Optical Methods for Solid and Fluid Mechanics

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Lecture - 15
Particle Tracking Velocimetry

So hello and welcome back. In the last few lectures we were priming ourselves for having a discussion on tracers and how tracers are important for us to visualize fluid flow. **(Video Stars: 00:17)** I had shown you this particular video which you also saw in the lab where you have these particles and we were discussing all these particles because as you can see in this image that the water is dark and has no signal as such associated with it.

And the particles which actually show up right here and then these are the particles which we hopefully will be choosing very judiciously so that they follow a particular flow in a given condition and then will help us visualize the flow itself. **(Video Ends: 00:57)**. So, when we had started the discussion we talked about tracer particles.

And how they should follow fluid flow streamlines and we had intuitively guessed that the particle density with respect here to the fluid density and the particle diameter those are very important parameters and we had seen this number called the stokes number which is an important determinant of whether the particle is the correct particle or not or rather the tracer particle.

So, I am going to use two or three different words, for example, tracers or seed particles or particles synonymously. So, if I use one of these words you should know that it is basically all mean the same. Now this is a very mathematical way of describing about the suitability of a tracer particle, but one should also have an intuitive understanding of the tracer particles relevance.

So, let us just have a quick rundown of what are some of the intuitive problems that we might face when we have tracer particles in a flow.

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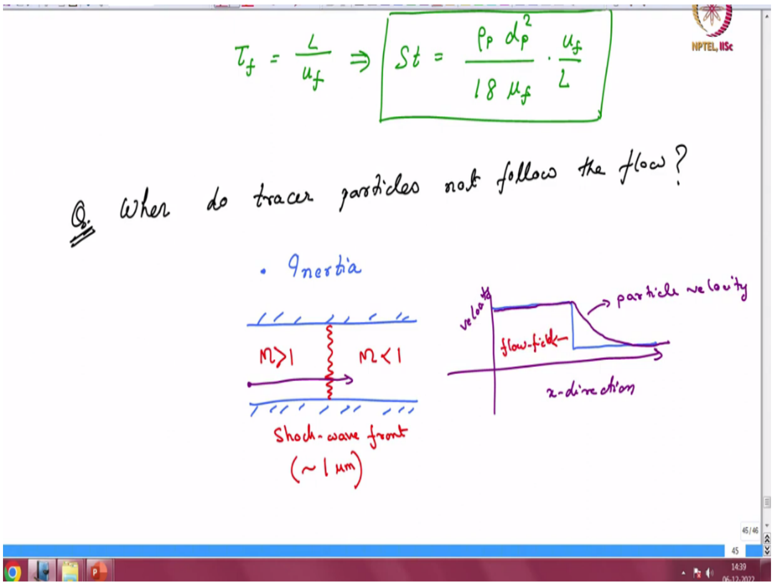

So, the question we are asking is when do tracer particles not follow the flow? By the flow I already mean the underlying flow field so that is sort of understood I do not have to elaborate upon it. So, intuitively the first would be the particle inertia. So, if the particle had no inertia at all associated with itself it would always follow the flow field irrespective of how sudden or how difficult the flow field is to follow.

A good example of this can be let us say a shock wave in a channel. So, let us say we have a channel here and there is a shock wave at some point and the velocity field is such that on this side you have a supersonic flow and on this side you have a subsonic flow M stands for mach number here. So, this shock wave is actually very small. So, this is actually shock wave front.

So, the shock wave fronts are geometrically they are very small perhaps of the order of few microns. So, if you had a particle here which wants to go from this side to that side what will it see? Well, with respect to the velocity so this is velocity and this is your x direction location. The real velocity is very high and more or less constant on this side and then it suddenly goes and decreases and goes on to some other, take on some other value on the other side.

Now a particle has to follow this, but the particles inertia is going to make its life a bit difficult for it to follow. So, what will happen is a particle will probably if it is a good tracer particle it will be able to follow the flow field on this side, but when it comes to this sharp change it will not be able to accelerate fast enough and instead will have a smoother transition into the other side velocity.

So, this is perhaps your particle velocity and this is your fluid velocity flow field (()) (04:54) mean the fluid flow field. So, this is a good example of how inertia is relevant. Now another problem that surfaces quite often I am just going to say again we are using blue dot.

