Optical Methods for Solid and Fluid Mechanics
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Lecture - 25
Basics of Digital Image Correlation

Now that we know the various steps we are going to follow in our discussion of optical methods for solid Mechanics for studying deformation in solid mechanics we will start with the basic idea behind Digital Image correlation.

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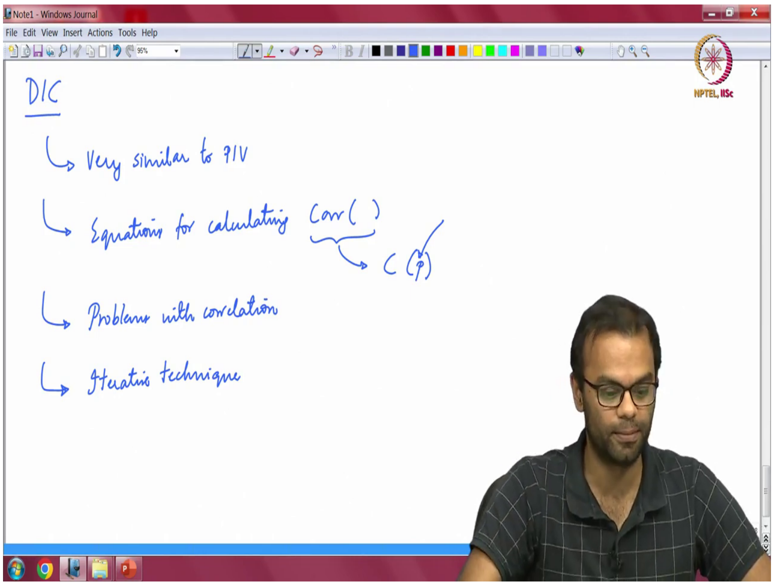

Now we have already seen formulation that is analogous to this in the context of fluids which is PIV. So, when we start I will tell you how this is different from PIV we have already touched upon some of this last time specifically that you have strain information and not just strain rate information as in the case of fluid. So, that has practical consequences that we will see now and I will also work out I will derive some of the equations for calculating this thing called the cross correlation coefficient.

You have probably seen this before in the context of PIV here I will refer to it as C. I will give you the definitions a little later on and it will be parameterized in terms of a vector as opposed to just positions and displacements like you had in fluid. So, some of those differences will be clear quantitatively clear as we work out these details I will also talk about problems with Computing this correlation maybe I will discuss that first.

So, that it gives you some motivation for why we need this type of elaborate scheme and then we will develop an iterative technique at the very end for calculating C of p and then from there determining what the minimum value is and what the corresponding p is. So, we will set it up and then in the next session we will look at the actual practical steps that go into a typical implementation once you know what the equations are about.

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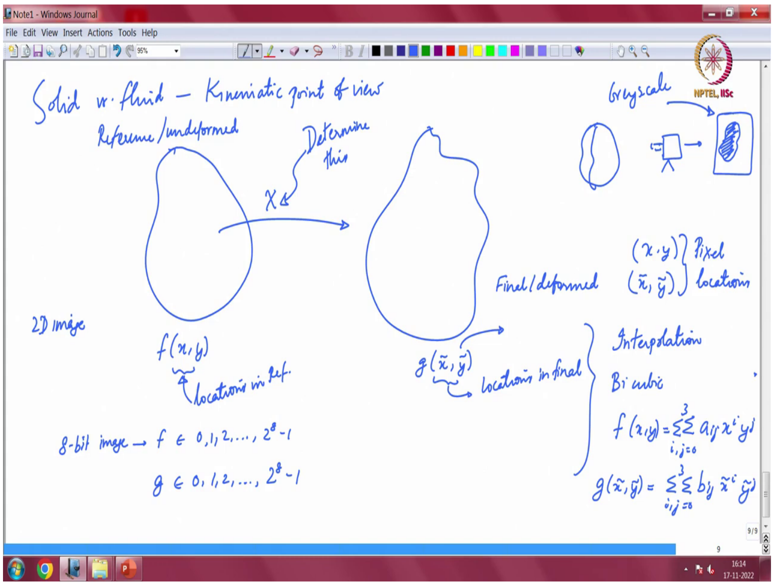

So, let me start with the basic solid versus fluid business now purely from a kinematic point of view. So, let us say we have an object that looks like this and it is deformed into some other shape okay. So, my object let us say is two dimensional okay and the I have an image of the object before it was different I have an image of the object after the deformation.

