Optical Methods for Solid and Fluid Mechanics

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Lecture - 27
Example Implementations

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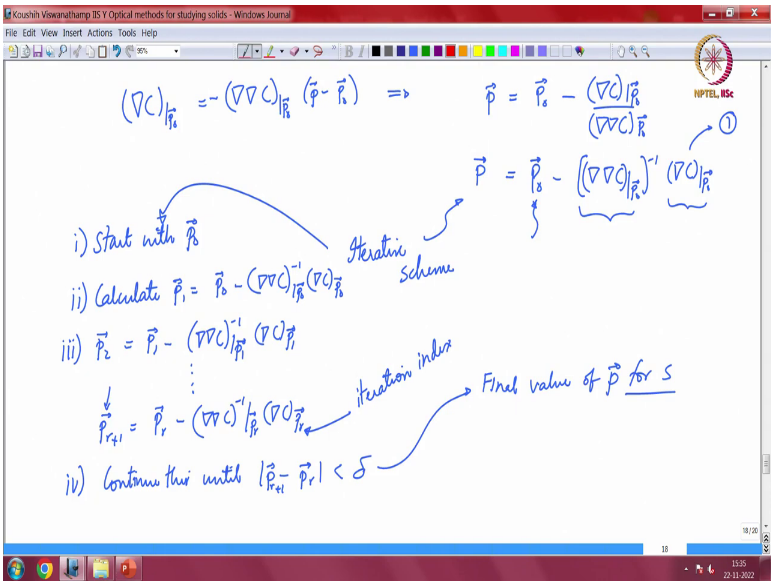

In the last session we left off with an outline of the iterative scheme for determining the six component displacement Vector with its gradients at each location. So, today we look at a practical implementation of how to set up a simple DIC scheme. Like I said there are many variants of DIC schemes that depend on some of the details that you choose to put in how you choose to interpolate how you choose to represent the grid things like that we will just see a bare bones implementation.

So, that if you are interested in pursuing some of these more detailed topics at a later stage you can you can do that with this as a good basis. So, this was the iterative scheme that we ended with again simply by just taking the gradient of C and then evaluating that at every single iteration step and getting a better estimate for the final value of P.

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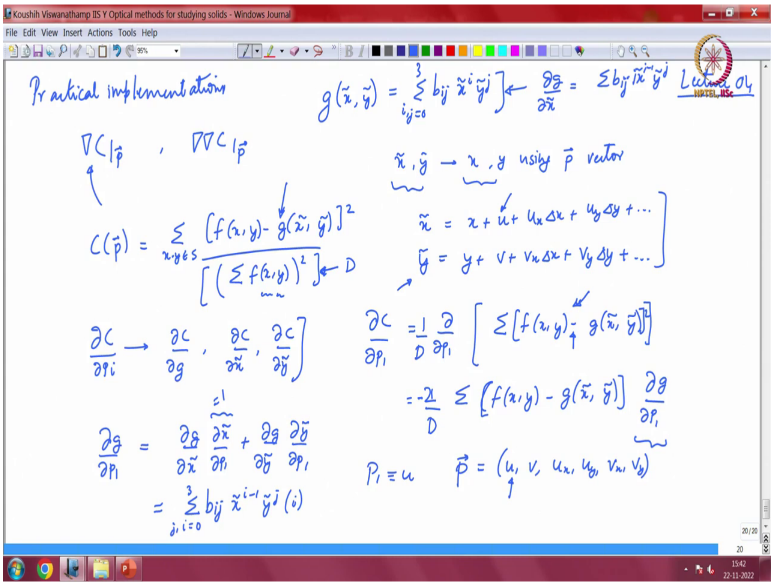

So, today we will talk about some practical implementations of this iterative scheme. So, the first thing you want to do is of course you have to be able to evaluate grad C at some value of P does not matter what P is it could be the first iteration or the 100th iteration but for a given value of P you should be able to evaluate grad C and you should also be able to evaluate the Hessian matrix. So, these are important coefficients or coefficient matrices that go into the iteration scheme.