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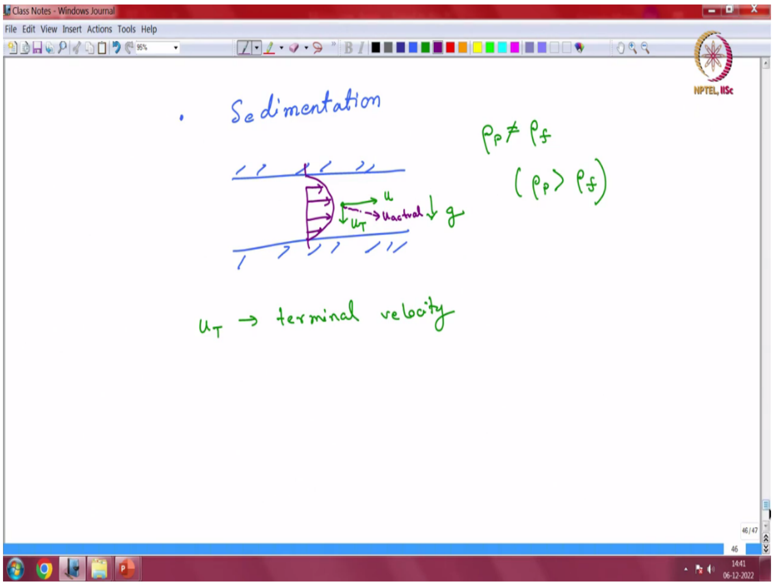

So, another problem that surfaces quite often is the problem of sedimentation. Now again I will take a particle in a channel. Let us say this time there is a parabolic flow through this channel. So, this is the underlying flow field and some particle now has to follow this. So, you have a particle you have a tracer particle that now has to follow so it is getting entered in this direction so it will have some velocity along your positive x direction.

So, this is maybe your velocity like this and then the particle also experiences so gravity is downward let us say and the particle density so the rho particle is not the same as rho fluid. So, in that case you will have a certain sedimentation and the particle density is let us say greater than fluid flow velocity then it experience a gravitational body force that tries to pull it downwards.

So, it will acquire another velocity which we often call the terminal velocity so this u T here is the terminal velocity. Now I encourage you to look up terminal velocity and solve that problem I am not going to go into the details here there is a certain amount of assumption that you understand certain fluid mechanical terms here in this particular course. So, can you do look up terminal velocity if you have not looked up and then once so this particle will have a terminal velocity in this direction it also has a velocity in that direction.

So, its actual velocity if you have to add vectors is going to be slightly off so this is your u actual. So, this also can interfere with your experiments and tracer particles ability to follow the fluid.

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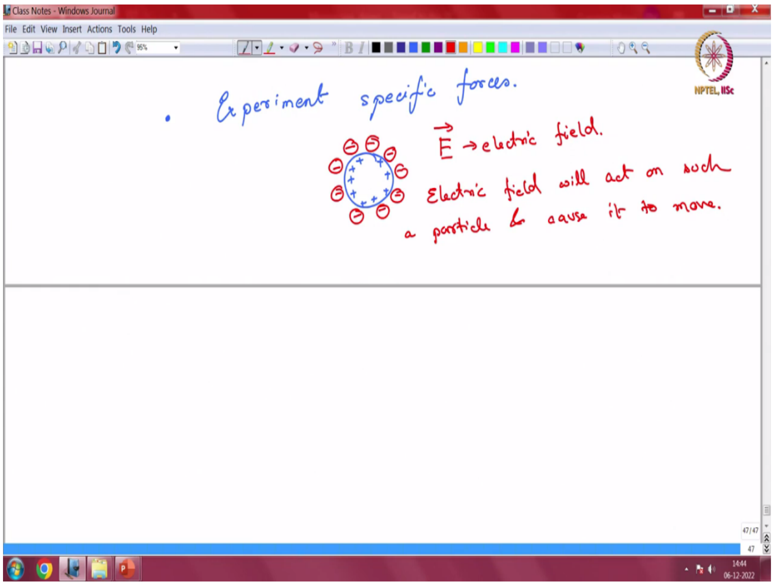

Other could be experiment specific forces. Now we cannot go into all types of specific forces that might be present it is not possible, but I will give you a illustration. So, let us say you have a positively charged particle that you have immersed in low conductivity electrolyte. So, let us say water with a very small amount of salt and so this particle here has once it is immersed in water it immediately acquires positive charges either it has positive charges already or it can acquire positive charges through absorption on its surface.

