Optical Methods for Solid and Fluid Mechanics
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Lecture - 20
Particle Image Velocimetry V

So hello and welcome. We have been looking at the technique of particle image velocimetry. And we will look at it in a little bit more detail today. Before that, I just want to have a quick recap of what we were looking at last time.

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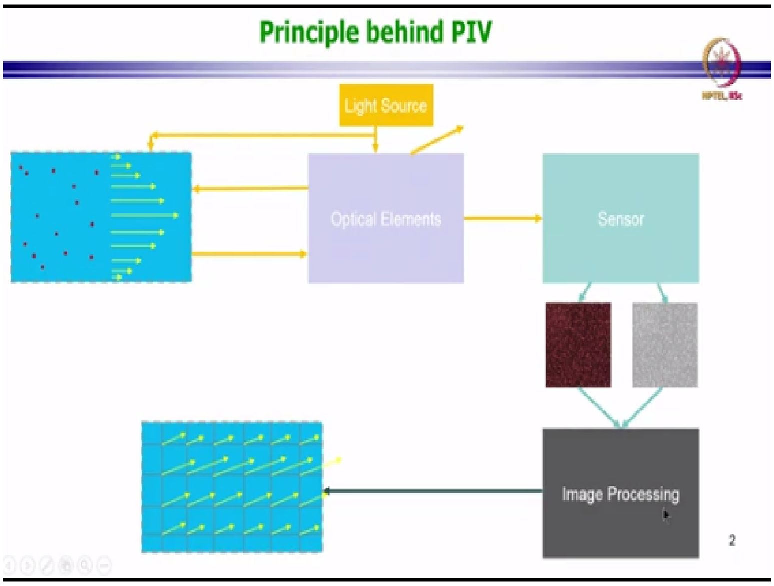

So what we were discussing, for example, was the overall idea of how particle image velocimetry works. We have looked, we have discussed quite a bit of details about tracer particles, the use of optical elements and then sensors. We have discussed to some extent that the fact that they take two of these, two images, which is also called as an image pair. And then the processing, image processing unit is what finally works on this dataset to finally produce a velocity map. We are now looking into more details about this image processing aspect.

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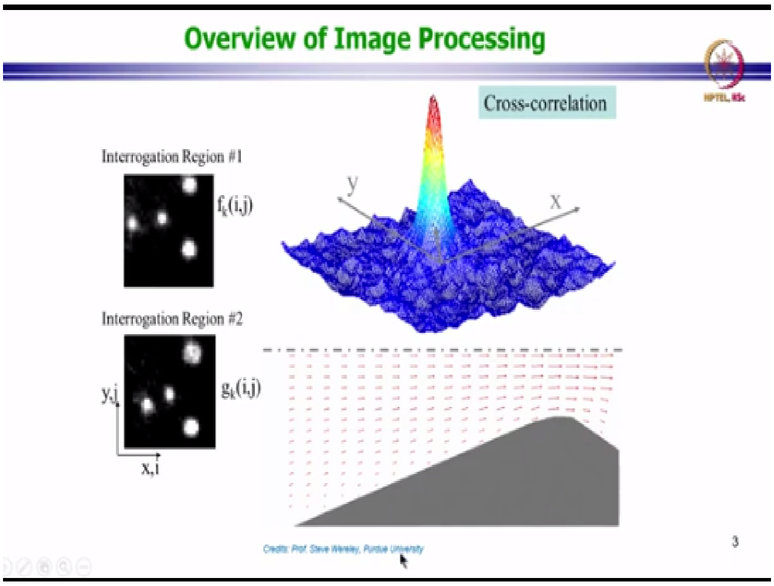

And we discussed about the idea of cross-correlation between different windows and how for example, we can use these sub sections of images and produce and calculate cross-correlation to find out the displacement of particles from one frame to another and how this cross-correlation peak tells us how much displacement has occurred between two images.

And once you do that, we will finally want to produce this kind of a velocity field. And again, credits for this slide go to Prof. Steve Wereley at Purdue.

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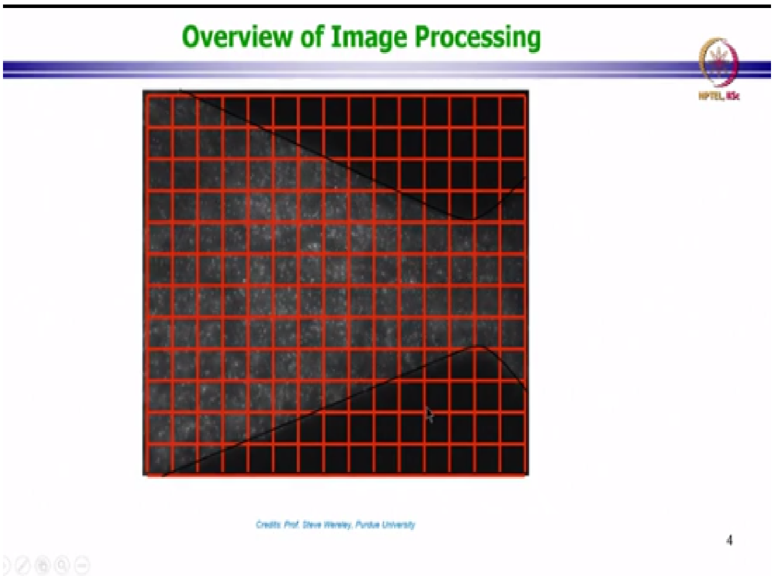

And then we discussed the overall idea of how we will break this up into grid points and then we will choose small windows and then see where the window moves in the corresponding image.