So, before I show you the sequence of steps for a couple of images that we have of our own couple of special patterns let us just see how you would actually evaluate these gradients. So, if you remember C of P was Sigma x, y in s f of x, y minus g of x tilde y tilde squared divided by f of x y squared now x tilde and y tilde are obtained from x using the P vector. So, any derivative with respect to P will only act on the x and divided not the x y right.

Because x y are in the undeformed configuration and x tilde y tilde R in the deformed configuration and you can only get from the undeformed to the deformed using p right we wrote down the Taylor series like this last time right and likewise for y tilde like. So, all the gradient terms will only act on g and they will not act on F because F has x and y and x and y do not depend on P then the undefined configuration.

So, anytime you want to do let us say dou C by dou p i you will only need dou C by dou g dou C by dou x tilde and dou C by dou y tilde okay these are the only things you will need the rest of them will not matter. So, as an example if you wanted let us say dou C by dou P 1 using this expression you will get dou by dou P 1 of we leave out this Factor we will call this

D this is the denominator it does not matter like I said for the derivative. So, we will take it out. So, only the numerator remains.

So, this is f of x y minus g of x tilde y tilde this is squared and this bracket is for the derivative. So, if you want to do this then you use the chain rule of course this goes in this will be one by derivative of this again you need to take it with respect to g and. So, if you do derivative of this guy you will get 2 times just that Sigma f of x y minus g of x tilde y tilde and because it is the chain rule.

You do dou g by dou p i or dou P one ok foreign because there is a minus sign over here. So, it will be minus 2 by D Sigma F minus g dou g by dou P 1 okay that is dou C by dou P 1. now you need to evaluate this guy of course. So, dou g Del P one remember what g was we had this Pi cubic interpolation business going so this is a double summation over I and J going from 0 to 3. So, this is power.

So, x tilde power 0 is 0 total term x to the power one is the linear term and then the square term in the cubic term um. So, if you want to evaluate dou g by dou P one this will be dou g by the chain rule by dou x tilde dou x tilde by dou P one plus dou g dou by tilde dou y tilde by dou okay and dou g by dou x tilde again comes from here by dou x tilde will be Sigma b i j i x to the power of I minus 1 Delta y to the power J.

Just the usual way in which you will take the derivative okay now P1 of course is u remember P bar was q v u x u y v x v y. So, P 1 is just U. So, dou x tilde by dou P 1 if you go back here 2 P one is this. So, dou x tilde by dou P 1 is 1 dou y tilde by dou P 1 is dou of this by dou P one dou by dou u of this which is 0. So, the only term that will survive is this and that is equal to one. So, this entire thing is just Sigma b i j x tilde power I minus one y tilde to the power j times i.

And this is of course summed summed over zero to three as before okay and similarly dou g by dou P 2 you can use the chain Rule and you can get the same type of expression okay dou g by dou p 3 into g by dou P 4 and so on. and those will come in to do C by dou P one dou C by dou P 2 and so on. So, this is how you evaluate the various derivatives that go into the gradient Matrix.

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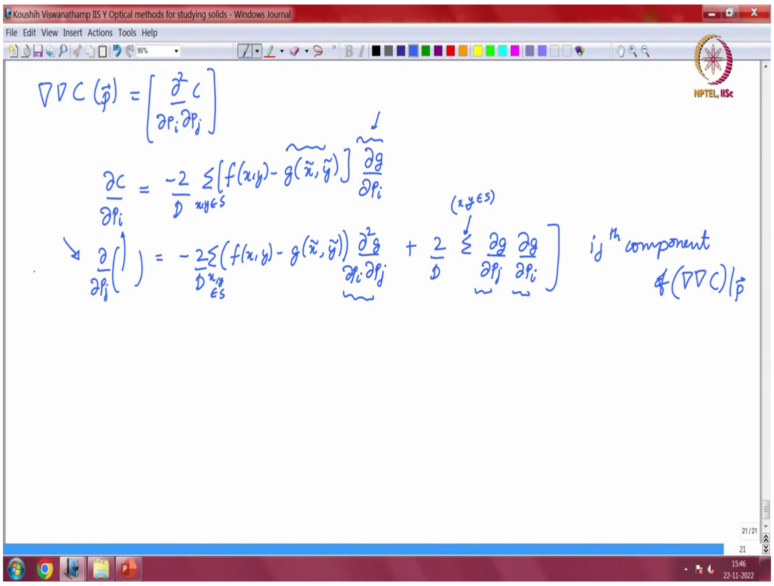