So, let us say once this happens then what will happen to preserve electrical neutrality you will have negative ions preferentially gather around the particle. There will obviously be positive particles also, but negative particles will gather preferentially. So, the moment now if you apply an electric field into this system. So, E is let us say stands for electric field the electric field is going to make this particle move.

So, electric field will affect even though the system is effectively charged neutral here. So, the negative ions have assembled in enough numbers to wipe out the charge due to the positive, but there is a special distribution of charges here. So, there is some kind of a potential field. So, the electric field will act on such a particle and cause it to move. This happens in electro osmosis for example.

You can have another obvious case which is you can have magnetic fields and particles which are polarizable and can develop a magnetic moment and then those particles will also not follow the fluid flow as much.

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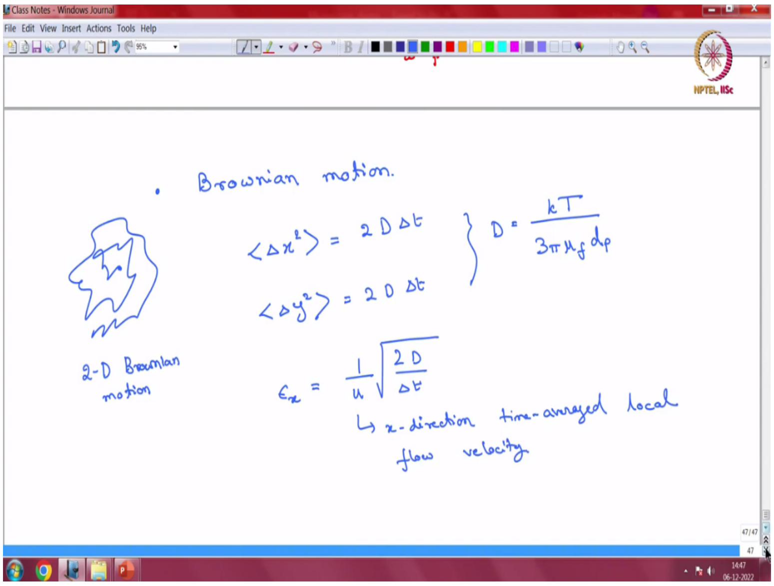

Another point is Brownian motion. Now I encourage you to read about Brownian motion by yourself, but now just to illustrate this particular point I will just go back to this particular video that I was showing you before. In this particular video you have already seen this in the lab right this is being taken under a situation where there is no fluid flow that has been imposed.

So, these particles are suspended in a fluid that has zero flow altogether, but I encourage you to look at this video one more time and you will see that the particles are moving, but they are not moving due to the action of any background fluid flow field that is there. They are moving because they are exhibiting Brownian motion. Now Brownian motion which again I encourage you to read up this I am not going to go into details of why Brownian motion happens.

But, Brownian motion is a motion of particles due to the thermal motion of itself and its surroundings and in this case you have all the fluid molecules bombarding these particles and causing it to move like this. So, this motion is present all the time and cannot be wished away. So, Brownian motion is a motion that is ever present and under no circumstances can you get rid of it.

So, if you cannot get rid of it the particle has its own velocity because of Brownian motion and how do you estimate the error due to Brownian motion. Now, there is a famous relationship which is the mean square displacement relationship. So, if the particle is moving in x y and z direction. So, let us say this is its Brownian motion trajectory and this is let us say a 2d Brownian motion.

Then the mean square displacements in x and y direction they are/have equal to 2 D times delta t where D is the diffusion coefficient and for spherical particles which are far away from any other particle D is given by the famous equation of k T by 3 pi mu f d p where k is the boltzmann constant, T is the absolute temperature this is obviously number 3 pi and this mu f is the viscosity of the surrounding fluid and d p is the particle diameter.

So, this governs the motion or the Brownian motion this equation governs the Brownian motion of the particle if you were to extend this to third dimension all you have to do is write a delta z square equal to 2 d t and if you had to add this (()) (12:50) of or the x square, y square and z square then this would become 6 d times delta t. So the error in the x direction due to this Brownian motion can now be estimated by this simple equation right here.

And here u is my x direction time averaged local flow velocity. So, since Brownian motion cannot be waste away you can only estimate the error because of that.

