Optical Methods for Solid and Fluid Mechanics
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Lecture - 24
Introduction to Optical Methods for Solids

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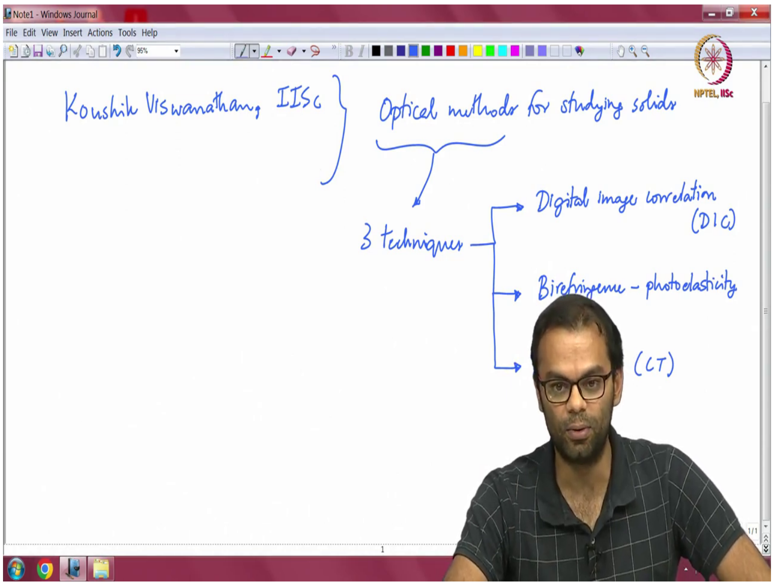

Welcome to the second part of this course on optical methods in solids and fluid mechanics. My name is Kaushik Vishwanathan I am also a faculty at the Indian Institute of Science. So, so far in this course we have been looking mainly at techniques for understanding flows fluid flows specifically which I suppose my colleague Alok has covered. In this part of the course over the next 10 hours or so, we are going to look predominantly at Optical methods that are tailored for studying deformations.

There is a difference between deformations in solids and fluids most of you are probably intuitively aware of this. We will make some of that clear a little bit specific and then we will look at a suite of techniques specifically three techniques for understanding deformation in solids. As you have probably seen from the outline these techniques are the following. The first is what is called Digital Image correlation or DIC.

The second one is based on a principle called birefringence and the method itself is called photoelasticity and the third technique which I guess most of you are familiar with but from a very different context is called tomography. So, or specifically computer tomography what people would normally refer to as CT, C being computed right. So, you have probably seen computer tomography when doing scans and so on CT scans x-ray CT scans.

So, we will talk about some of the methodology that goes behind this some of the theory and the formulation behind these techniques and then we will see how they are applicable to studying deformations and solids. So, that is broadly the outline of the topics I am going to cover will spend about three to four sessions per technique. So, that will give us some chance to go into some of the details before we see how they are applied in a particular context.

And what type of information you can get with each of these techniques. Now they are all complementary. So, in the sense that the information you can get from DIC is very similar very different from the type of information you can get from computer tomography and that is very different from the type of information you can get from photoelasticity. So, in that sense they are all complementary.

The techniques are not always applicable to all materials unlike fluids I should say the wider variety of solids and wider variety of deformations that you can come across in solid mechanics and. So, depending on what the end application is what the end material is what the end field is that you are looking for one of these techniques is probably best suited and we will go through some of those decisions systematically over the course of this next 10 hours.