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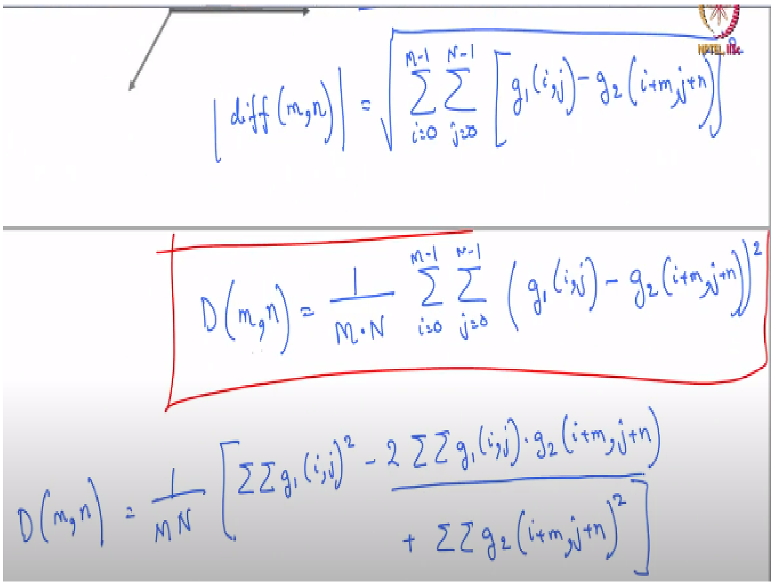

And going back to our discussion that we are doing, we discussed in detail some of the formulas that are used.

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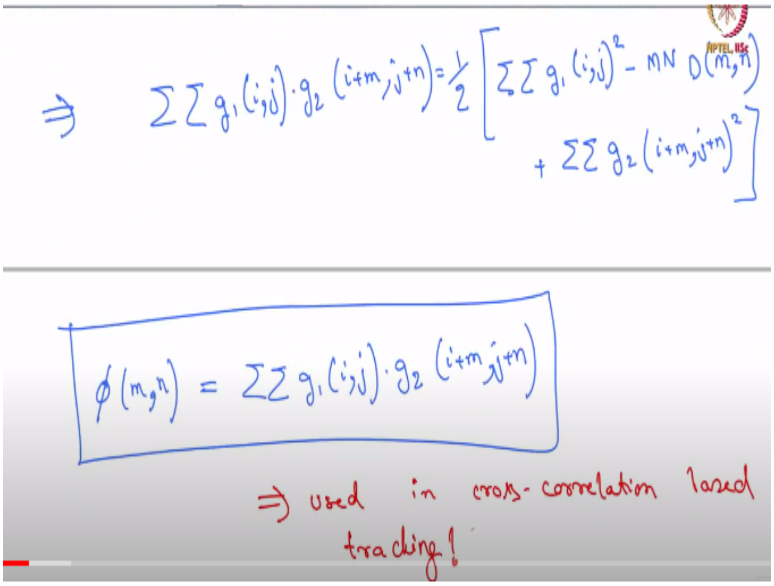

And I talked about the MQD method and also the use of the cross-correlation based tracking. You can use either of the two but most softwares today implement cross-correlation based tracking.

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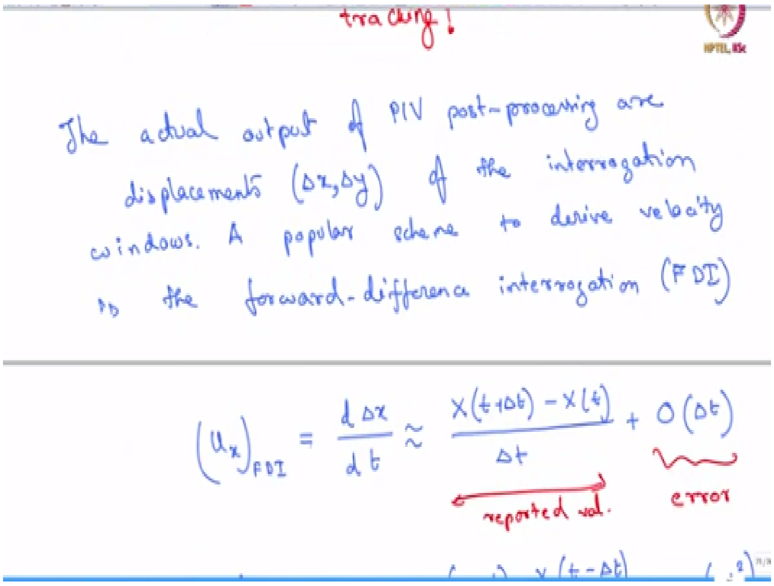

And finally, when you have the actual output, you do get displacements delta x and delta y. And now you have to use numerical schemes in order to get a velocity field, which is a derivative of the displacement field and the time derivative of the displacement field to finally get the velocity map.

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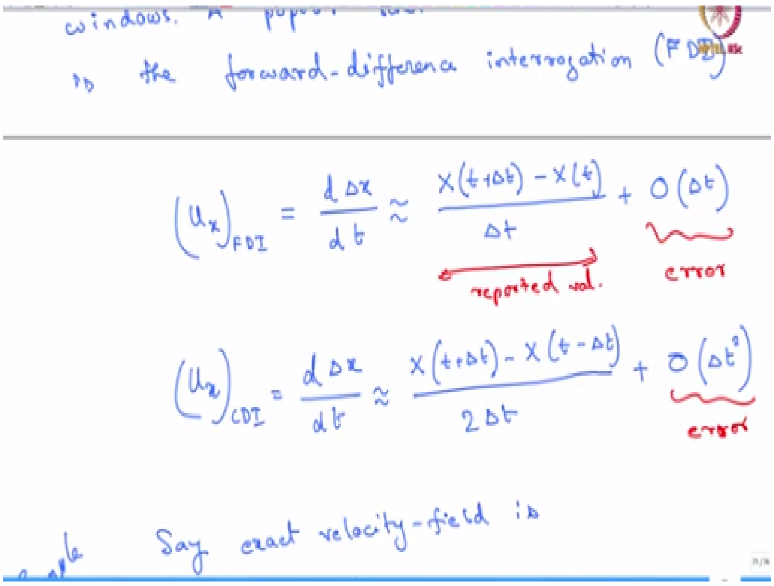

And we discussed how we can do a forward difference method or a central difference method and we get these two different type of expressions with different types of errors in the two cases.