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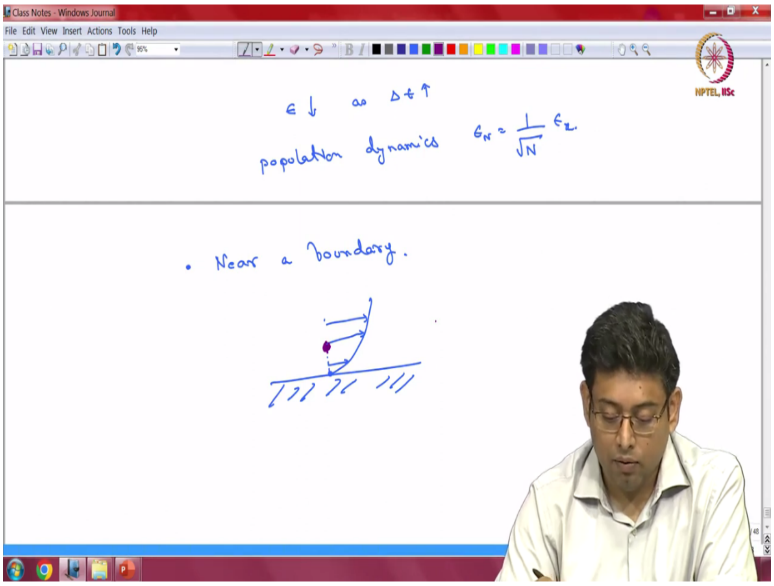

And for population and what you will notice is that this error it goes down as delta t goes up and for population dynamics which means you have N tracer particles the error actually scales as root over N of epsilon x. So, these are some results that I am using and I am sharing with you without going into the exact details, for example, I am skipping the exact all the details of Brownian motion because that is something you will have to read up.

And then is one more final issue of why particles do not follow fluid flow is near a boundary. Now boundaries are always special so if you have let us say a boundary here the boundary imposes a very strong condition on continuum flows and I hope you probably know of the very famous condition called the no slip condition which means that the particles right next to a wall have zero velocity or the velocity of the local wall velocity.

So, the relative velocity between the fluid particle right at the boundary and the wall is 0. This condition can be violated in under certain cases so you can have slip, for example, in nano channels or if the flow is very sparse you can have a slip there and there are some other cases, the no slip condition is a very common case in continuum flows and that is usually almost, always used.

So, under this condition what happens is a particle near a wall can experience other than certain specific attractive forces that if the wall is charged and the particle is charged you can have short range attractive forces there etcetera, but the particle here experiences a very strong gradient of the fluid flow and you can now have a lift force that is created because the flow field at the top of this particle and the bottom of this particle are very different.

And because of that you can have lift forces generated and you can have motion of the particle away or towards the wall, most of the time it is away from the wall. So, in fact if you are having flow in a micro channel people usually use this to focus the particle along the center line of the flow field. So, this is one more reason by which particles may not follow streamlines the local flow velocity.

Now one more thing that I just wanted to let you know is that once you add tracer particles of underlying flow field which will see what the tracer particles are they can change the local viscosity of the fluid. So, you have to be really careful in choosing the particle because it should not change the fluid flow velocity. The entire idea behind using a tracer particle is that it should not change the properties of the fluid itself.

So, if you add so much particles that it changes the properties of the fluid itself then it causes a problem. So, that is one of the unsaid rules of adding tracer particles. Again from illustration purposes I am going to use this okay wait before I go here I want to talk about flow measurements.

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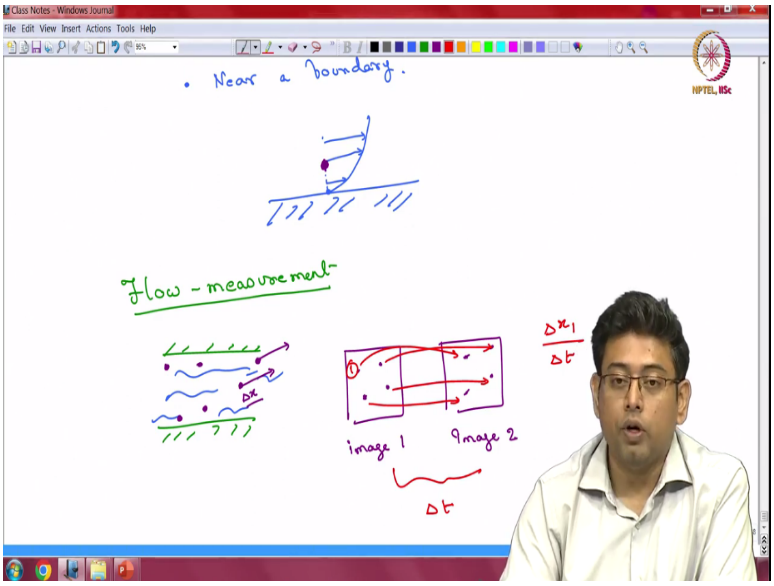

So, we are now going to get into flow measurement so we understand that we are going to measure flow by the use of proxies which are tracer particles and these proxies are going to help us visualize the underlying flow field. Now one of the most obvious ways of measuring a flow is to let us say you have a condition like this where you have some fluid and then you have these particles that are there.