Now the Hessian matrix is analogous it is a little bit more tedious but it is analogous. So, to evaluate to evaluate this you have to do dou Square of C. So, this is a matrix I, J going from one to six of course. So, six by six matrix. So, you have to take a second derivative. So, if you remember the first derivative look like this. So, I am just going to reuse that expression s, for some arbitrary component.

So, let us say p i was minus 2 by D Sigma f x y minus g x tilde y tilde dou g by dou p i and if you want to take a dou by dou P J of this you will have to do it on the right hand side remember again D does not depend on any of the P's because it depends on your x and y in the undeformed configuration. So, that does not matter the only thing you will have to know room. So, I will have the 2 by D out the only thing you will have to now remember is that.

Now you have 2 terms that contain P J one is this and one is this. So, we will act on them one after the other. So, if you act on this guy first you will continue to have f of x, y minus g of dou Square g by dou p i dou p j that is the first term and you will have a second term which will have a minus sign. So, it will be plus 2 by D. But now the second term will act on the g inside the bracket. So, you will get a do g by dou P J doji by dou p i ok.

Now this summation of course remember this is x y within the subset x y within the subset x y within subset and so on. make sure you distinguish this summation from the summation in g because summation in g is over indices this is over locations inside the subset right. So, whether it is exact pixels or whether it is sub pixels depending on how much accuracy you need this is the type of sum that you have to do over here both are double summations but the indices are very different.

So, this gives you the ijth component of the Hessian matrix at whatever value of P right and again dou g by dou P J is the same you do the same thing like we did last time for this for this and for this as well now you will have I times I minus 1 and so on. Depending on which p i and PJ you take. So, it is the same as before. So, this basically now completes our entire scheme we know how to evaluate grad C from here each of these components we know how to evaluate the Hessian from here to each of these components and the iterative scheme needs only these 2 matrices.

So, once you have these 2 matrices populated for a given value of P you can just put into the iterative scheme get another p and keep repeating this process okay. So, I will show you a practical implementation of this with an image with our own image and so that you can see some typical steps that go into the actual calculation.

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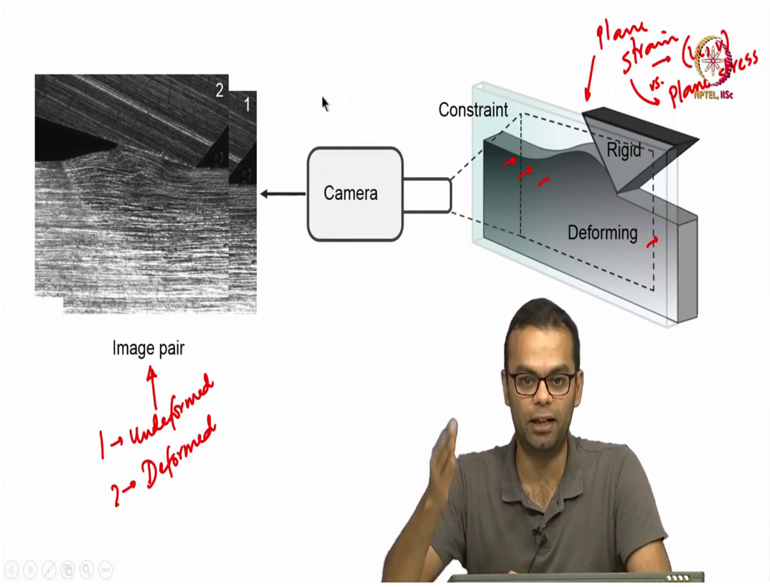

So, how do we first get the images. So, right now we have been talking about this entire scheme assuming that we have 2 images One F and one g but then how do you get the images. So, here is a typical experiment right you could think of any experiment I am just using this particular configuration and explain what it is in a second just. So, that you see the analogy with fluids.