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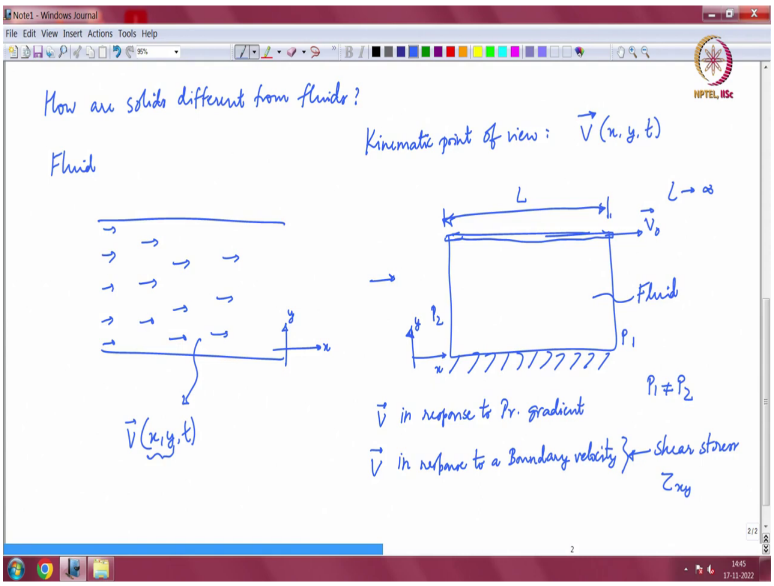

So, before I start lets very quickly go down and see how are solids different from fluids.. Now obviously this is a very naive question you know if you have go back to your high school textbook or something you will see that solids cleat in their shape liquids take the shape of a container and blah blah blah and so on. But there is a very specific way. Now that we are sort of familiar with the Machinery of tensor calculus and Vector calculus matrices linear a one some of which was covered a little earlier in this course.

We can give a very specific distinction between these 2 types of materials. Now usually when you talk about a fluid let us say you have a channel and you have fluid that is flowing here the flow corresponds to a velocity field. So, you have a velocity field this is let us say a 2 dimensional. So, you have x and you have y and you have a vector that gives you the velocity of the fluid particle at a particular location xy at any point of time right. So, we can also have a time dependent velocity Vector field.

And this basically characterizes the flow. So, from a kinematic purely kinematic point of view the only thing you need for a fluid in this case 2 dimensional three dimensions you will have one more variable you will have one more component for the vector is this velocity field right. So, if you know the velocity field you know everything about the kinematics of the fluid.

This is of course assuming that the fluid is Newtonian that is not viscoelastic and so on. but nonetheless. Now if you step back a little bit and you ask the; question if this fluid started like this. So, initially it was all stationary. So, you have a top wall in the bottom wall and this Channel or the length of the channel is L which is very large let us say very large Channel and you have a fluid here.

If you want to get a flow in the fluid there are usually 2 configurations that you probably see in multiple times in the fluid mechanics course the first is of course you can apply a gradient in the pressure in the X direction again this is X this is why the origin of coordinates do not matter and you can apply a pressure gradient. So, if the pressure here is P 1 pressure here is P 2 and you have P 1 not equal to P 2.

Then you can get the fluid to flow either left to right or right left depending on which one is larger right. So, you get a flow in response to a gradient in the pressure that is one way of course. And the other way you have probably seen a quick flow solution plain coat flow is you take one of the walls let us say the top wall in this case and you apply a velocity to it I will just call this V 0 bar to distinguish it from the bulk velocity.

So, if you do this then you can also get a velocity in the fluid. So, you get the fluid to flow in response to a boundary velocity and this boundary velocity is applied in the form of a Shear. So, you are sharing the fluid. So, you are applying a shear stress on the top wall. So, I will call this Tau in this case it is in the XY Direction that is the only Shear component for a 2d system and so you apply a shear stress and the fluid will flow.