And just so, that we have some concrete notation I am going to give some symbols here. So, I am going to define this as an intensity. So, let us say if your object is some three dimensional object like this when you look at it with a camera then you are going to get a two dimensional image with some intensity variation as a function of x Y location right as function the pixel location. So, that is basically what I am going to call f f of xy.

So, if you have a 8 bit image F will be integers from 0 to 256 if you have a 10 bit image it will be 0 to 1 0 2 4 and so on. and the deformed image I am going to call G okay just to distinguish between the two I am going to use x and Y for locations in this image the first image this is called the reference or the undeformed image and this one is called the final or the deformed image.

And just so, that we make a distinction between which points we are referring to whether it is in the reference image or as in the final image we are going to use xy for the reference and x tilde y tilde for the final. So, these are locations in reference and these are coordinates or locations in the final. Now typically like I said if you have an 8-bit image f is an integer. So, it is either 0 1 2 blah blah blah up to 2 to the power of 8 minus 1.

And likewise if you have a higher bit image you will have more values for f but they are all discrete right and the same applies to G these are the only values at each of these locations point locations can take okay and you have also seen some basic Notions of what a discrete image looks like. So, the x, y are pixel locations typically. So, xy and x tilde y tilde are typically pixel locations in a digital image.

So, that means there are also integers they go from one two three four up to the size of the image if it is a 500 by 500 then those are the limits on x and Y and. So, you have three integers x y and the actual value or the intensity of the image. Now we are assuming of course that this is a gray scale image will work only with grayscale images okay. Now just so, that we can do some calculus.

So, we can take derivatives of the intensity derivatives of the values we will make an approximation. So, this is typical of most of these image processing schemes will use an interpolation method and the method we are going to use here is by cubic which means I will represent f as f of x y is like this okay this is a double summation I, J equal to 0 1 2 and 3 and likewise for g. So, this is the basic framework we are going to use.

So, you have a map let us call this map Chi which takes the undeformed image and creates a deformed image out of it our whole aim in this DIC is to look at the two images and determine this map okay that is the basic problem we have to solve and all the correlation coefficient calculations minimization all the schemes iteration etcetera is all finely towards doing this okay that is the end goal.

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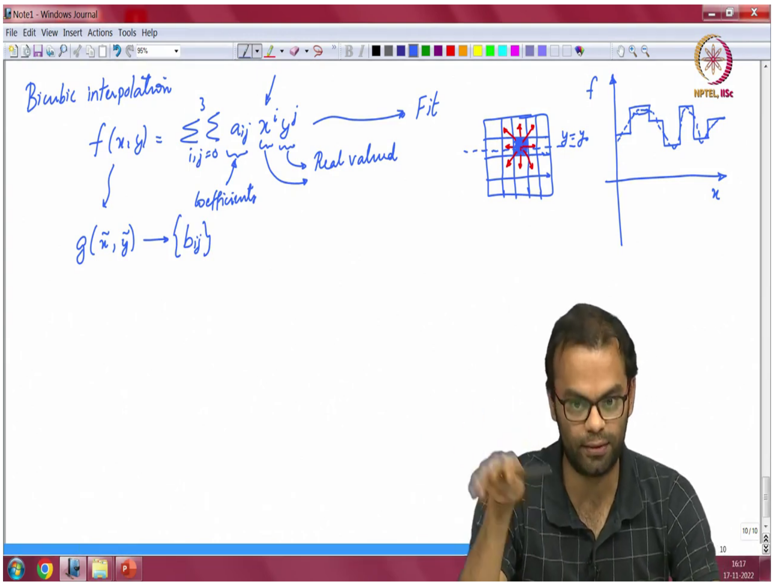

So, now let us rewrite this by cubic population explicitly again you will see why this is needed because you will have to take derivatives of the image intensity to be able to evaluate the cross correlation coefficient and if you have discrete values the higher order derivatives you take the poorer your approximation becomes the fewer points you will have. So, doing interpolation saves you this trouble.