So, for instance if you had flow past the body in a in a fluid mechanical problem you would look at you know particles that are seeded in the Upstream flow and then look at where they go as they flow downstream and then from there you get the velocity field and so on. Now an analogous thing can be done when you are deforming solid. So, for instance if you assume that this is a rigid body analogous to your; bluff body and fluid Mechanics for instance.

And you assume that you have a solid that is continuously deforming this is analogous to your fluid now you want to get the deformation field in the solid six components of course as opposed to 2 you do not need the rigid body because by definitions rigid body is not going to deform and it is kept in place right. So, it is not even going to translate and rotate. So, you do not have to worry about it.

So, if you want to do that the first thing you want to keep in mind is you need a 2 dimensional deformation. Now a 2 dimensional deformation or what is called a plane strain deformation means that you cannot afford to have material coming out of the planar view the moment you do that you are going to start recording velocity components or displacement components that are not exactly in plane components the projections of the actual velocity into a plane of viewing of the camera.

So, that will give you incorrect results. So, typical way in which you would make something plain strain is put a constraint. So, here I have a constraint which is a glass plate it is just pressed against the deforming object to make sure that deforming object does not come out of the plane of viewing now just. So, that we are again on the same page I will just make a quick distinction about this situation this is called plane strain.

I think I mentioned this in the introduction this means that you only have components u and v you do not have a w or out of plane displacement component. So, you do not have gradients also out of plane and you must contrast this with a plane stress problem right in solid mechanics you have you have to make a distinction between these 2 to a plane stress problem means that you do not have an out of plane stress component.

But you can have an auto plane displacement component a plane strain problem means that you do not have an out of plane displacement but you can have an out of plane stress and that is the case here because the constraint is applying a force on the object and preventing it from coming out of play right. So, because we are measuring the kinematic parameter which is the strain in this case we do not have to worry about the stress component it makes our life easier.

Anyway so, the constraint is there of course the conditions have to be transparent because you have to look through right. And it is there here and it is fixed to the deforming workpiece to prevent deforming material here to prevent out of plane motion and you have a camera of course and that looks at a particular field of view. And from the camera you get a sequence of images if it is a high speed camera it is going to record at a particular frame rate. But let us say for Simplicity we have 2 images from the sequence which we are going to label one and 2.

So, this is our image pair. So, one will be like a undeformed configuration like we drew before and 2 will be like a deformed configuration. So, we are only going to be looking at strains incrementally from one frame to the other. So, if you have a sequence typically you do not look at frame number one and frame number hundred because the displacements in the stains are going to be very large.

And the Taylor series expansion you write truncating is the first term or the second term is no longer will be valid okay. So, that is why you take incremental images take a sequence and then look at every pair subsequent pair or you know pairs with one left in between and so on. and you get incremental strain values incremental rotations.