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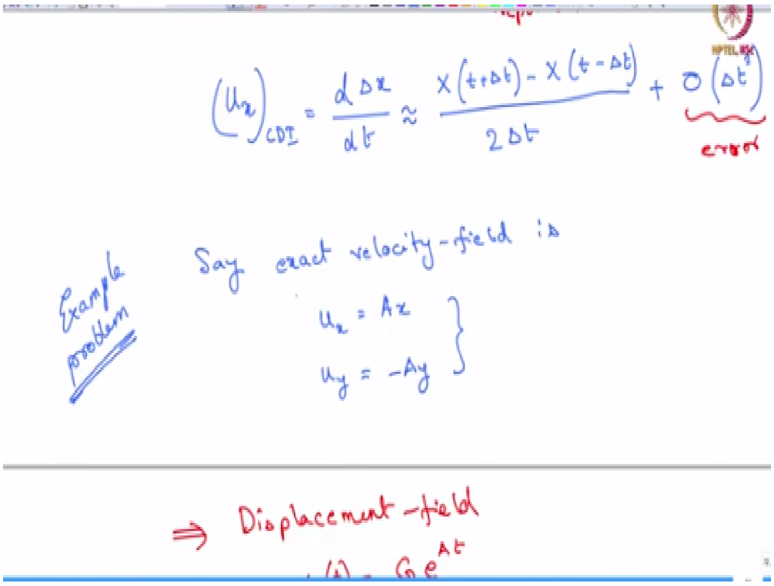

We want you to understand this issue of difference between the forward difference and the central difference techniques a little bit more. So we started off on this particular problem last time, which is that you have a known velocity field u x equal to A times x and u y is equal to minus A times y.

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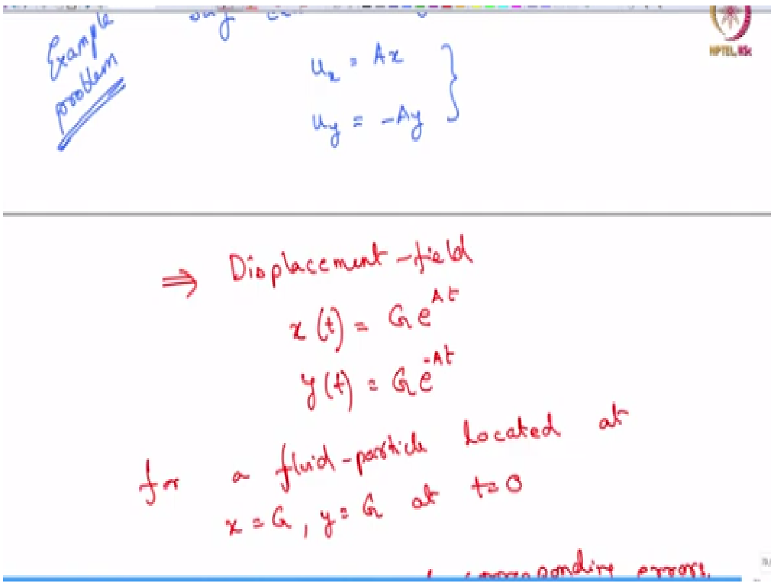

And obviously, by integrating these two velocity fields, you get the displacement field as x t equal to G e to the power A t and y as a function of time is equal to G into e to the power A t. I am sorry, this is there is a mistake here, this should be, there should be a negative sign here, okay. So now that we have this, what we want to do is we want to understand how we calculate the numerical portions of the velocities from this, in a situation like this, and also the corresponding errors, okay.

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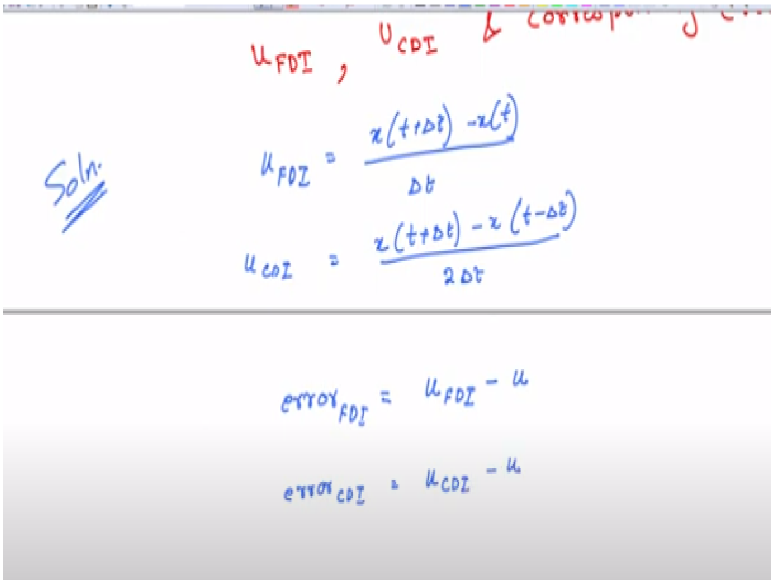

So we have already discussed, okay so we now are ready to discuss the solution of this. So the velocity of the forward difference method, that we had already written down the formula for it, which was x t plus delta t minus x t by delta t right? And we also wrote down the formula that we are going to use for the other case, which is t plus delta t minus x t minus delta t by 2 delta t right?

Now the error in either of the two cases. So the error for the forward difference method is nothing but the estimated numerical velocity which is this quantity minus the actual. So you actually know the real velocity right here, right now right? So you can calculate the error correctly, exactly. And similarly, for the other case, you have this as the error.

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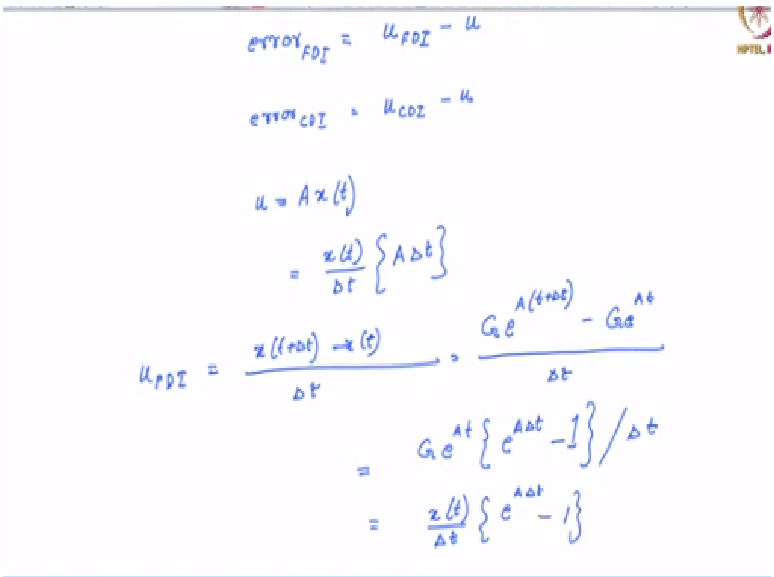

Now and for my u I know that it is A x to the power t, right. I am just using this expression here. I have dropped the subscript x, because otherwise, just going to become a bit more complicated. And then this is I am just going to rewrite this a little bit, which is as delta t and A times delta t okay? So this is nothing but the original expression which has been rewritten.