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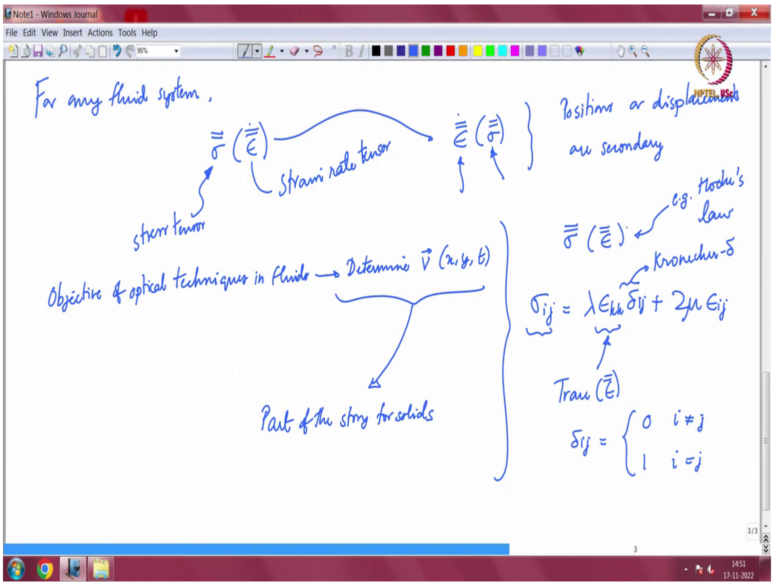

So, in general for a for any fluid system this is also applicable to gases by the way when I say fluid it could be liquid could be a gas. For any fluid system the stress tensor is a function of this train rate tensor to make my notation consistent. So, this is your train rate tensor and this is your stress tensor it could be any of the substances could be any of the strain rate tensors depending on which configuration you are looking at but those details are not important for us basically the fluid flows or you know you could also think of this as analogous as this.

So, the fluid flows in response to an applied stress shear stress boundary velocity pressure gradient and so on. So, the idea of positions or displacements are secondary. So, the velocities are primary the idea of having a displacement you never talk about a fluid having a particular strain right because the strain does not contribute to any energy change in the fluid because if you apply a stress you do not get a strain you get flow right.

So, you get dissipation directly there is no stored energy again assuming these are all Newtonian fluids they are not viscolastic and cannot store energy and so on. So, the main objective of most Optical techniques in fluids is to determine the velocity. So, everything is centered around velocity determination PIV PTV all those techniques are based on the fact that you need to determine the velocity.

Once you know the velocity you know this you can sort of infer what the stresses are by taking gradients and evaluating strain rate tensors and so on. Now this this is only part of the story for solids the reason being in a solid the stress tensor is usually a function of The Strain tensor not the strain rate tensor. So, this is for example a very well known example of this is Hooke's law which you have probably seen before.

Where you have in hooks law you know that the stress tensor with indices i j is related to The Strain tensor right and this of course is the trace of the strain tensor and this is the Kronecker Delta function just so, that we are all on the same page the Kronecker Delta function is 0 if I is not equal to J and is one if I is equal to J. So, it is basically like an identity Matrix. And it does not matter if you have not seen this before at least you know about Hook flow I am guessing the force is proportional to the displacement for instance.

So, the stress is proportional to the strain and that tells you that the stress is a function of the strain tense.

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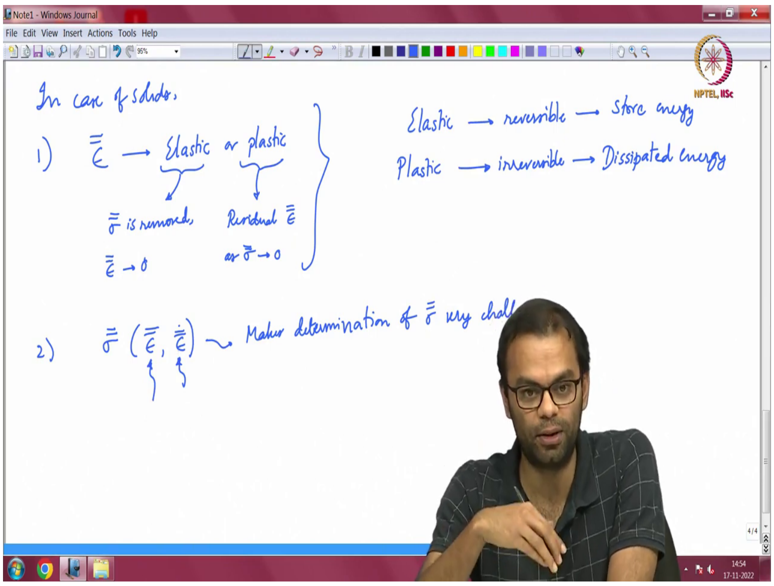

