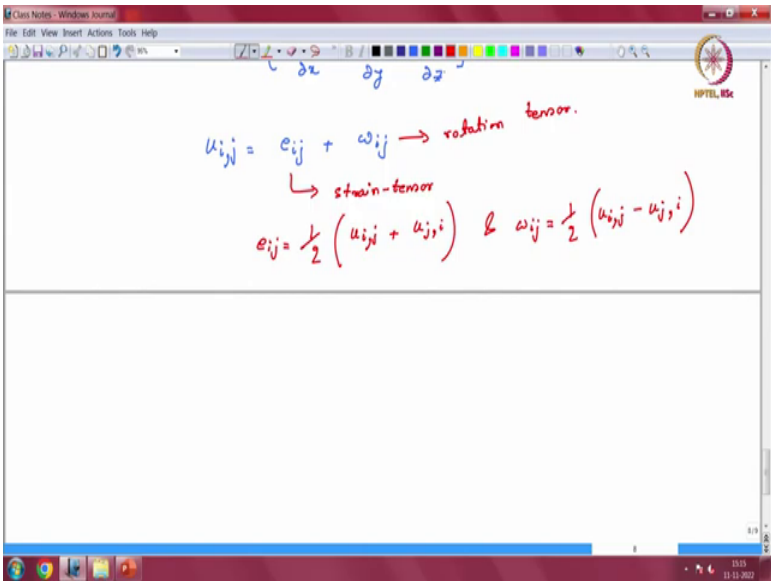

Now it is usual for us to write down this particular tensor. The displacement gradient tensor as a sum of two other tensors shall call e ij + Omega ij where this is also called the strain tensor. And this is also called the rotation tensor where e ij is defined as half of u i, j + u j, i so, the order is reversed and Omega ij is nothing but u i, j – u j, i. And actually, one of these matrices is a symmetric matrix, the other is an anti-symmetric matrix.

I urge you to read up on that now in this particular course we cannot go. There is a fair bit of mathematics that is required in some sense in order to appreciate the experimental techniques in full detail but this is not a course on the mathematical preliminary is required for that. I we are just going to try and give you a feel of what are the important things. So that you can go ahead and read them by yourself and make sure that you are familiar with the required amount of mathematics.

Because we are going to need that quite need that to understand the output of the experimental techniques. So, this is something that you should just make sure you do. I also switched off to an integral notation right here, just to make my life easy in writing down all those terms. So, we had set out to understand deformation and how deformation is relevant in the sense of tenses.

And here we have two tensors which are very important which we saw the strain tensor and the rotation tensor. Now, in fluid mechanics, you do not see the strain tensor so much. In fact, what you do see is the strain rate tensor which is nothing but the time rate of change. So, basically, this is the strain tensor is something that you will see in solid mechanics quite a bit because that is what usually is the required output.

Then you want the strains to be known f at a given location, whereas in fluid mechanics what we are looking at are the strain rate. So, we are usually not dealing with the displacements themselves, what we are dealing with velocities. So, in that case, the important tensor is the time rate of change of that sensor. So, it is e ij with the time derivative of that.

So, here you can these do not u does not remain just the displacement term but becomes the velocity term. Sorry, the time derivative term also comes in. So, what we will do is? Today we will stop here next time I will try to give you a little bit more of the geometric interpretation of these deformations. And these tenses and from the geometric deformations that will help you get a better appreciation of why this is needed.

Especially in a course that is designed to help you do experiments and analyse experimental data. So, it will help you with that and then from there on, we will introduce ourselves a little bit more to the optics that are usually going to be very relevant to some of the experimental techniques. Before we fully start on to understanding some of the experimental techniques such as PIV. So, we will stop here today and thank you very much.