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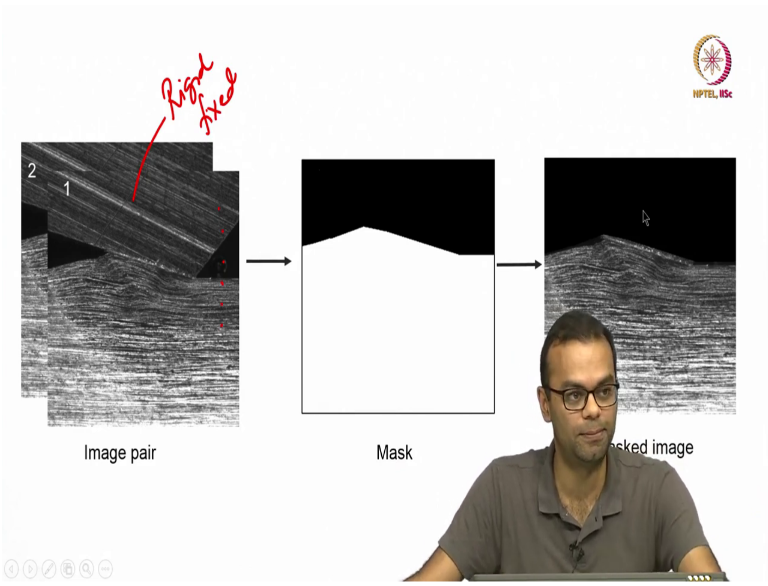

So, we start with the image pair and usually if there are regions that are not of interest for instance here we know that this fellow here is rigid and fixed. So, we know we are not going to get any displacement fields in that region right the region of the rigid object in this case looks like a wedge it is not going to matter. So, whatever we are going to do and we are going to. If we are going to run the algorithm over the entire image remember I said that you have to get it Point by point at every location.

If you do not run it everywhere here you are just wasting your time and your computer's time. So, you might as well take this section out. So, you typically would use something called a mask and black regions are regions there are lot of interest white regions are regions of Interest. So, you just do a point wise multiplication of the image with the mask and you get this and then you can look at a subset and if a subset is fully black you do not have to worry about the subset at all.

So, you can just simply touch the intensity of the subset and see if it is within the region of Interest or not.

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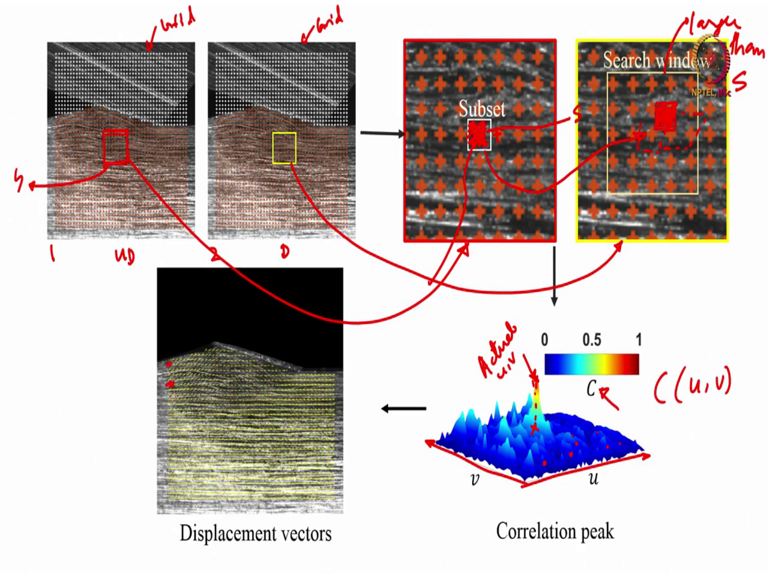

So, here is our typical algorithm goes. So, here is our image one here is our image 2 undeformed and deformed and we have a grid. So, this is a uniform grid that is placed here. And we have a uniform grid also placed here the second grid is a little bit Superfluous but nonetheless for completeness I am including it here now in the grid we already know from the masked region which is the region of interest.

So, we can multiply this grid with the mask again and then the white regions we do not have to do any correlation and in the red regions we have to. So, that makes our life a little easier. So, the first step is to identify a subset. So, you have a subset s in the undeformed configuration if you zoom in the subset test is going to look like this. So, this is one region that we are zooming into that comes up here.