Now the thing that makes solids a little bit more complex to understand in terms of their deformation is the following. So, in case of solids the strain itself can be either an elastic strain or a plastic strain ok., this means that when the Sigma is removed this guy also comes back to zero that is what the last extraneous the plastic strain means that there is some residual Epsilon as the applied load goes to zero right.

So, now you have a situation where you have to distinguish or at least be able to distinguish between elastic strains and plastic strains or an elastic deformation and a plastic deformation you did not have this issue in the case of a fluid because everything was the equivalent of a plastic deformation. Now the main distinguishing factor of course is that elastic deformations are reversible which means they store energy.

So, if you have an elastic strain and you have a corresponding stress developed in the object in the material or the solid then the potential energy of the solid goes up but on the other hand you can have plastic strains after stresses are removed. So, you do not have a strain energy you have some energy that is dissipated in the process of plastic deformation. So, this is dissipated energy. So, this distinction has to be made.

So, there are some techniques for instance photoelasticity that we talk about has the word elastic in the title right. So, you cannot use it directly for situations where the strains are plastic but you have gone beyond the yield limit for instance right. So, you have to be a little careful about the nature of the deformation where it is coming from what you know if it is reversible irreversible and so on.

But in addition to this so, this is first complication in addition to this there is an additional complication which is that the strain sorry the stress can also be a function not just of the strain but also of the strain rate so this makes determination of Sigma very challenging ok. So, even if you can get even if you get information about the kinematics from either the strain or the strain rate field it makes it very difficult for you to put that together and determine the stress. So, that is another challenge that you will have to put up with in the case of solids.

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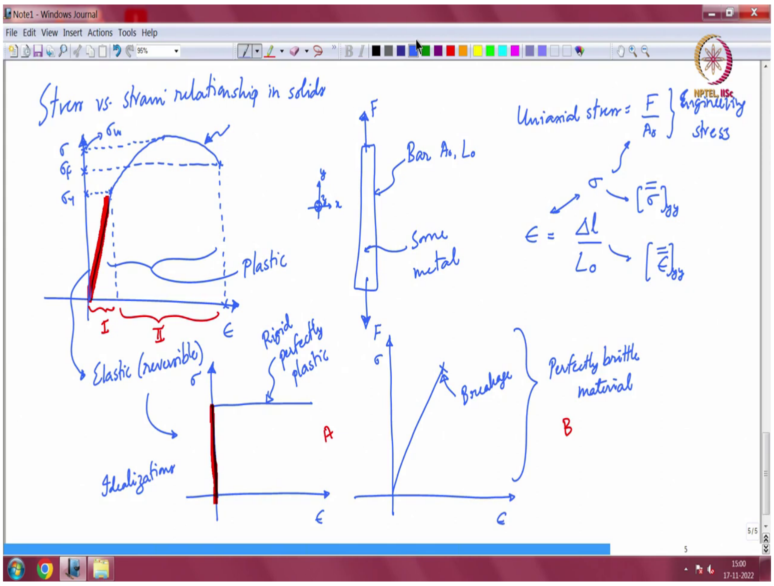

Now let us take a step back and just look at a typical stress versus strained relationship in solids right and again this if you think about it a little bit this goes back to our you know High School notion of solids being able to retain shape and liquid is not being able to retain shape. So, if you apply stress liquids will flow. So, they will not create they will create a strain rate and response compared to solids.