So, this typically these are coefficients that are determined from the image intensity and these. Now are not integers but are real valued. So, they go from 0 to the maximum size of the image without any integer values. So, the continuous and. So, that allows us to Define intensities between pixels and things like that okay the same thing applies to g. Now this is basically nothing but a fit right.

So, you take sets of image sets of pixels around a certain location you have x and Y going let us say you have this part of the image you have some intensity here but you want to basically interpolate from here all the way to here from here all the way till here from here all the way till here from here all the way till here and so on. So, you want to get a continuous variation in the intensity if you were to plot the image intensity along this line. So, the non interpolated value will look let us say this is f and this is along this line let us say this is y equal to y naught.

So, we are going along x. So, if you plotted x in the uninterpolated image it will look like this Maybe right but then. Now our aim of being able to do this is to use cubic interpolation to capture this variation okay and. So, this allows you to take second derivatives and you will see second derivatives will become important in the iteration scheme like I mentioned okay. So, even though f is an integer valued function we are now going to assume that it is a continuous function that can take values on continuous coordinates.

So, the three integer sets that we had before are now all real numbers okay for the rest of the formulation. So, please keep that in mind and the same thing of course like I said applies also to g and this is done in terms of the b ij. So, the bij are all fits aij are all fits they have to be determined for each image completely you will have a whole set of a ij b ij from each location you will have a set of them good.

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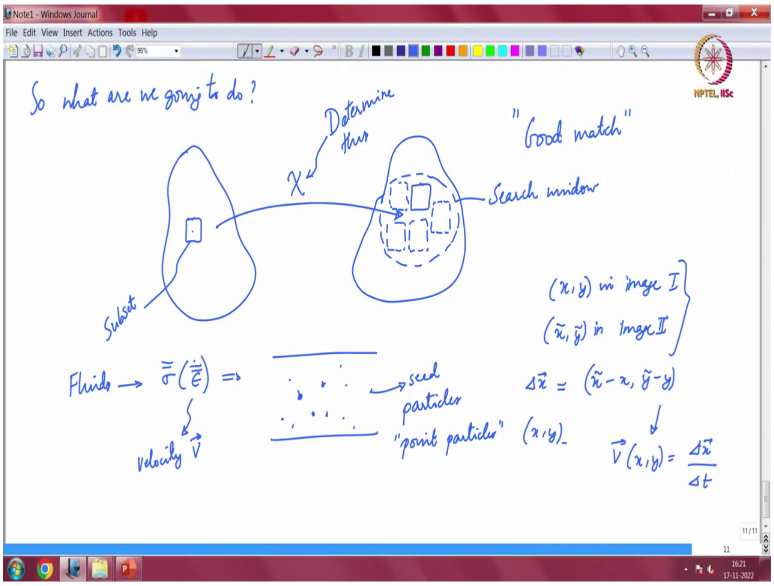

So, what are we going to do if you recall I drew this deferment potato image and this is chi our aim again is to determine this. And the way in which we will do it if you think about it naively is you take small sections of this and you match with candidate sections here everywhere in the deformed image. And wherever you get a good match right whenever you get a good match it is very likely that this has been deformed into the matching subset here.