Now this quantity, we can calculate simply by evaluating we have now since we know x t exactly, because we integrated it and we said it is equal to some G times e to the power A t, we know that the first expression can be written as, rewritten as this. Just a second, A times delta t, sorry A times t by delta t, right? So now I can simply go ahead and I can expand this expression, which is G e to the power A t.

And in brackets, you can have e to the power A times delta t minus 1, right? So I can rewrite this entire thing as nothing but, so just if I go up, we have this expression for G e to the power A t in terms of x. And I have forgotten a delta t here. So let me incorporate that. So this entire thing can be rewritten as delta t e to the power A delta t minus right?

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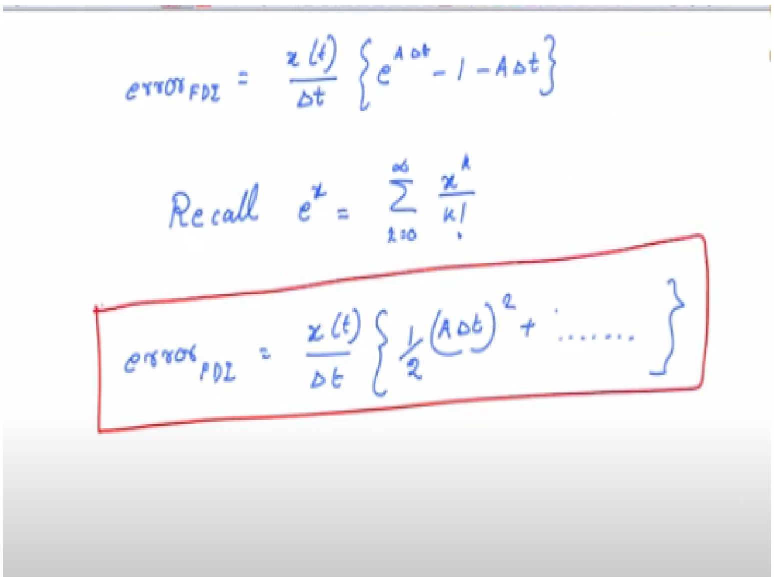

So now that we have the expressions for both the numerical estimate of the velocity and the actual exact velocity, we can go ahead and calculate the error, right? It is rather straightforward, because we have already written down what we have to do. So it is nothing but, sorry wait a second. This is A times delta t minus 1 and then minus A to the power A into delta t, not A to the power, A into delta t, right?

So I have just gone ahead and plugged in the two expressions that we have already obtained. And now recall that e to the power x is nothing but equal to summation, let us say some dummy variable k which goes from 0 to infinity x to the power k by k factorial, right? So we know how to evaluate e into A to the power A delta t. And the first term for this is going to be 1, the second term is going to be A delta t.

So when you do this subtraction, the first two terms disappear leaving you with, so once you apply the series expansion for the exponential, what you are left with are the terms starting with A delta t square. And in this particular case, you will have half A delta t square plus all the other terms going up, right?. So the third this second number will be 1 by 6 A delta t cube and so on.

So you can just go ahead and calculate this series solution. So my error FDI looks like this particular series. So this is a infinite series. This is what it looks like, it will converge to a particular value, okay. So I have my error FDI. Now this, just remember this, we will come back to this expression in a moment.

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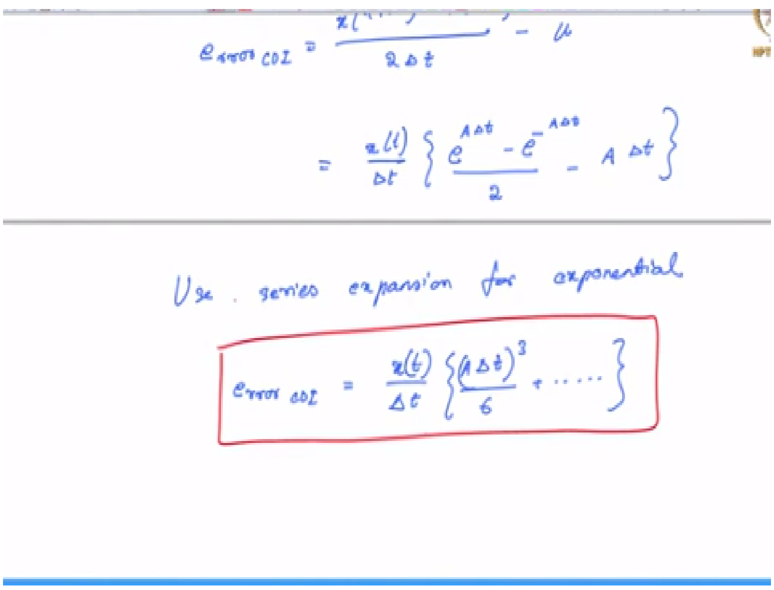

Before we do that, let us go ahead and calculate the error and the central difference method, which we already wrote down as x t plus delta t minus x t minus delta t 2 delta t minus my u right? And we have already evaluated these quantities before. So if you just go ahead and try to do that by yourself, what you will find is this starts to look something like this e to the power A delta t minus e to the power minus A. Okay, I am just going to take this 2 out.

Or rather I can take this 2 inside, wait a second. This is A. So if you go ahead and evaluate this expression, if you evaluate these two again using the series solution, so use the series expansion for exponential and you get the error as x to the power x t by delta t into delta t times A cube by 6 plus other quantities. So this because my error in this particular case.