But let us take a look at a typical what is called a stress strain curve right again this is something you have probably seen before but just in the interest of completion I am going to put it out. So, that we know that everybody is on the same page ok. So, this is your stress this is your strain this let us say I have a bar that looks like this and I am pulling this bar in tension right.

So, this is a bar it has an initial area a cross section area a naught and initial length L naught and then I am applying a load here let us say this is the y axis the x-axis and the z-axis is out of the board out of the screen and if I calculate the what is called the uniaxial stress. This is just f divided by a naught this is called the engineering stress if I calculate this and I denote this by Sigma and I look at a plot of Sigma versus Epsilon.

I am now defining Epsilon as the engineering strain. So, as I pull the length increases right in tension in the y direction and I take the original length I have a capital l take the original l l naught original length L naught and then I divide the change in length by that and then I plot these 2 quantities right. So, this is if you look at your actual stress tensor this is Sigma YY and if you actually look at an actual full strain tensor then this guy is Epsilon YY.

So, if I plot this here I am going to get curve that looks like this typically if let us say my material were a piece of metal let us say this is some metal is going to look like this and again I am sure most of you have seen this before this is of course the yield Point Sigma y you have a maximum here this is the ultimate tensile strength we will call it Sigma u and this is the failure strain or the failure load Sigma f.

Now all of the elastic stuff happens before the yield or in the proportional limit for instance before yielding actually occurs. So, all of this remains elastic. So, these are elastic strains. So, for instance if you could do photo accessory you can do it here right these are elastic and the rest of it is all plastic ok. So, it is irreversible. Now if you take this curve. So, this is a typical curve that we all know and love if you take this curve.

You can make certain approximations depending on how big the elastic range is depending on what the modulus is and depending on how much ductility is there in the material and so on. So, here are 2 idealizations the first one is what we will call this. So, this is a what is called a rigid perfectly plastic material. So, basically what this says is that the elastic strain is almost zero. So, if the elastic region for your particular material is small you can approximate it like this and the material does not have strain hardening.

So, it does not this curve is sort of flat. So, if it is reasonably flat you can make it look like a step function like it is here and this model is idealized model is called the rigid perfectly plastic model that is one end of the spectrum. The other end of the spectrum is of course this end of the spectrum is this end of the spectrum is when the material can deform quite a bit. So, it is very ductile you can use rigid perfectly plastic assumption.

At the other end of the spectrum you have a material that does not deform plastically enough and that basically goes like this breaks. So, this is your Sigma this is your Epsilon this is a point at which breakage occurs this is called a perfectly brittle material. So, it does not show any plastic deformation it is completely elastic and it just breaks. Now depending on how small this region is and how small or large this region is we can use either this or this to approximate the behaviour.

So, that is the basic idea. So, we will see that these models will come in handy at some point and we will see how to use them when we analyze stress fields. So, just something to keep in mind as we go forward.

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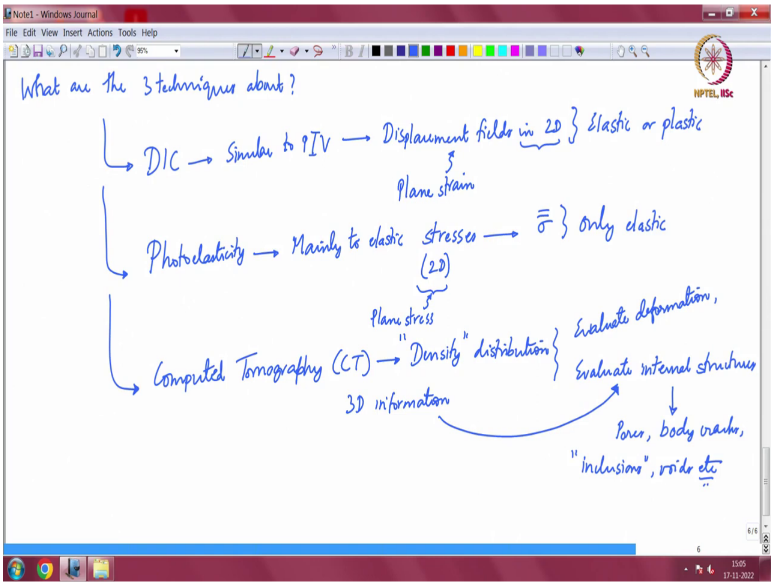