So, this is called a subset and this region is usually referred to as a search window again many of these things are similar at least in construct in a broad construct for PIV except for how you look and how you do the searching okay. So, remember we said in fluids you have Sigma as a function of Epsilon Dot right. So, this implies that instead of looking at chunks of fluid what you will typically do as you would do in a PIV scheme is you would seed particles OK the aim is again to determine the velocity field I think I stressed upon this at the very beginning for a fluid case.

So, you seed particles in the fluid and then you image and see where the particles are going and these particles are assumed to be Point particles typically. So, the point particles are just characterized by that location right. So, you only have x, y. So, you look at one frame see where the point particle is you look at the next frame see where the point particle is. So, if it is an x y in frame 1 or in one image let us let me say image one and if it is at x tilde y tilde in image two then the displacement between the two images is x tilde minus x y tilde minus y.

So, this gives the Delta x bar for the particle for one particular particle and if you know the time distance between these two images depending on the frame rate at which your Imaging was done then you get the velocity at that particular location x, y as Delta x till Delta x bar divided by delta T right this is typically what you will do in a PIV scheme and you do this for every single particle and presumably you have lots of particles then you will calculate the velocity field and so, that will give you the flow field and the fluid right.

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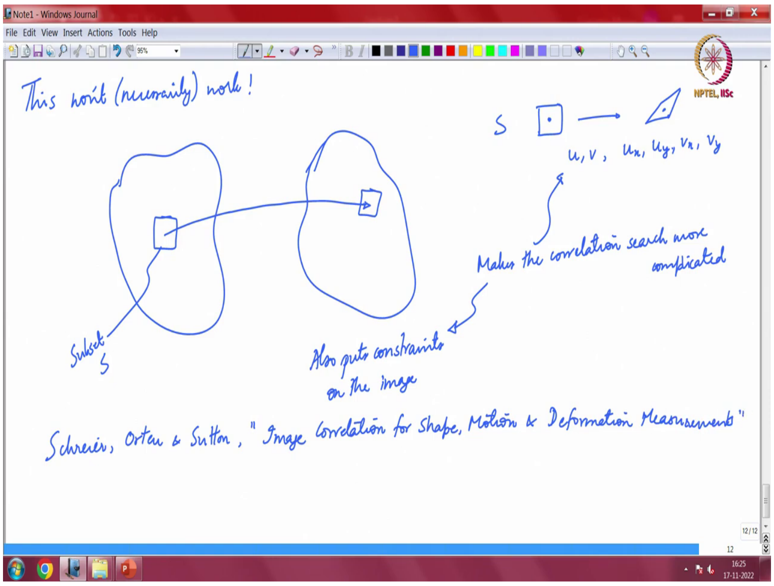

Now notice we said that for solids because of the strain dependence this scheme will not necessarily work the reason being if you have a region here in a solid and you have a corresponding region here the solid is not just the section of the soil is not is getting translated the u and v are the x components that we had the x tilde. So, if I call this u and I call this V just say you know for velocity components the u and v are not sufficient to determine the shape of the deformed subset.

So, the subset not only is getting translated but it is also getting rotated presumably it is also getting stretched it could also be getting shared and so on. right. So, the subset if I call this s. So, s if it was like this in the undeformed configuration in the different configuration it could look like this under a different location. So, not only do you have to figure out where it is moved where the center of a subset is moved but you also have to figure out what shape it. Now has taken in the deformed image right.

So, now you automatically realize that the search scheme is not just looking for uv but is also looking for gradients of v and u and these also become important because the gradients are what tell you what the final shape is. So, this makes the correlation search more complicated. So, that is something you want to keep in mind. Now not only does this make the search more complicated but it also makes the image you know it puts some stringent constraints on the actual image itself.

So, you cannot always do this type of correlation I will give you a couple of examples. Now if you do not have certain types of features in the deformed image to look for on the undeformed image or vice versa right. When you do correlation so it puts these constraints on what you have what you need to have in the deforming body types of features you need to have in the deforming body that you need to look for finally okay and this problem does not come up in fluid mechanics it does not come up typically in PIV.

