Optical Methods for Solid and Fluid Mechanics
Prof. Aloke Kumar and Koushik Viswanathan
Department of Mechanical Engineering
Indian Institute of Science – Bangalore

Lecture - 14
Tracer Particles for Flow Visualisation

Hello and welcome. So, today what we will get started with is the topic of flow visualization. We have already seen quite a bit about imaging and some of the aspects involved there and now we can start getting involved in the flow visualization finally leading up to quantifying the flow fields. **(Video Starts: 00:20)** What I would like to do now is I would like to show you a video where we will see flow in a micro channel.

The relevant literature is right here this is actually taken from one of my papers which is published quite some time ago, but you have flow in a micro channel and so you have flow that is going this way and this is a microfluidic channel and you have these walls right here and your flow is going this way. The flow is seeded with particles in fact it is seeded with so many particles that it is difficult to make out the individual particles.

So, in the last lab session you saw what happens when I we put one micron particles into water and this is what happens when you put a very large number of that kind of beads into the flow. So, you can see the flow to some extent it is difficult to make out the single particles, but you can see there is some amount of churning happening here and this sort of a image actually allows you to do important quantification for understanding fluid flow.

Now this is again a fluorescent imaging so the background water is actually black. This is just because there are so many particles that what you see is basically a very. veru strong white intensity across the entire image. So, just to explain what the actual flow in the system what you had is this kind of a channel and the flow was coming in from this side and exiting from this side and you had these channel walls right here at an angle.

And in this particular image this is a numerical simulation of the actual fluid flow. In the red you can see the streamlines in the flow and the streamlines also show that there should be these vortex structures that should be there in this small gap and when you analyze the data through particle image velocimetry techniques you get this kind of a vector field which actually shows you that there exists a vortex.

So, you can see that this is the Eulerian vector field of the fluid flow and you can see this type of a flow that is there. So, you can use your experiments to validate any numerical simulations. Now the key here was the use of particles in water and we have seen this image before. This is from one of our lab sessions. Thanks to Praveen and Abhinit's fantastic work in the lab.

And I want to before I get started with some of the important aspects of flow visualization there is something I wanted to hit upon here. Now in most imaging to get good signal-to-noise ratio, for example, in microscopy the background liquid is often transparent. So, if it is a gas it is not going to interact very strongly with light. If it is a if it is something like water because water is transparent, it has the no color.

So, it does not also interact very strongly with light and you in many of our imaging the background just basically looks black because it is not interacting, but the problem is to visualize the flow field of the water or the gas. So, something that is not interacting with light enough you have to visualize and quantify the flow of that. So, how do you do that? You cannot visualize flow easily in such a case there are some techniques like Schlieren which allow us to look at flow patterns qualitatively in gases.

And use some of the weak interaction to visualize, but if you want very strong quantitative data like in particle image velocimetry we want the velocity field it becomes very, very difficult and that is where we have to use particles like this and this becomes our tracer particles. So, what we want is to seed the flow with tracers particles which will help us illuminate the flow that is one of the backbones of techniques like particle image velocimetry.

So, that is going to be one of the first things we would want to learn when we start to learn particle image velocimetry because it is going to be important to get good images where we can visualize flow. **(Video Ends: 05:18)** So, with that I will going to change to my notepad. So, the problem at hand is flow visualization and we are going to consider the use of tracer particles, but even before we do that there is a couple of important aspects that need to be mentioned.