Let us start with the first sort of overview of the three techniques that we are going to discuss. So, what are these three techniques about the first the DIC technique that I mentioned. So, this is very very similar to PIV in spirit in implementation and so on. There are some differences we will discuss that we will start discussing that today. But it is really our segue in into determining displacement fields in 2D.

So, 2D displacement fields in a solid we will use this method called DIC determine and again here it does not matter it can be either elastic or plastic does not matter which displacement field you are talking about or which what the state of stress is below the yield strength is it above and so on. The second technique called photoelasticity this applies mainly to elastic stresses. So, here you directly get the stress tensor or in some reasonable form you get an expression for the stress tensor.

And this is only elastic predominantly there are ways to adapt photolasticity to non to inelastic deformations but we will not discuss them here and many of them actually constitute active research topics this is a very very old technique it is been around for a very long time and it is actually much more useful than you think and. So, we look at how it is how it is done what the implementation are what the calculations are and so on.

The third technique that I spoke to you about which is this computer tomography. So, this basically determines it is a little bit different from the other 2 in that it is used to determine what I would call a density distribution. The density within codes as we go along I will define this a little bit more rigorously and then you will see what this means. And you can use this to evaluate deformation fields if needed right.

CT is much more than just deformation Fields or stresses and strains it gives you actual density information you can use however if you have a pair of CT volume data sets you can use that to evaluate deformation this is something we will discuss at the same time you can also use it to evaluate internal structures. Now another thing I should mention perhaps here in this context is both of these I have written the word 2D here.

And I have now just added the word 2D here both DIC and photolasticity are predominantly 2 dimensional techniques. So, in here you have what is called a plane strain problem and here prototypically again not critical but prototypically you have something called a plane stress problem. So, since you are directly determining the stress the system has to be 2 dimensional presumably this is a plane stress system and the first one is a plane strain system photography is completely three dimensional.

So, you get full depth information once you run a single CT scan and evaluate the information from here and. So, this allows you to evaluate internal structures and by internal structures I mean things like pores body cracks inclusions voids and things like that. So, some of these will be clearer as we introduce this idea of doing inversion in tomography but I will leave them within quotes like this right now so our outline is the following.

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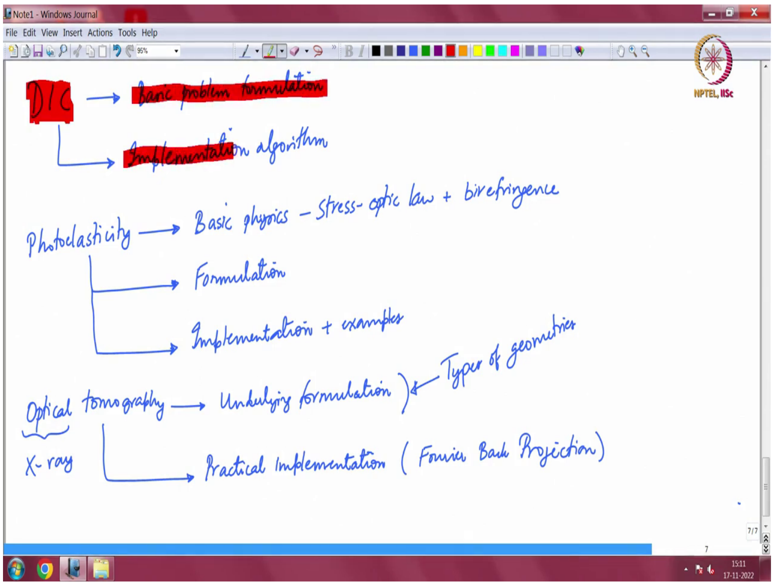

So, for DIC we are going to look at the basic formulation. So, this includes the some notational ideas and then of course the derivation of the fundamental correlation relations. And then we look at given the short time that we have at hand then we look at one concrete Implement let me just say implementation algorithm I will show you a sequence of steps. So, we will take a pair of images and then we will do a subset search.