Before I illustrate this I will show you a couple of images the images are taken from a book and I will also give you the reference for that book the formulation that I will present is also taken from the same book it is by Scheirer Ortan and Sutton it is a very standard reference book it is called image correlation for shape motion and deformation measurements. This is a very standard reference you will find most of the information that I am discussing in some form in this book.

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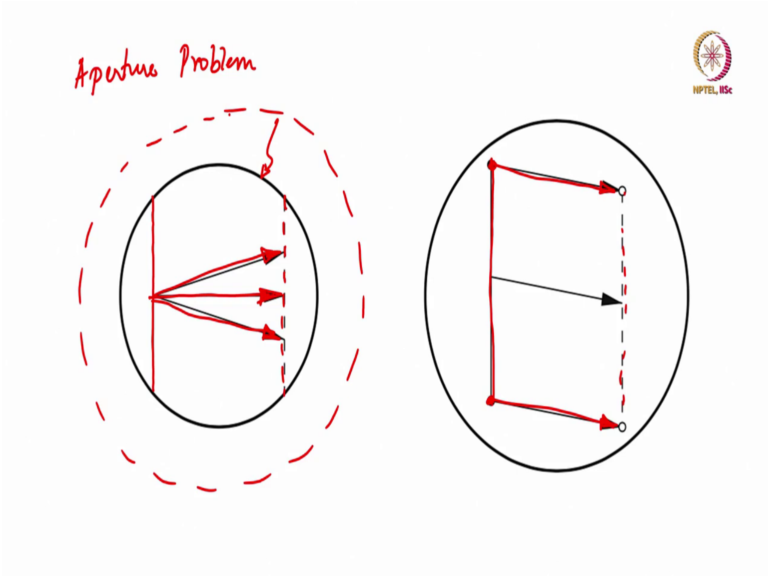

I will show you a couple of images that are also borrowed from this source that discuss what type of constraints you need when performing this type of correlation calculation or this type of matching on subsets in a solid. So, here is the first example. So, this is called the aperture problem this is the first issue that you will encounter the name is sort of self-explanatory.

I think which aperture basically if you you know if you have worked with the camera before you know aperture is the size of the opening through which light is allowed to enter and go into the sensor. So, assume that you have this is your camera. So, this is your lens and you are seeing through this we will do not worry about the right hand side image first I will talk about the left hand side image.

So, let us say you have a line the left hand side image in the undeformed configuration and the line in the deform configuration is here the dashed line. Now your objective is to find out what the displacement of the line is right. Now the obvious thing that you would do is say okay the line is moved to the right like this like it indicates here. But because the aperture you have is very small there is no way you can tell if it moved horizontally like it shown here or if it moved up and horizontally it moved down and horizontally.

Now notice that again in PIV since you are looking at typically tracking points you do not have this issue because the point is inside the field of view you know where it is moved as a direct correlation and you can actually calculate what the displacement is uniquely however here since the line is going out of the field of view there is no way to tell exactly what the displacement is in a unique way right.

So, this is called an aperture problem the solution is fairly mundane and simple you just make the aperture larger in which case hopefully for a reasonably large enough aperture the end points of the lane will also. Now be within the field of view and. Now this issue will not come up right. So, if you know that it is deformed like this you know that the displacement has to be like this and likewise if there is a rotation you know that there is a translation and rotation simply because you can correlate what the endpoints have done.

And then you can correlate every single point on the line with an analogous point on the deformed configuration. So, this is the first problem you will encounter that you have to keep in mind while working with DIC with solids.