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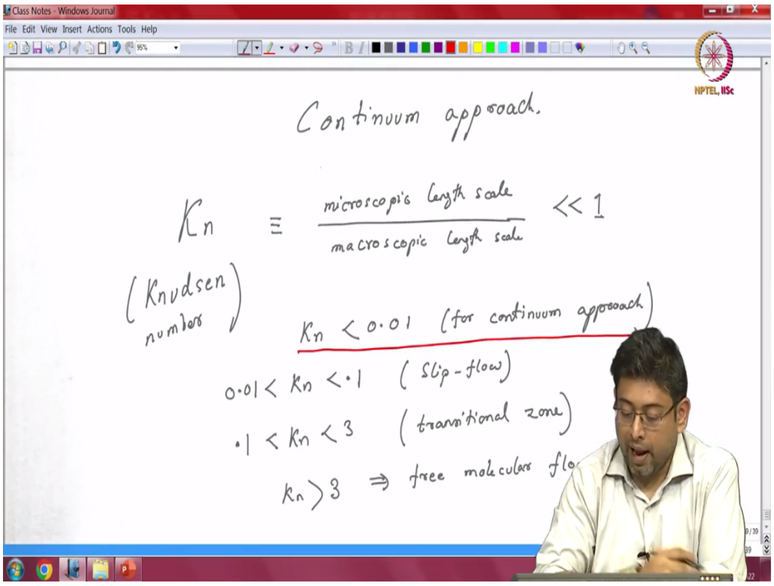

Now in theoretical fluid mechanics there is an idea called the continuum approach which is used and in the continuum approach basically the idea is that the fluid is in a way an extension of the idea of space and basically in any neighborhood of any given point there exists a collection of an uncountably infinite number of points. So, it is a mathematical idea which allows tools of calculus to be applied to certain problems.

Now in practical considerations when it comes to experiments, when it comes to visualizing flows and quantifying them there is an important number which allows us to validate whether the continuum approach is correct or not. This number is called the Knudsen number. It is usually denoted by the letters K n so this is my Knudsen number. It is defined as the ratio of two length scales.

And on the top you have the microscopic length scale and I will explain what that is in a second and in the denominator you have the macroscopic length scale. Now the microscopic length scale typically is the mean free path between molecules as they collide with each other because that gives us an idea of how much free space is there in a fluid whereas the microscopic length scale is the length scale that is associated with the flow.

So, if you have flow over let us say a ball, the diameter of the ball is the macroscopic length scale or if you have a flow inside a channel length of the channel wall can serve as a macroscopic length scale. So, this number should be much less than 1 if my continuum approach has to be valid. In fact we say that for continuum approach to be valid where Knudsen number should be less than 0.01 for continuum flow.

If Knudsen number is greater than 0.01, but still less than 0 then this is in the domain of what is often known as slip flow where the continuum approach is more or less still valid, but you might encounter slip at the wall. So, the no slip condition at the walls might actually be not exactly true, but as this number grows so if this number is let us say between 0.1 and 3 then you end up into a zone called the transitional zone.

And if Knudsen number is now greater than 3 then you have something called free molecular flows. In the free molecular flows you have non-interacting molecular very weakly interacting molecular impacts. Molecules usually do not interact with each other that much at all and hence the name free molecular flows, where would you encounter such flows? Well, I hope you remember the image of the nebula that I showed you in one of the classes that would fall, you know, Knudsen number which has a high value to it.

And hence the traditional techniques of tracers etcetera cannot be used. So, when we talk about tracer particles we will assume that our flow is such that this condition holds and our continuum approach is valid and this is quite important because the very fact that you are using a tracer particle means that, you know, how I actually have a proxy for the actual fluid particle.

The fluid particle itself is not visible and you want to visualize it through the use of a different particle and obviously the idea is that the particles should follow the fluid flow properly and we will come to that how to figure that out we will come to that also, but once you do that just like we showed you here in this particular image. Here you have in the red what you have is the other fluid stream lines which by the way are have to be computed in a given experimental technique what you usually will get is the Eulerian velocity field.

From this Eulerian velocity field you will have to create these stream lines, you can also create other visualization lines such as the streak line and the path line and we will just quickly go over them as well.