And then I will show you how to implement it what all you have to do to get a typical DIC scheme running of course DIC itself is also an active area of research. So, there are lots of very fancy algorithms you can do to deal with specific cases notorious test cases you have very sharp gradients you have evolving interfaces things like that. But you will have at least the hope is at the end of this you will be able to write your own scheme that can do DIC by itself.

Including understanding what the scheme is based on the basic equations. For photoelasticity I will do three things first we will discuss the basic physics so this is based on something called the stress optic law and the phenomenon of by refrigerence. So, we will discuss this at length and we will understand why some materials are biopringent by some materials are not and why lots of materials especially polymeric materials polymeric solids show birefringence they are also optically transparent and so on.

So, some of that discussion will happen here then following that we will have a formulation and I will show you how to derive the equations basically quarter wave plate halfway plate things like that. So, we will discuss all of that here and then I will show you and implementation plus some examples of how you can actually use them for a very specific problem again the hope is at the end of this you understand the basic idea behind photologicity.

And how you can actually implement it on your own you can actually see Photon elasticity quite easily if you just maybe go to a you need you know what is called a polarizer sheet you can take 2 polarizer sheets and cross them and look through maybe any glass window near the ends you will see you can even look at a phone screen for instance right and you will see fringes colored fringes those fringes give you information about the stress that is there in the system in this in the case of the phone glasses the residual stress.

And so, you can actually extract information from those colour fringes exact quantitative information and that is what hopefully you will be able to do after the second bit on Photo velocity the third one the third one is a little bit like I said different from the other 2. So, the optical tomography so, notice I have used the word Optical the same ideas apply also to x-ray tomography it also applies to gamma ray tomography or you know any type of tomography for this we need to spend a little bit of time on the underlying formulation.

If you have ever had a CT scan done a medical CT scan you know that there is usually an axis symmetric scanner system and then you have to be inside and the source and the scanner go around 360 Degrees we will work out the Maths of why that is important and why that forms the basis of reconstruction algorithms. So, the formulation becomes a little bit a little bit involved and little bit unconventional compared to the others that are listed above.

So, we will spend a little bit of time on that and then I will show you a practical implementation. So, how do you write for instance a scheme for doing reconstruction. So, the most common method is called Fourier back projection. And I will discuss this method. So, that at the end of it you can always pick up some open source data set x-ray data set and you can mess with it and try to do your own reconstruction.

But in the process hopefully it will be clear how full volumetric information is obtained how the resolution changes as a function of the scan parameters as a function of the image Z Direction thickness and so on. We will also look at for instance in this process types of geometries. So, something called a parallel beam geometry something called a cone beam geometry we will discuss some of them the mathematics changes a little bit depending on what geometry you use.

Basically this tells you whether your source is a cone beam Source or it is a paddle beam source and things like that but the scheme of reconstruction and interpretation remains more or less the same irrespective of geometry. So, some of those details will come out and at the end of it hopefully you will have a better understanding of how tomography works and what you can do to make it better right in general.

So, that is the broad outline for what we are going to do. So, in the next session we will start with a with our implementation of DIC. This will be the next 2 sessions first one on this and the second on this. And at the end of this entire sequence of discussions we will have a demonstration a lab demonstration that also shows you how some of these can be implemented how what are what type of Hardware you will need to implement some of this in practice.