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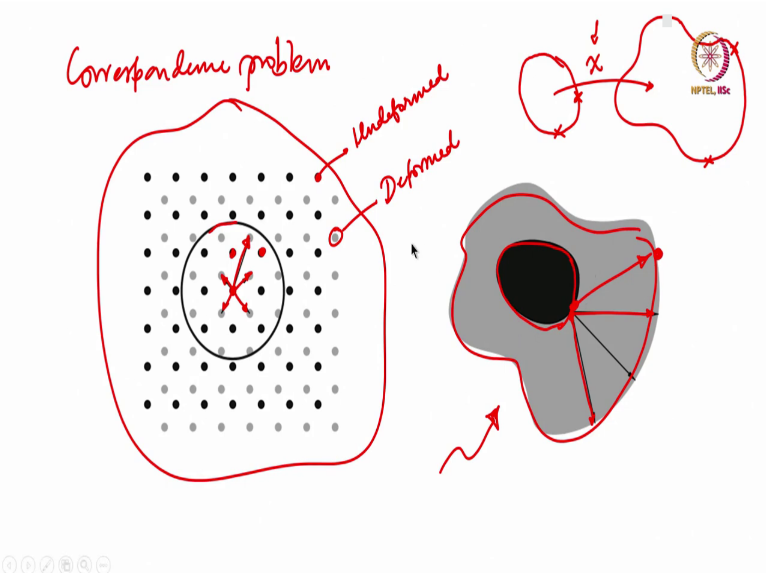

The second problem is the following this is called the correspondence problem and it typically is the following. So, let us say you have a uniform structure on the deforming object. So, let us say your object is like this and it is fairly large and your aperture is this the circle that shown here. And so, the aperture is picking up one part of the object and on the object you have a uniform grid that is comprised of these gray and black dots.

And on deforming you know that gray dot has gone some other location black dot has gone to some other location right the only problem here is again it is similar to the aperture problem which is that there are an infinite number of gray dots and. So, correlating one grade dot uniquely with another gray dot on an infinite lattice or large lattice which is not whose endpoints you don't know also poses a uniqueness issue.

So, you cannot tell for example if you have gone let us say if the blacks correspond to the original configuration the undeformed configuration reference configuration the grays are the deformed configuration then if the gray line the black lattice let us say is the undeformed and the gray is the deformed. So, within the field of view you know that the black has to go to one of the Grays right because that is the new image you have but you do not know which of the Grays is going to right.

So, for example the central black we do not know if it corresponds to this gray or this gray or this gray or this gray because you could have a solution for all of them or even to this gray unless you know the end of the lattice or the end of this grid there is no way you can tell right. So, this correspondence problem is also something that you will come up against. So, we know that you need to have some finite you need to have some end points or some unique points within the field of view which is the aperture problem.

You also know that you cannot have a uniform grid and look at a uniform grid and make a unique correspondence. So, these two issues are there. So, if you do away with any of these features if you say okay you know I do not want any features can I just look at the object and say something then again you are confronted with an analogous problem which is shown in the right image. So, here for example if you know that the black egg shape has deformed into this larger amiboid shape the question is.

Now is this point does this one correspond to this point or does it correspond to this point and correspond to this point and whatnot right. So, there is no unique map one-to-one map from a point over here to a point over here. So, if you thought if I drew this back in our; if I carefully draw this back. So, this is the black oval that is in the middle and this is the gray thing that is outside and if you recall I have that map guy remember our aim is to find Chi right this map by looking at these two right.

Now the if there is nothing that tells you that this point somehow has gone to this point or this point has gone to this point there is no way you can get a unique Chi out of this right. So, this is again a analogous to the correspondence from that we saw earlier. So, what do we do to get around this these two issues. So, first the opposite problem like I said is easy to reasonably easy to correct you just make the aperture larger include features that are within the field of view the correspondence problem is gotten rid of by using something called a speckle pattern.