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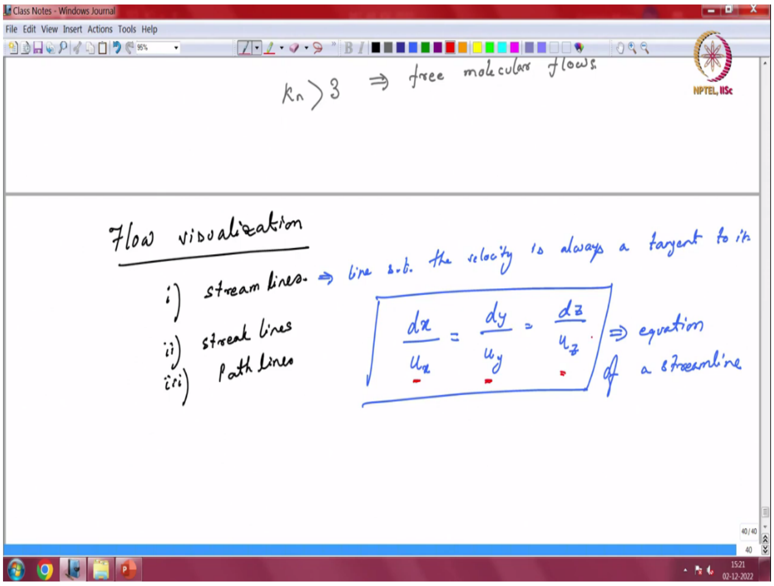

So in flow visualization these three lines are often referred to the first being the streamline and the possibly one of the most important ones the second being a streak line and the third being path lines. There are some other lines as well but these are the most common. The streamline is very important because this line is such that the flow field is everywhere tangent to this line.

So, streamline is a line such that everywhere it is tangent such that I will write such that as s.t such that the velocity is always a tangent to it. You can show from this that the equation for a stream line is my d x by u x direction velocity d y by u y and d z by u z. So, this is the equation which governs streamlines I am not going to go into the derivation of it hopefully you are aware of it. So, this is my equation of a streamline.

So, if you have an experimental technique in which you are able to calculate u x u i u z then you can use this particular equation now to calculate your stream lines in terms of x y and z. So, x, y and z an equation connecting them that would probably represent the streamline in this particular case.

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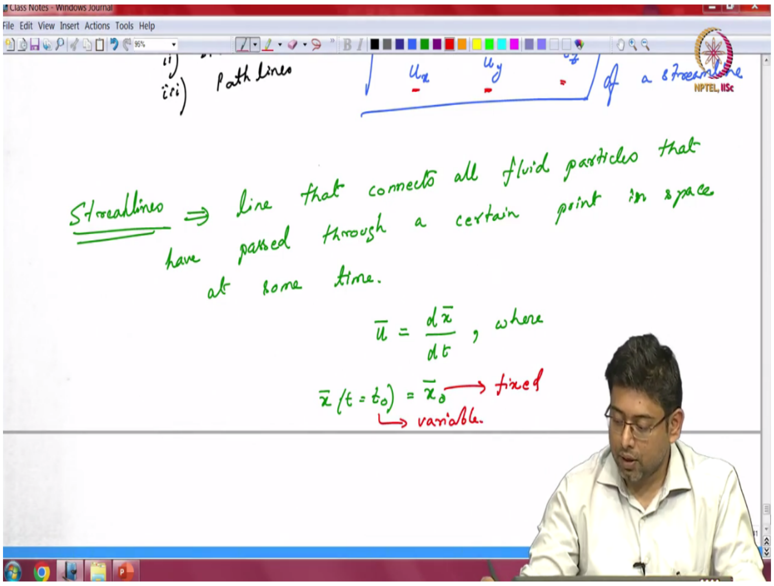

Then now comes the topic of streak lines. Now streak lines are a line that connects all fluid particles that have passed through a certain point at some given time in the past. So, that connects all fluid particles that have passed through a certain point in space at some time either in the past or the future so that is the key here. Now it connects all fluid particles I did not say some other massive particles.

So, streak lines refers to fluid particles that have passed through a certain point. Now how do you describe a streak line? Well, if it is a fluid particle then you are following now the fluid particle as it moves along in passes through different points in your observation window and you have to equate now the velocity. So, at any given point that it passes through the, Lagrangian velocity of the fluid particle will be same as the Eulerian velocity that you measure.

So, the equation that describes them just says that you have to equate your Eulerian velocity field to the Lagrangian velocity of the fluid particle where this equation which represents an ordinary differential equation has to be solved under a certain condition So where your x is at some t equal to t 0 was a specified value called x 0t where x 0 is now fixed and t 0 is some variable.