If the particles are sparse and if you are able to identify each particle and how it is moving in the next frame you might be able to find the local velocity. So, if you if you can get the local delta x of the particle and you know if you have two images of this. So say let us say you take one image in which the particles are somewhat like this and then the next image the particles have moved a bit.

So, I know that this is let us say this is image 1, this is image 2. I can say that this particle corresponds to this, this particle corresponds to this and this particle correspond if I am able to determine this correspondence then I can find how much let us say this is let us say particle 1. How much particle one moved and what is the time gap between these two images and then this is divided to get an estimate of a velocity.

Now this velocity is a Lagrangian velocity and then what you do with this velocity is you are able to get some idea of the local fluid flow. Now this idea it is a very simplistic and very intuitive idea, this idea of using this kind of a correspondence between particles in different images and using it to calculate local velocities

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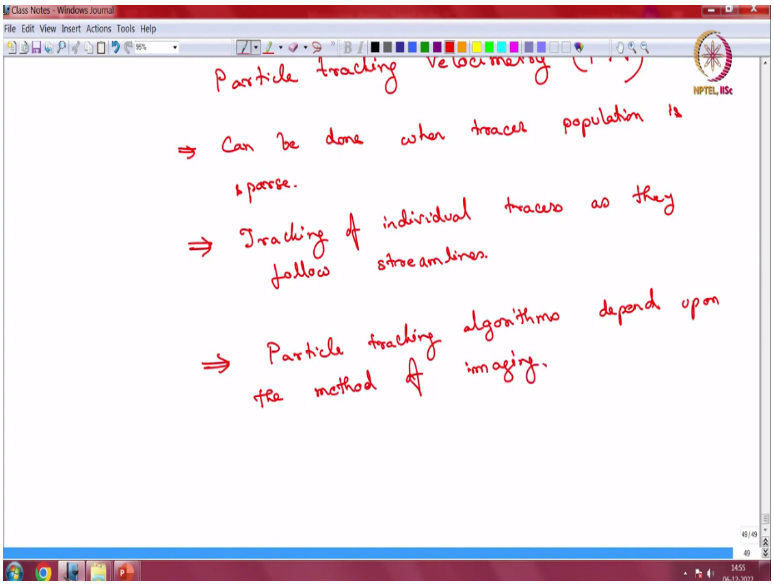

This is also known as particle tracking velocimetry. Now the reason I want to introduce PTV is also when I introduce PIV I am going to make a very strong distinction between PTV and PIV. So, this can be done when tracer population is sparse and we will we will see what sparse means when we encounter PIV we will see that the population there is no longer sparse.

Here the fluid flow is measurement is done by tracking of individual traces as the follow streamlines. Obviously, we are already assuming that you have already been judicious in selecting the correct tracers so that we are not going to go into and now the particle tracking algorithms the one of the issues with PTVs is that the particle tracking algorithms depend upon the method of imaging, what do I mean by that? Let us go back to this images here.

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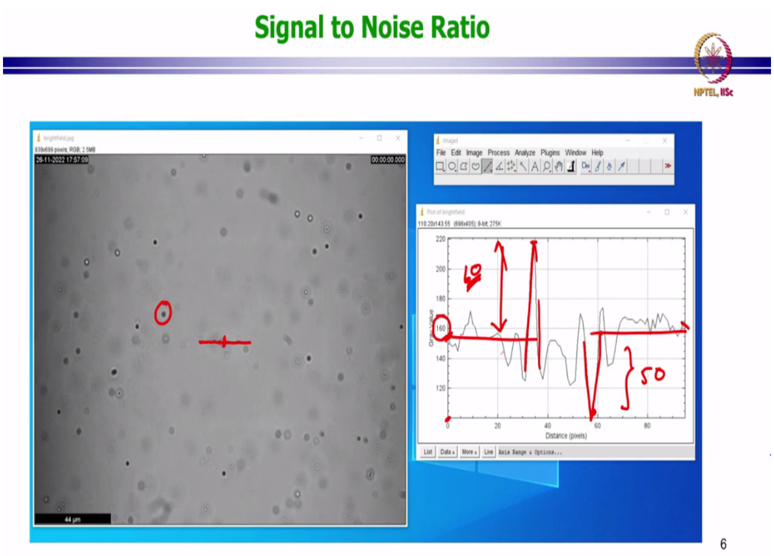