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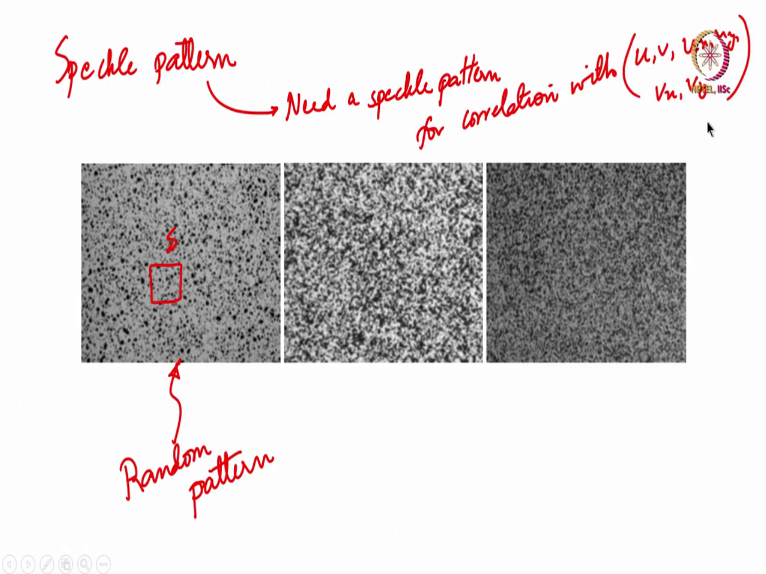

Now this is typically these are three typical speckle patterns that you can see here and this is typically a random pattern okay the random pattern alleviates the correspondence issue and the fact that you have sections you can for example take a subset like this if I call this s like our original subset you have lots of little features here that you can easily map into a deformed image and get some estimate not only of the translation but also hopefully of the rotation and shear and stretch and things like that.

So, that is basically the starting point. So, you need a speckle pattern for correlation evaluation with not just displacement but also its gradius. So, that has to be there. So, any DIC scheme should start with a cycle pattern.

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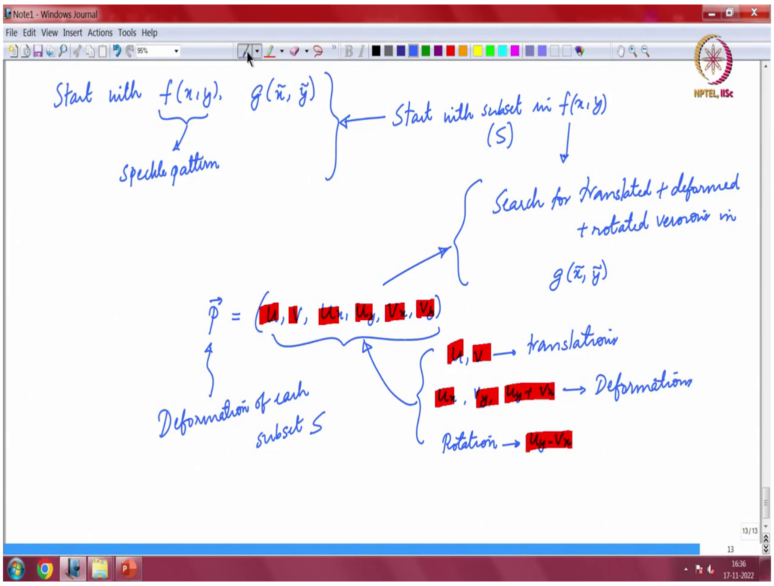

So, if we go back and sort of summarize what we have we have to start with f xyand g of x tilde y tilde and this will have a speckle pattern on top. There is a random pattern usually people will you know there are all sorts of ways to put special patterns you can use a spray paint or you can use some abrasion and so on. anything that introduces random features that that can be distinguished in the form of distribution of points considered the speckle.

I suppose and now you start with a subset in F and you look for I should say search more technical term I suppose you search for translated and deformed and rotated versions in the g. So, you have an image the cycle pattern you have bicubic interpolation which is assumed I am going to assume that. So, you can take derivatives you have these two sets of functions and you start with one small part of F.

And then you look for a match within codes for f in a translated deformed rotated manner in g how you evaluate that match is the subject of you know some formulation I will give you that formulation in a bit. But that is the basic idea once you know that this will give you basically something that will call a vector p it has six components UV u x u y p x v y okay. So, this vector characterizes the deformation of each subset.