So, by making t 0 a variable you allow for that particle to have pass through x 0 at any given point either in the past or in the future that is what a streak line is.

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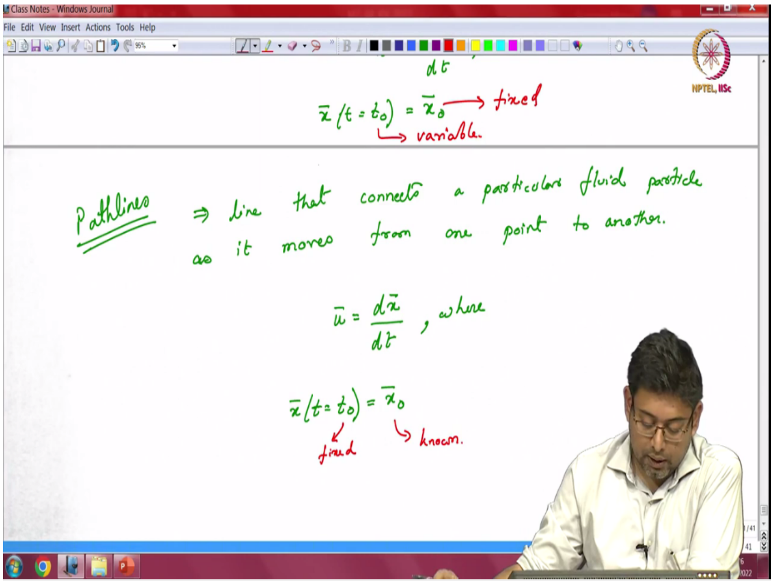

And then we come to path lines. Now path lines are probably the most intuitive of all of them and the reason for that we will see because path lines is basically a line just like the previous one it is a line that connects fluid particles that have not move through a point at any time, but instead it is the movement of a given fluid particle as it passes through different points in space right.

So, let us write down the definition for this. So, this is a line that connects a particular fluid particle as it moves from one point to another. So, I am referring to fluid particle again and again is simply because if it is a massive particle then you will actually have to solve for the the entire equation of motion and you cannot just equate the velocity of a massive particle to the Eulerion velocity field at that location because they need not be the same.

But if it is a fluid particle of the same density of everything the same then they will be equal. So, the governing equation in this case is actually exactly the same. So, you again have the local Eulerian velocity is equal to the Lagrangian velocity of the fluid particle which you are interested in and this now has to be solved such that your x is at some t equal to t 0 where t 0 is now fixed is equal to some x 0. So, you have to give the right initial condition.

So, this is now fixed and this x 0 has to be known for you to be able to solve. If you have a condition like this let us say you had tracer particles in a flow and if there was a flow impose in this case there was no flow impose, but if a flow was imposed here and you took a long exposure photograph then you would have let us say these particles moves like this and you took a long exposure photograph of this.

You will get this entire line which is actually the path line. So, the path line is probably the most intuitive of them to understand, but the other more important flow visualization quantification is in terms of the stream lines because they are very important and relevant in fluid mechanics. So, what we want to do is now we want to be able to use tracer particles to understand flow in a given fluid.

And we are going to look at how we can analyze some of the images, images of the type that I have introduced or I have shown you here this is I will just show you again. There are tracer particles that have been seeded in the flow and by analyzing the traces I want to analyze or understand the fluid flow behavior.