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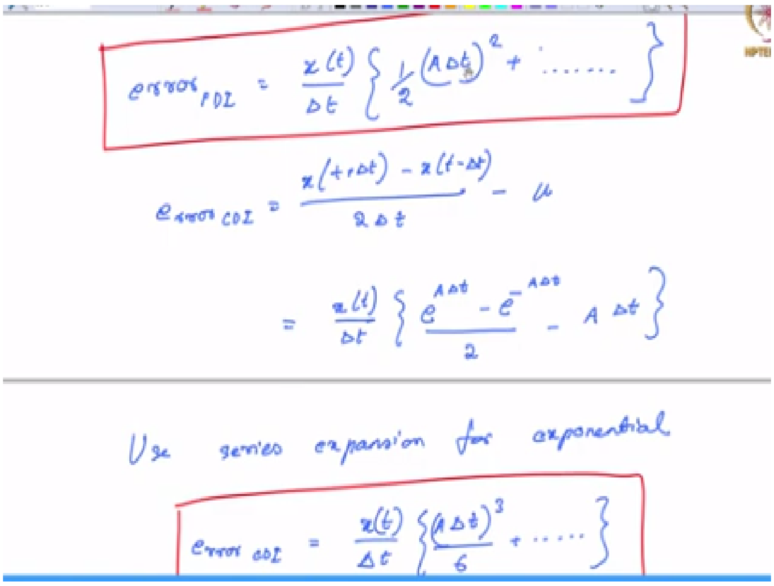

So now if you just want to compare these two, I have both the expressions written up on the screen. So if you go ahead and compare the two expressions, what you will see is, in this case, both of them have the same multiplier outside which is x t delta t, but the expressions inside here these are two different expressions. So here you have a delta t square, here you have delta t cube, you also have a delta t here, so you can see that the error here is of the order of delta t.

So the error in the finite the forward difference method **is,** is of the order of delta t. And if you do the error in the central difference method, that order is of the is of delta t square, which just goes back to exactly what we are saying in the very beginning that the error in this particular case is of the order delta t. And in this case, it is of the order delta t square. So this just verifies that claim that we were making.

And it also gives you an idea of how these velocities are calculated. So **so** now we have basically discussed much of the idea behind particle image velocimetry. And overall the idea is rather straightforward.

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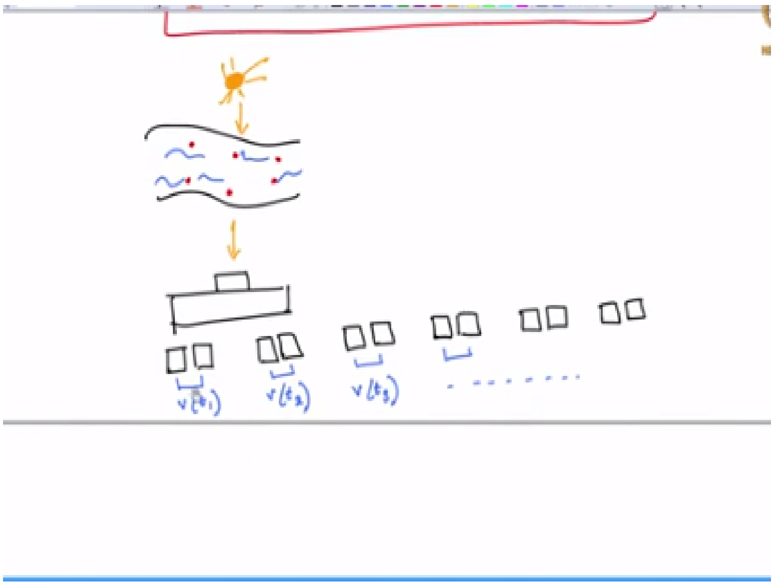

You have let us say, some system in which there is a flow that is happening. And there are some tracking particles, seed particles, there is a source of light somewhere. And this light is eliminating our fluid. And then finally, this is being captured by some camera. And hopefully, it is a special type of camera where it is clicking an image pair. And then after some time, another image pair, after some time another image pair.

So these rectangles stand for images, right. So this way, you are just going on and on and taking different image pairs. And when you do the calculation, what you will be doing is you are going to be processing two images at once. When you do that, what you end up getting is one velocity field. So maybe this is at time t 1 and you get one velocity field for this.

Then you calculate the velocity for these two image pairs and then these two and so on, right? So now you can see that two images here they yield one velocity field. Now I have deliberately created a situation where this distance between this image and this image is more than the differences between this image and this image, right? You can see, so this is sort of deliberate and the reason for that is in normal particle image velocimetry an image pair is taken very, usually very close to each other.

And that is why you need special PIV CCD cameras most of the time where the image pairs are taken almost immediately, whereas the difference between two image pairs can be much more. So this distance in time can be significant, whereas, this need not be so. However, nowadays with the popularity of high speed cameras, a lot of people do use, have started using high speed cameras for image processing and particle image velocimetry calculations.

There some of these CMOS based high speed cameras may not be able to take these image pairs, but what they can do is that they take many images spaced equally from each other at very high speeds, in which case you will do this calculation just like this, but this distance will change. So again as I said, the actual implementation on the ground depends a lot on the actual flow. It depends a little bit on the hardware, and it actually also depends on what you really want.

So if you are implementing a particle image velocimetry system, you must ask yourself, what is it that you need, what are the, what is approximately the velocity field. You must have some idea of the scale of the velocity which you are going to measure and then you must ask yourself what is the kind of hardware that you need.

And there is no universal necessarily hardware because I have shown you also that in microscopy, the type of lighting is very different and the hardware can be very different for microscale flows versus lab scale flows versus very high extremely large scale flows, the hardware can again be quite different, right? So this is what is usually done. So you do need, most PIV conventional PIV operations you do need image pairs like this.

However, for when the flow is laminar and steady, you might be able to get away with just a normal DSLR camera and just a traditional recording in that case. This usually happens for microscale flows quite a bit, where the flow speeds are not necessarily very high.