So if you recall we had shown you this particular image which was a bright field image and we had seen that the particles here were exhibiting a hill or a valley in the intensity scale.

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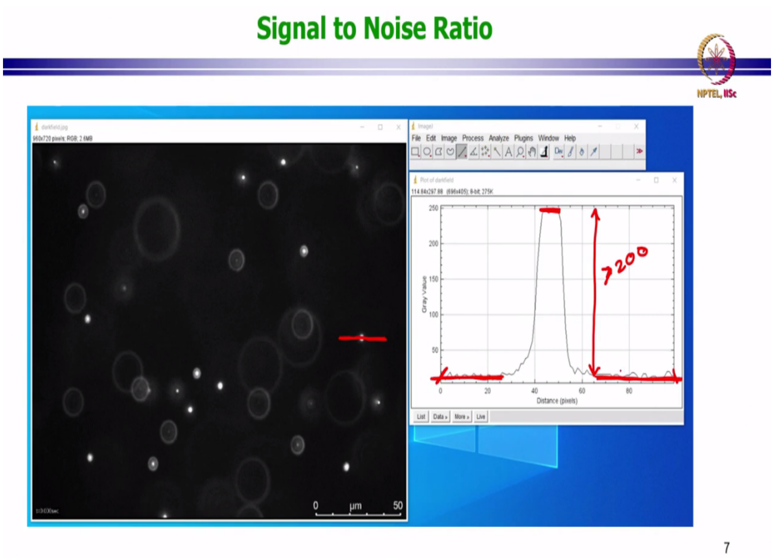

And the dark field image had a very different intensity profile it had a flat top and it had a very sharp rise very low noise and this is the fluorescent imaging case where you have the intensity profile very nice and smooth it almost looks like a Gaussian curve and also it has a very nice good signal-to-noise ratio. So, it tells you that different types of images can have very different intensity profiles. So, if you are tracking the particle you must know how to track the particular particle.

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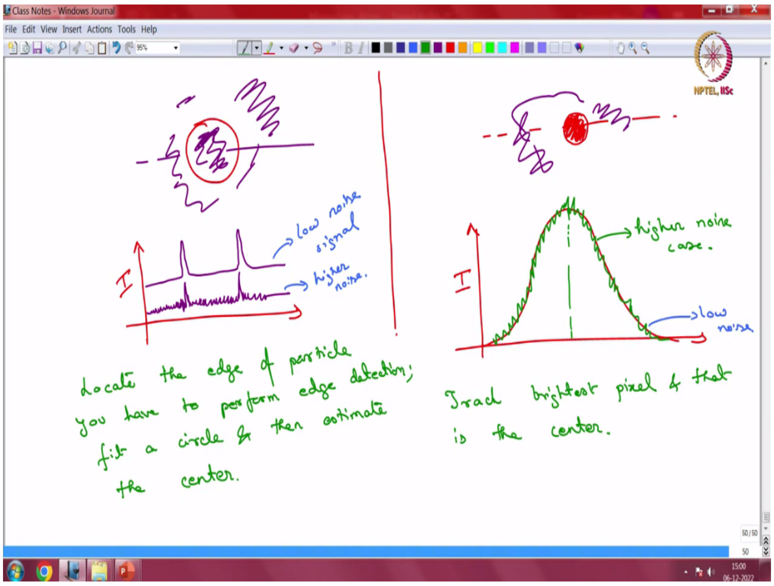

So, I will discuss two cases, for example, if you have a case of dark field imaging and then if the particle is large enough in what you can often see is that on the intensity profile so the particle can often look like this where the outside so if I try to color this the outside and the inside of the particle do not look very different, but the, periphery is nicely visible. So, there are certain imaging, for example, as I said in bright field imaging you can see situations.