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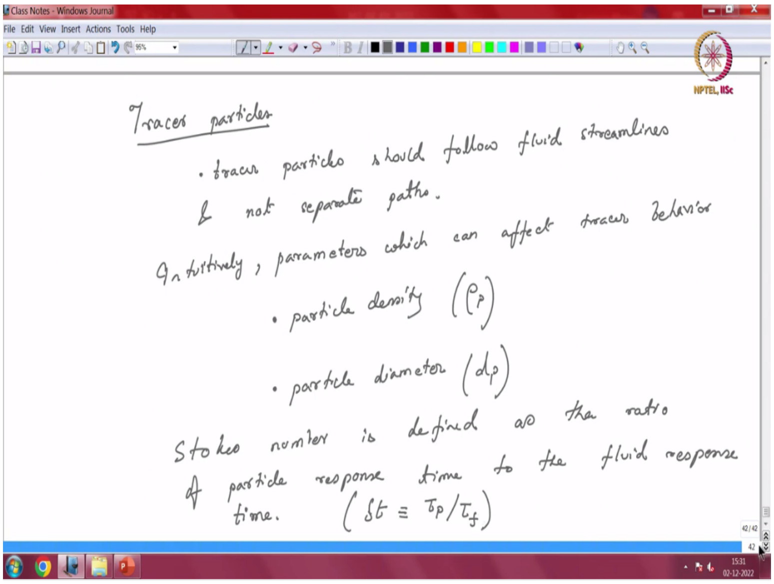

So, what I need a tracer particles and assumption is going to be that tracer particles should follow fluid stream lines or fluid particles as they move and not separate paths and not some of its own path. So, intuitively what are the things that can affect this? So, intuitively the parameters which can affect tracer behavior would be particle density. So, my row of the particle of the tracer so I will just put a p here.

And my even the size of the particle so particle diameter; diameter should be as small as possible. So, you have these different parameters which will actually impact how faithful a tracer particle is in following fluid flow and intuitively you can say that there are difference some of these quantities like the particle density and the particle diameter will have an impact on the behavior of the tracer particle or rather the goodness of the tracer particle.

Now there is a way to quantify this further and that is through a number called the stokes number. Now the stokes number is a number which is defined as; so stokes number is defined as the ratio of particle response time to the fluid response time. So, my stokes number I am going to define it as some tau p which I will call the particle response time versus some tau f which is my flow response time and I will see what each of these quantities are.

Now my response time is basically a measure of how quickly a something a quantitive response to change.

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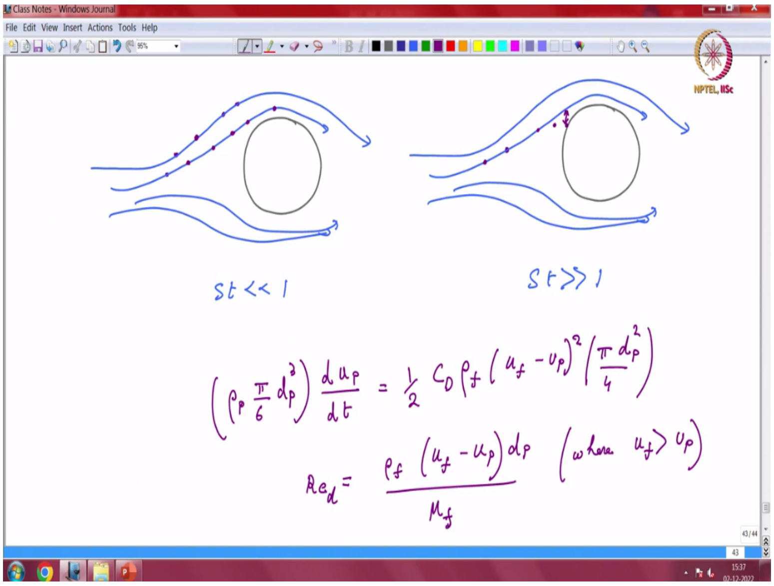

So, what happens when my stokes numbers are of values that are greater than or less than a unity. So, let us say you have flow around some cylinder and these are some of the stream lines associated with it. So, on both sides here is the same actually let me just copy this. So, will try to show what happens when you have stokes number much less than 1 and you have stokes number much greater than 1.

So, we will try to understand this number intuitively before we actually come up with the equation for it. So, let us say there are these particles in the flow. Now stokes number much less than unity means that the particle response time is much smaller than the fluid response time. So, the particle actually responds very fast. So, as the flow changes so the flow has some response time to this, but the particle changes its own direction faster than that.