The S is within this and each of these points now you can see are actually represented by plus Marks here. So, in this case we take a small subset that is centered around one grid point okay. So, this is the actual subset s now this is the yellow in the deformed image zoomed in and this is our search region. So, you see the search system is clearly larger than the subset because if you deform the subset.

Let us say ideal match is somewhere over here is going to fall out of the original region which was here okay. So, you need a larger search window the larger the search window the more time you spend in searching but the likelihood of getting a true Peak or a true minimum or a true maximum for C of p is higher. So, once you do this you evaluate the correlation coefficient and this is what C is its normalized.

So, you do not have to worry about the exact value and you notice it is shown here only as a function of u and v not as a function of a six dimensional Vector because then you need a semi-dimensional space to represent it which we cannot obviously do. So, we just restrict ourselves to u and v for now okay just for illustrative purposes. So, if you do that basically you have a set of values of u and v which you are going to search for you take any particular value let us say you take a point here evaluate C.

So, what you are doing now is you are taking the subset you are moving it out translating it to some other location and looking at the overlap. So, is the destination image or the destination thingy below the translated subset is it matching up well with the original subset is the question right that is that is what C of P measures that is what we saw earlier and for this value clearly it does not this value clearly does not does not blah blah blah.

And finally you see that here the overlap is very high. So, this gives you the actual u and v for this particular point. So, you know that at this grid location the U, V is this the horizontal coordinates of this peak now are six dimensional searches you know an algebraic version of this a Motion Physical algebraic question of this and once you get this you have the displacement vectors of course here on the grid we have only evaluated it as a function of u and v.

So, we represent the displacement vectors at each of these points you can see the little arrows at each grid point superimposed over here.

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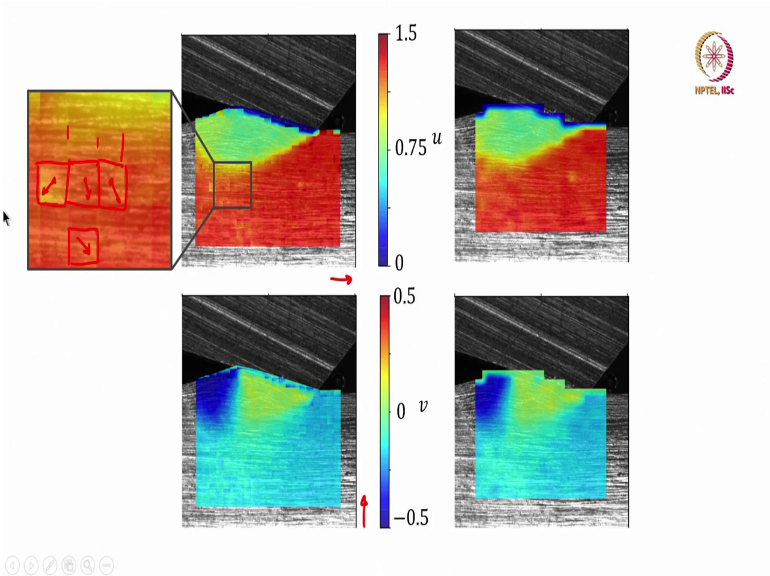

This is what the actual values look like now there is a small problem that comes up in the case of solids which is not there in the case of fluids because you are working with these gradients now in the fluid case since the particles are by themselves and you do PIV typically you do not have to worry about continuity issues from one frame to the other however in a solid case if you zoom in to this field you will see the displacement has jumps between adjacent windows.

This is the full reconstruction with the six component vectors. So, the displacement is not constant inside each subset you can see there is a variation going from here to the bottom left as a variation going within here for instance and even in this window you can see that there is a variation going this way and so on. but the key thing is that the variation is not continuous and you can see these boundaries very clearly in the image right.

Now if you think back a little bit there is nothing that we do to enforce that the displacement is continued from one window to the next right we are only evaluating things at each grid location we take the window there subset we go and search for it and do our iteration scheme we get some values and we put it there the next one we do the same thing and so on.. So, there is really no guarantee that the displacements are reverse the gradients will be continuous from one point to the other right.