Like this where the particle is if it is large enough inside the particle the kind of signal that you get is the same as similar to what you get in the outside. So, what does it look like and the intensity if you take a cut so if you take a cut the intensity can look like something like this. You can have the two periphery is very clear and you can have a signal something like this where the particle itself with the middle of the particle there is not much of a signal.

And then the major signal comes from your periphery of the particle. This is a case where let us say there is not much noise so this is low noise signal and in another case you can also have if you have a lot of noise you can have a situation like this where something of this sort. So, this is a higher noise case. In both the cases now your problem if you are trying to detect the center is to locate the edge.

So, to locate the edge of the particle you have to perform edge detection then perhaps fit us a circle to it to the periphery that you have already detected and then estimate the center. On the other hand if you do something like the fluorescent imaging that I had shown you your particle may appear such that you have a significant signal from inside the particle and the outside of the particle looks very different right.

And we have already seen what an intensity profile in this kind of a situation will look like. So, if you draw your intensity profile across a certain cut like this you will get us a nice curve like this. This is again a low noise case so we have low noise signal and then if you have a higher noise then this signal can look something like this. So, this is a higher noise case. Here if you wanted to track the center you have to track the brightest pixel.

And that is the center. So, in order to do particle tracking properly you have to know exactly what your signal looks like and that depends on the imaging mode that you had chosen and in different imaging modes things can look very different. So, we come back to this particular case where you have to look at the center of the of the particle. So, this is very common, so this type of imaging or this type of intensity profile you see is quite common in particle tracking. So, I will discuss that particular case.

Now if you had to detect the center of the particle you have to first detect the brightest pixel and the brightest pixel is usually the center of the particle. So, now this particular pattern here.

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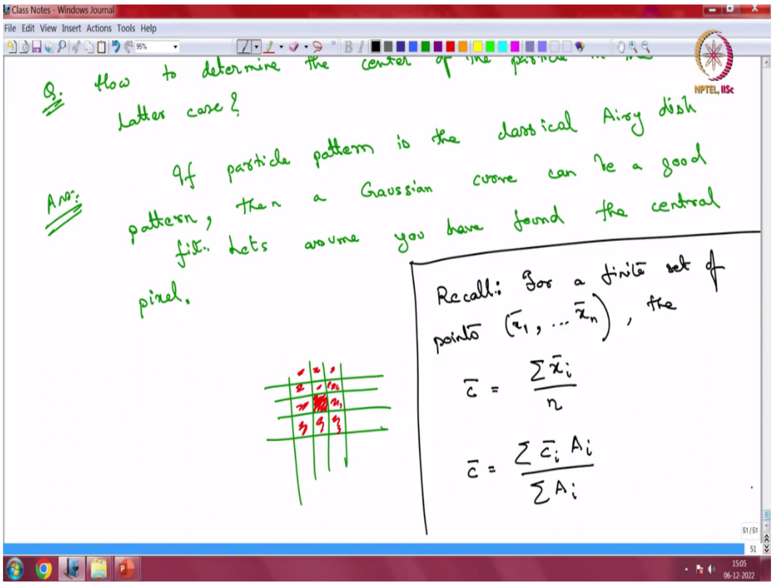

So, the question we are trying to ask is how to determine the center of the particle in the latter case and with sub pixel accuracy. So, what you can do is if the pattern of the image is what is known as the classical airy disk pattern then a 2d Gaussian curve can be would fit. So, if particle pattern is the classical airy disk pattern. Now, again I am not going into the detail of the airy disk pattern because that is something that you would encounter in optics.

So, if you are not familiar with it then you can just look it up then a Gaussian curve it does not have to be 2d it can be 3d Gaussian also and then a Gaussian curve can be a good fit, but how do you find the pixel center? The center of the particle with sub pixel accuracy. So, let us say you have found the center pixel. So, let us assume you have found the central pixel this central pixel I mean the center of the particle.

So, the pixel which corresponds to the center of the particle so let us assume you have found a central pixel by maybe locating the brightest point. Now you will have a situation where let us say these are pixels. So, I have been able to look at let us say the central pixel which corresponds to my particle. Now I also know the values that are there in pixels close by and I can apply what is often known as the centroid method for finding the center of the particle.

Now if you recall so I am just going to take a quick mathematical sorry detour here recall for a finite set of points x 1 to x n the centroid is if c is the centroid then it is nothing, but summation x I by n and if you want centers of n areas which are A 1, A 2, A 3 etcetera you want the centroid of the of n areas each of which centroids are known then the centroid is I am not writing everything down hopefully you are able to follow.