So, the stokes number is much less than 1 the particle is able to follow these stream lines very nicely whereas if the strokes number is much greater than 1 then as the flow changes the particle is not able to change its location properly to align with the fluid flow and you will have a situation like this where it can just go and hit the cylinder. So, it separates out from the flow and this is not appropriate anymore for good flow visualization experiments.

So, your tracer particle has to be chosen carefully and as a stokes number is great much greater than 1 particle is unable to follow streamlines. Now, let us see how you can quantify this number I mean we have just quantified it as a ratio of two time scales one being the particle time scale and the other being the flow time scale. Now in a fluid flow what does happen?

So, if you have a particle let us say the particle started at a initial condition of when it was at rest with respect to the fluid and then it has to accelerate and achieves the fluid flow velocity. So, how does the particle do that? So, we have to write the equation of motion for such a particle. So, we have to basically write f = m a for a particle of density let us say rho p my total mass is rho p pi by 6 d p cube.

This is your mass and I have to now multiply it with the d u d t which is my acceleration. So, this is my m a part on this side now I have to write the force. So, what is providing the force for making my particle move along with the fluid flow. It is a fluid drag which enters the particles and makes it move along with the fluid flow. So, on this side we have the fluid drag which I can now write as half C D C D being the drag coefficient.

My density of the fluid which I will write as rho f. So, you have two rows here they are differentiated with respect to the subscript. So, rho subscript f is the row the density of fluid a row subscript p is the density of the particle u f -u p square where u subscript f is the is the velocity of the fluid u subscript p is the velocity of the particle and this by the exposed area which for, let us say a sphere is pi d square by 4.

So, I hopefully all the quantities here are well understood and just to repeat one more time d p is the particle diameter, rho p is the particle density, rho f is the fluid density and similarly for the fluid quantities here. So, to simplify this is an equation that we have to solve. but to simplify the right hand side we are going to introduce a quantity called the Reynolds number with respect to the particle diameter.

I am writing small d so let me be consistent and just use small d here. So, my here you have the relative velocity d p by mu f where this is the viscosity of the fluid. I have assumed that the flow velocity is greater than the particle because I am going to assume that the particle started out with a initial velocity of rest with respect to the fluid and then has to accelerate to the local velocity of the fluid.

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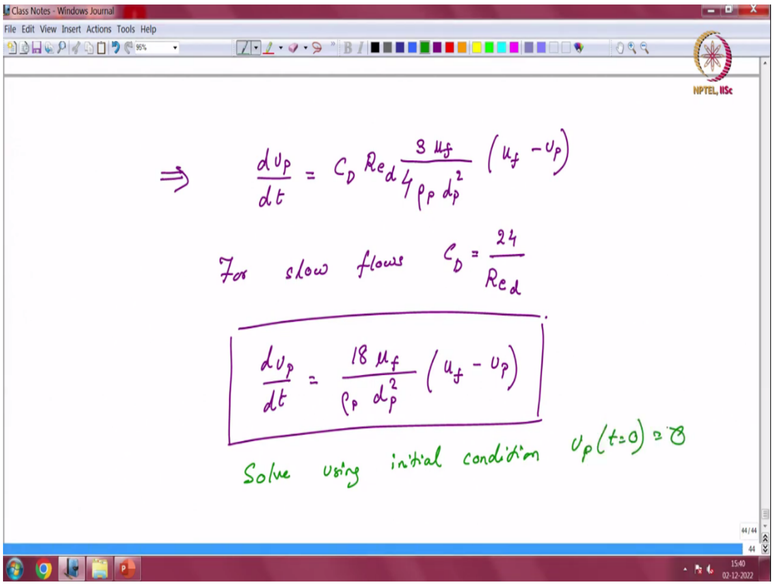

Now this equation this now can be simplified and I urge you to simplify it yourself and check with me. On this side I am just going to retain my d u p term which is my time derivative of the particle velocity and here I am going to write my other terms which will now include my drag coefficient, my Reynolds number and once you do this simplification I am going to just simply write the completed expression right here and I urge you to do this and make sure that my equation is correct.