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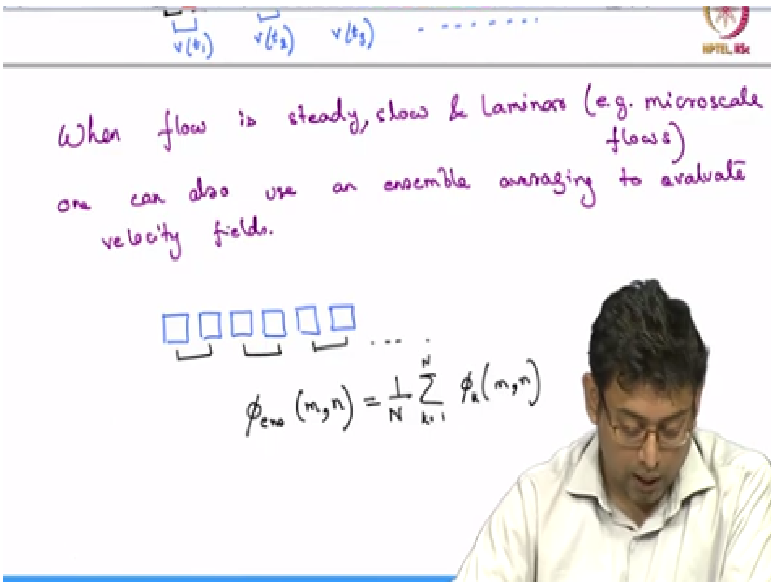

Now one special case I want to discuss is what happens when flow is steady, slow and laminar, example microscale flows. One can also use an ensemble averaging to evaluate velocity fields. What do I mean by that? Well, so in such a case, you can use a normal camera and then you can take images such that the images are equally spaced away from each other.

So let us say you have taken a sequence of images. And then you can go ahead and you can calculate let us say the correlation peak between these two images, then subsequently these two, then subsequently these two and so on. And you can now describe or define an ensemble averaged correlation peak, which is going to be equal to some averaged. So some 1 by n where you choose the number of image pairs that you are going to average over.

And here it can be some dummy variable which goes from 1 to n and this could be the phi is the phi k. So phi is the correlation quantity and this phi here, so this is on the left hand side you have the averaged file, on the right hand side you have phi calculated for each of these image pairs. So you have these image pairs, you calculate, in this particular case you calculate a correlation and that becomes phi 1.

Similarly, you calculate a correlation between these two images, that becomes phi 2. And you do this over the second pair and it becomes phi 3 and so on and you end up with this kind of a expression. Now this actually leads to significant signal enhancement in correlation calculations and can give very good results.

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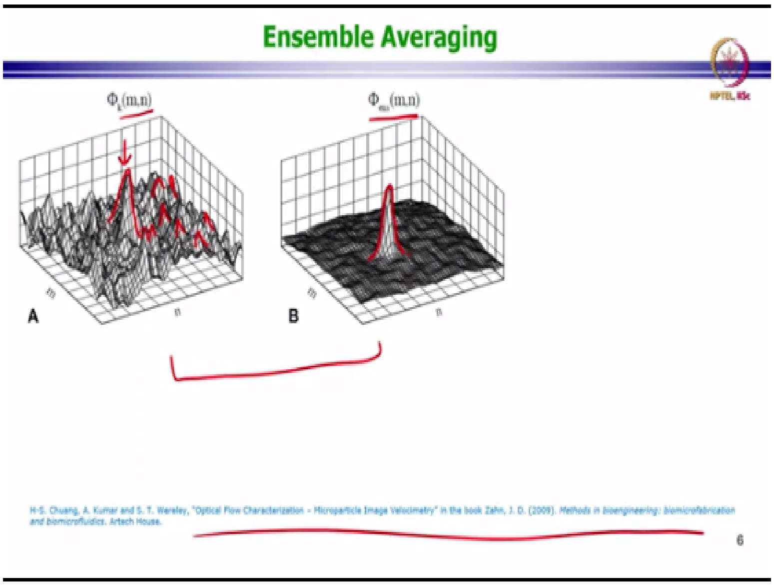

So let me show you one such system. This again it is a figure which is taken from this book chapter which I have already introduced to you before. So this is for example, an example of a correlation plot taken between two images, you can see this the correlation peak which exists here that is not very high compared to the background. So these are the background other peaks that are there, which is not the correct peak, this is the correct one

So you want to evaluate this, but the noise levels are high, which can happen in a noisy system. But then when you averaged over a number of different images to get one correlation peak, you actually get a very smooth and nice correlation peak. So here the signal to noise ratio is much better and you can use this now to evaluate an image pair.

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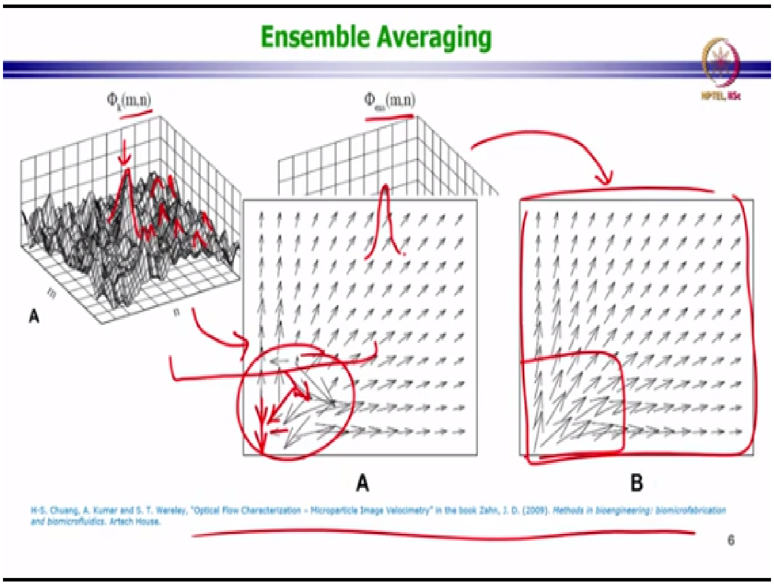

So for example, this is the result of this velocity field is gotten from the correlation, the correlation calculation on the left, whereas here this image this velocity diagram is a result of the ensemble averaged correlation. So you can see here there are a lot of vectors which are spurious vectors right. So you have this flow, but then you suddenly have a vector that is pointing in the opposite direction.

Here it is pointing in some random directions. Whereas here you look at the same location on this side of the image you see how much smoothed out the entire system is, right? And this happens, so these vectors are spurious vectors, they are incorrect vectors. And they have resulted in this situation because the noise level in the system is very high and you have not been able to calculate the correlation peak properly in these cases. Okay.