If the centroids of each of these areas is known then this is how the centroid is found. Now this formula here this should give us an example of what we can do. So for a case like this now intensity so you can see that the centroid here in the second case it is weighted by the areas that it represents.

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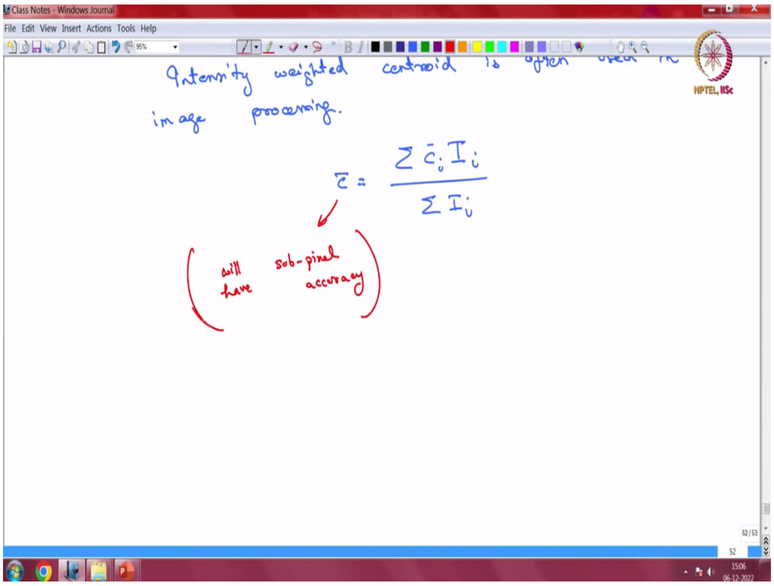

So, the intensity weighted centroid is often used in image processing and then your centroid will simply become summation c i by summation I and this value now will give you the location of the center you are through subpixel accuracy, will have sub pixel accuracy. I will discuss one more method before I stop today.

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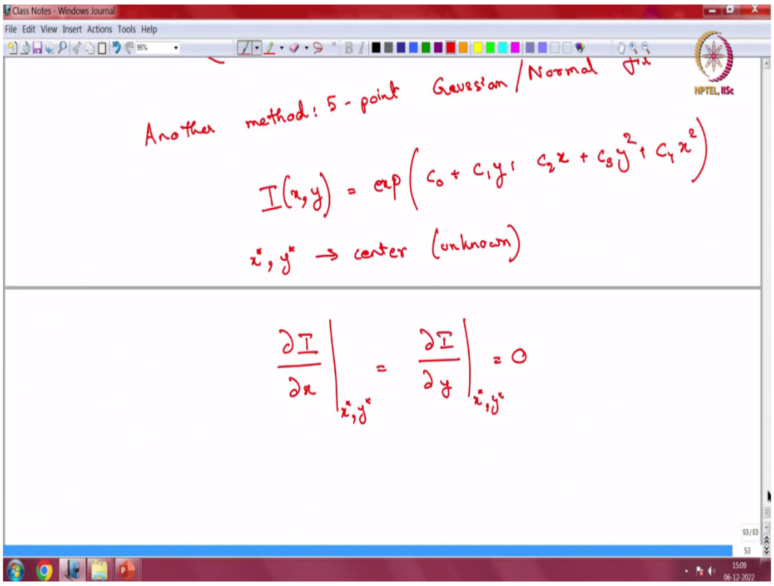

Another method is often called the 5 point Gaussian slash normal fit. So, given the intensities what you can do is you can fit a curve which is let us say the exponential and which has let us say the form c 0 c 1 x + c 2 x so square sorry wait a second let me correct this the first terms I will just put it as let us say this is y this is x then c 3 y square + c 4 x square and I hope you can see why it is being called a 5 point.

And let us say excess are x star y star is the center of the particle which is unknown. Now in order to find the center all you have to do is you have to do at the center your curve will show a maxima. So, when you impose these conditions on this above equation you will be able to find out c 0 c 1 c 2 c 3 and c 4 and hence be able to find out the location of the center of the particle.

So. all this we are doing in order to do particle tracking and we were discussing particle tracking velocimetry and that is why we discussed the intent or the idea behind finding the particle center with so much rigor right now. So, we will stop here today and we will carry on from this point in the next class. Thank you.