So, for each subset call it s like this if you do this for every single subset inside the image the undeformed image you can get a full displacement field for those individual subsets in terms of these six components okay and you notice that u v of course if it is not already obvious uv correspond to the translations the deformations correspond to u x v y u y plus v x right. So, these are typically if you take a half for one of these this will become the strain components you have three components for a strain tensor in two dimensions and the rotation will be u y minus v x.

So, your four components are here in terms of these four right. So, once you have one two three four five six for determining one two three four five six okay. So, that is basically what a typical DIC scheme entails I will get that formulation started and then we will complete it in the next session.

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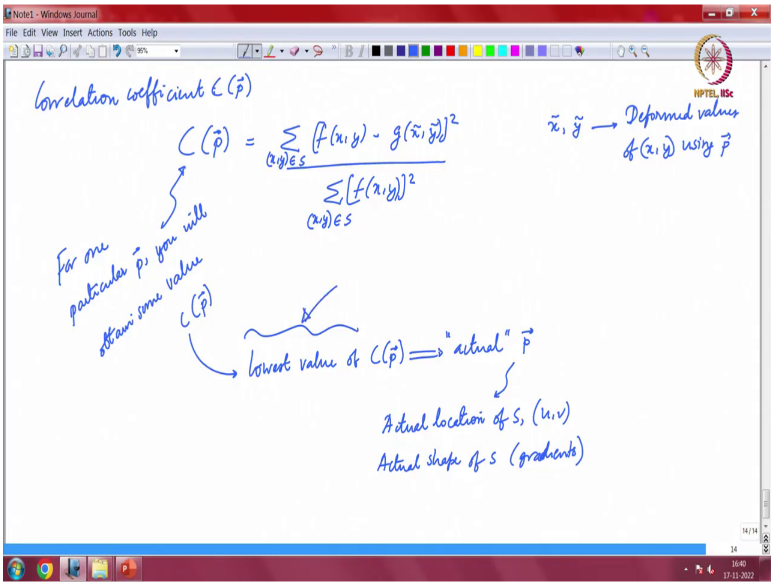

So, that we do not rush through the entire thing the formulation is basically dependent on cross correlation coefficient there are multiple ways of defining across correlation coefficient we will use this formulation. So, we will call it C of P again is that Vector P that I defined on the previous slide and you look for the summation is done for all xy inside the subset the subset right.

So, you have discretized set of values for x and y at every single value you calculate the C of P correspondingly by assuming again that x tilde y tilde for the deformed values of xy using p. So, you translate from x Y using u v and then you change shape using u x u i v x v y and so on. Like I discussed earlier and then you basically calculate this value for each combination of P right. So, given one p so, given six components one single six Road Vector for instance you will get one value of C.

So, for one particular P bar you will obtain some value C of P bar. So, if you change P bar by little bit you give different values for the six components you will get a different value of C and so on. So, you can keep doing this at infinite term and the lowest or I should say extremum value of c bar of a c of P bar this gives the actual or an estimate of the actual Peak okay just like you would do correlation minimization in PIV this is analogous but only. Now you have six components instead of two.

And then this will give you the actual location of s and the actual shape location of course comes from UV in the actual shape of s in the deformed configuration that comes from the gradients. So, if you were very naive you could just go and use you know all sorts of combinations for the six components if p and then calculate C and keep track of every single one of them and then see which is the lowest and that will give you the value that is a naive way to do it of course that is not the most intelligent way to do it.

So, that is a very simple scheme that we can derive for evaluating this lowest value ok. So, we will develop an iterative scheme for doing this. So, that is what I am going to work out in the next session and we will take up we will we will continue from where we have left off over here in developing this scheme. And I will also show you a typical sequence of implementations I will show you a sequence of images on how each step makes gives you slightly more information slightly more information.

What you are doing visually when you are actually doing the search in the iterative scheme and how you will do a little bit of post processing to get a continuous field continuous gradients and things like that.