So, you will get typically a blocky field like this everywhere this is of course the x component U is the x component and V is the y component and they're all normalized the scale bars are in pixels. So, it does not matter what the exact value is as long as you can see the variation for now and um. So, in order to get a better version of the actual displacement field you have to do some post processing.

And typically a type of post processing you would do is do linear interpolation or you know higher order interpolation if you want smoother fields now the only problem the main problem with this of course is that when you do interpolation you are losing some information. So, remember at each of these points this is accurately determined from an actual cross correlation match right for a six dimensional vector.

But when you do interpolation you are losing that information which is recovered from the search scheme. And so, how much interpolation you do is a trade-off between how much you are willing to sacrifice at the expense of a smoother field right. So, that really constitutes the basic a set of things that you will do for getting a typical displacement field in typical strain field.

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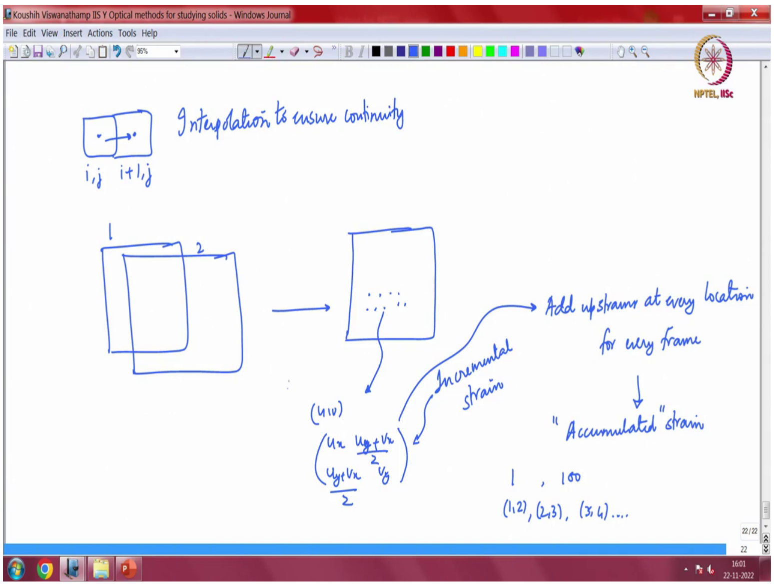

Now the strain field is analogous I am not going to show you that here. But the method for evaluating The Strain fields the method for evaluating The Strain fields are analogous once you have the strain fields you will have this discontinuity between cell i and cell i plus 1 for instance, J or window or subset ions are located at i, j located I plus 1,j for instance and you have to do some interpolation to ensure continuity and typically what you will have to do is remember we said that there are 2 images that you do this for.

I showed your image pair one and 2 and if you have a sequence you get a strain field at every point some value of the strain tensor basically. So, now you will get the vector UV and the strain tensor u x u y plus v x this right at every point but remember this is the incremental strain. So, you are actually using another image every time to be the reference configuration or the undeformed configuration as you compute the Strain.

So, it really is the strain between 2 frames or between those 2 successive images it is not the strain from the undeformed original undeformed configuration to the present time right. So, what you actually have to do is to add up strains at every location for every frame and this gives you what is usually called the accumulated train. So, if you have image 1 and image hundred you do incremental strain between one 2 between 2 three between three four and so on.

And then you add up the strains depending on where the particular material point is gone you add the strain in the subsequent pair to that particular value and then you keep adding up as you go along. So, this will give you not only the strain at any point but the strain as a function of the deformation during the deformation itself. So, that is a typical implementation now there are some open source codes that you can use for doing some of these calculations if you are not up to writing your own DIC scheme.

So, what we will do in the next session is we will have a little demonstration of one of these packages and a typical set of images showing you in a lab setting how the images are obtained and also the images are processed and how you can use that package for instance for getting these displacement constraint Fields. So, we will start off with that in the next session.