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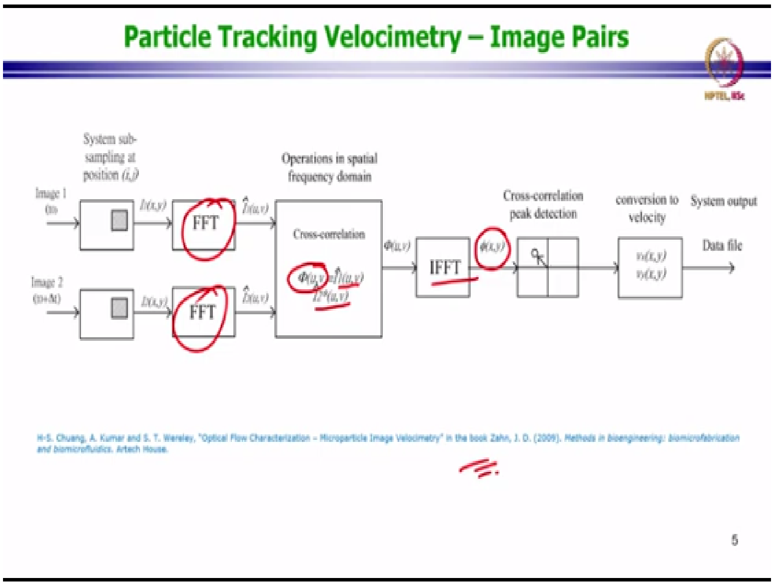

Now one more quick thing about calculations is that you have to calculate the correlation quantity, which you saw was a summation, right? So now that quantity you have to calculate for each grid point for every window you have to do that for within a search window and that can be computationally quite expensive.

So when you are implementing PIV codes which should be able to run in real time, you are at least not take too much time for the evaluation. You have to switch to what is known as the fast Fourier transforms FFTs where the images are taken and then they are converted into the fast Fourier transforms. And then in that case, the correlation cross-correlation quantity in the Fourier space becomes very easy to evaluate, because, all you have to do is to multiply these two together.

And then you do an inverse Fourier transform to get the cross-correlation that you were looking for. Once you have this real cross-correlation, then you can just go ahead and do this detection thing that we have talked about and then you convert it to the velocity, finally get to the data feed. Again this image was published in this book chapter. So if you want you can look it up more.

Now as you have seen, just I will just quickly go back to this image here. These velocity fields are two dimensional, right? So all these images that we have been discussing, which are used for, so these two images that are being used, these are representative of two dimensional systems. So conventional PIV is two dimensional in nature, which means that if you have a significant out of plane motion then that does create a problem.

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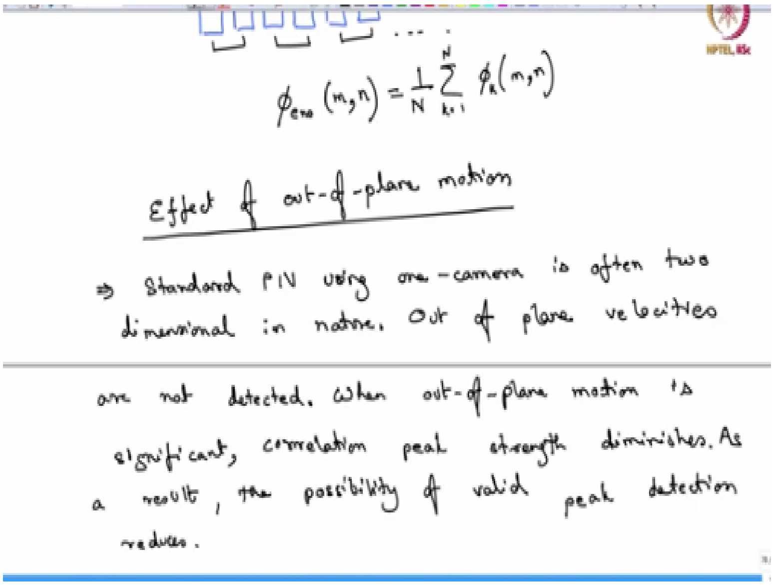

So the question is what is the effect of the out-of-plane motion? Now standard PIV using one camera is often two dimensional in nature. I am deliberately writing often because there are cases where you can use one camera and still get some 3D information, okay? So now out-of-plane velocities are not detected. And when out-of-plane motion is significant, correlation peak strength diminishes, right?

And as a result, what can happen is the possibility of valid peak detection, of valid peak detection reduces. So experimentalists have tried to do different things in order to reduce errors in such case.

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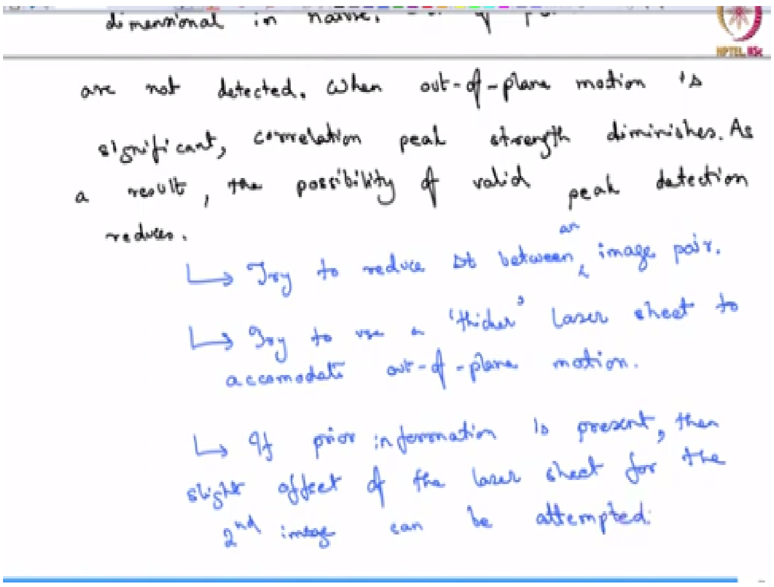

So if you have a situation where you have significant out-of-plane motion of the tracer particles or the basically the flow is three dimensional, strongly three dimensional in nature, what you can try to do is, one of the first things you can try to do is try to reduce the delta t between image pairs. And deliberately between an image pair, okay? So this is the delta t between two successive frames

Another thing could be if you are it is a conventional large scale PIV, then we recall that we are going to use a light sheet for imaging the particles, right? So if the light sheet is very thin, then as the particle moves out of the light sheet, it is no longer illuminated so it is no longer captured on the screen.