Now what is the value of c d? Well for slow flows we can assume that my drag coefficient for a sphere is my Reynolds number by 24. I am sorry this is the reverse is 24 by Reynolds number. If you introduce this value of c d into the above equation you know you can see why we introduced R e d so it cancels and you end up with a more simplified expression of my u p which comes out to be 18 times mu f.

There is a mu here missing sorry 18 times viscosity rho particle size d p particle diameter square u f velocity of the fluid minus velocity of the particle. So. this now becomes an ordinary differential equation that can be solved. So, solve this, solve using initial condition u p at t = 0 is equal to 0 and if you solve this what you end up with.

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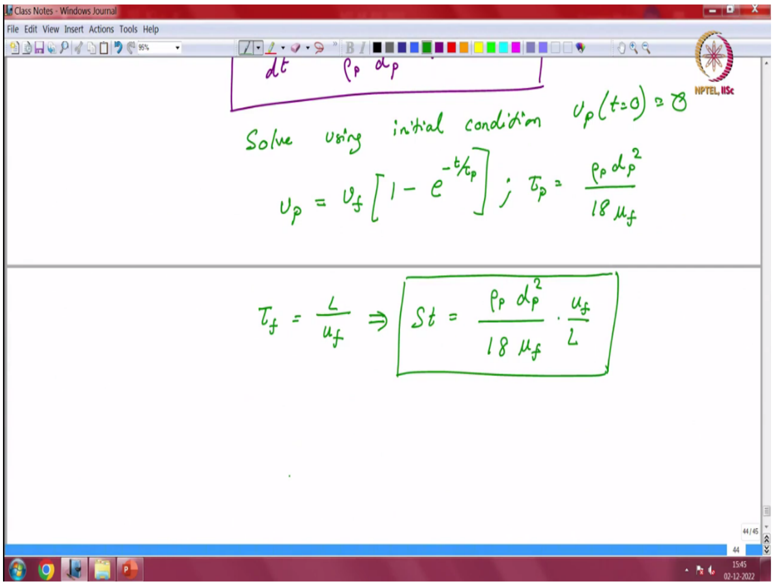

The solution of this comes in the form of my particle velocity is my fluid velocity times 1 - e to the power - t by a time scale which is my particle time scale. So, you can see this is the tau p that we are after and this tau p now where this tau p is rho by 18 to mu f. So, now I have an algebraic expression for my particle response time which if you recall is how we had started off with the definition of the stokes number where the particle response time was in the numerator and now we have to figure out what the fluid response time scale is.

Now the fluid response time scale is actually rather straightforward. So the fluid response time scale or tau f is nothing, but my length scale of the flow divided by the flow velocities scale. So, if you combine these two this gives me a stokes number as rho p d p and please correct me if I am wrong I am just going to already derive this before. So, this is a very, very important number.

And you have to make sure that you before you start using a tracer particle you calculate this particular number for the tracer particle and you will need information about the flow. So, you can see the in the stokes number the approximate idea of the flow is already built in that brings us to the question as to how do you know the velocity scale beforehand you have to know what type of fluid velocity that you will get even before you actually do the experiment that you can get by various physical considerations.

Let us say by imposing mass continuity this is not by the way the local. This is just a velocity scale in the system. So, you have to estimate approximately what is the velocity scale of the fluid flow and then you use that number here. The geometry of the system already defines or determines L which is the flow length scale and then you have to experiment with different type of particles to figure out what particle to use row p will come from that.

So, if you are using let us say polystyrene that from a certain manufacturer they will already specify the density for that. So, you will have this information beforehand usually for particles you can choose different diameters so this is something that is totally in your control. Finally this viscosity term is already predetermined by the fluid that you are going to do, so it is predetermined by the experiment. So, this is a number that you have to calculate and you have to appropriately choose a tracer particle so that the strokes number is much less than 1.

So that the flow field is accurately followed by the particle. So, in this particular video that I showed you that calculation had already been done and these are polystyrene particles in water which are actually behave very well when especially polystyrene is a very nice tracer particle when it comes to water its usually neutrally buoyant and follows the flow fields very well.

So, I hope we learned an important lesson about choosing the right tracer particles and we will follow up with other aspects of flow visualization in the next class. So, with that thank you very much.