So if you could make the the laser sheet a little bit thicker, then the particle stays in the laser sheet illuminated for a longer time. So that one other thing could be try to use a thicker, and I am just going to put thicker in quotes because what exactly is thicker is something you have to decide in a given situation. Try to use a “thicker” laser sheet to accommodate out-of-plane motion.

And finally, if prior information is, prior information is present then slight offset of the laser sheet for the second image, for second image can be attempted. This works well when you have the out-of-plane motion is sort of constant.

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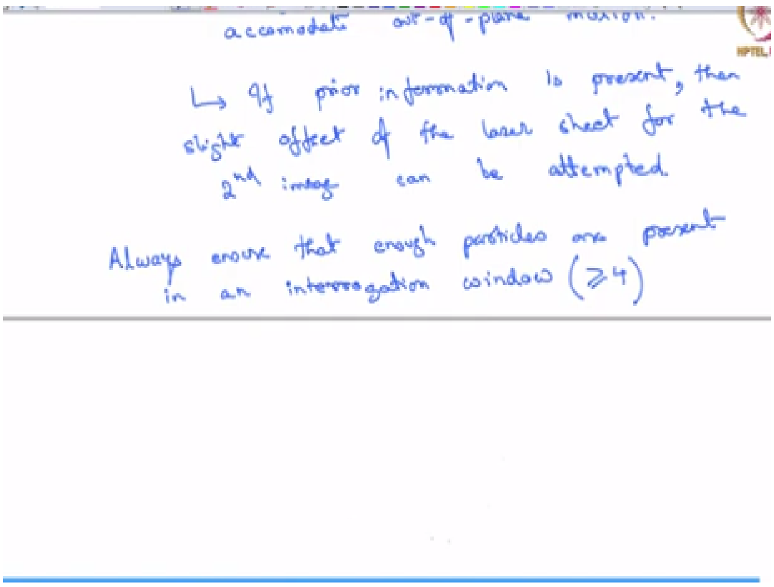

And always you have to ensure that enough particles are present in an interrogation window, in an interrogation. And enough particles usually means that greater than 4 equal to or rule of thumb 4, 5 the more the better. But it should not go below 4 or 5. Then it does become a problem.

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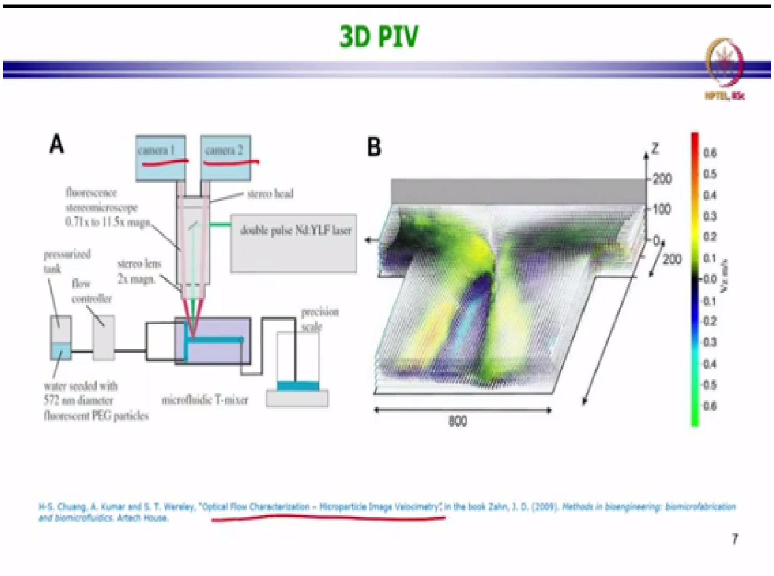

So with that I just wanted to also let you know that there are different techniques which do use three directional particle image velocimetry. One such method is the use of confocal imaging, where you can image different planes successively, and make sure that every plane has a very small depth of field and then you go on tracing different z planes with your microscope.

That is usually done for confocal, laser scanning microscopy can be done for microscale flows, because you can have confocal microscopes which take very sharp images of a given plane. Alternatively stereoscopic PIV which uses two cameras at slight angles, that also can be used.

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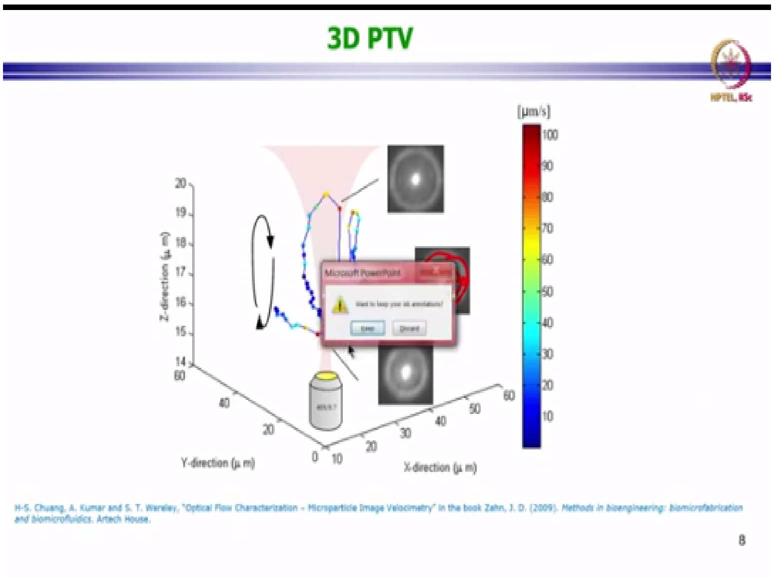

So again, there is a small discussion on 3D techniques for microscale image velocimetries is in this book chapter, including particle tracking methods where you can use for example these airy disc rings. And you can use the size of these rings to tell how far a particle is away from the focal plane. And you can get 3D dimensional information from that.

However, we are not going to look at three dimensional techniques in this particular course that much. So we are not going to discuss that in too much detail. For large scale flows stereoscopic PIV, where two different cameras are used with small angles between, small angles subtended between them. That is usually the most common. Then in that kind of a case you do require algorithms that will take in the data taken by the two cameras, synthesize it, and create a three dimensional data out of it.

Even more complicated techniques can use holograms for eliminating and recording data and holographic systems do exist that those also are there. Again, those are quite intricate and not always used. So they are not the most standard application. So we are not going to go into that here. Okay. So with this, I will close today’s class and I will see you in the next one. Thank you.