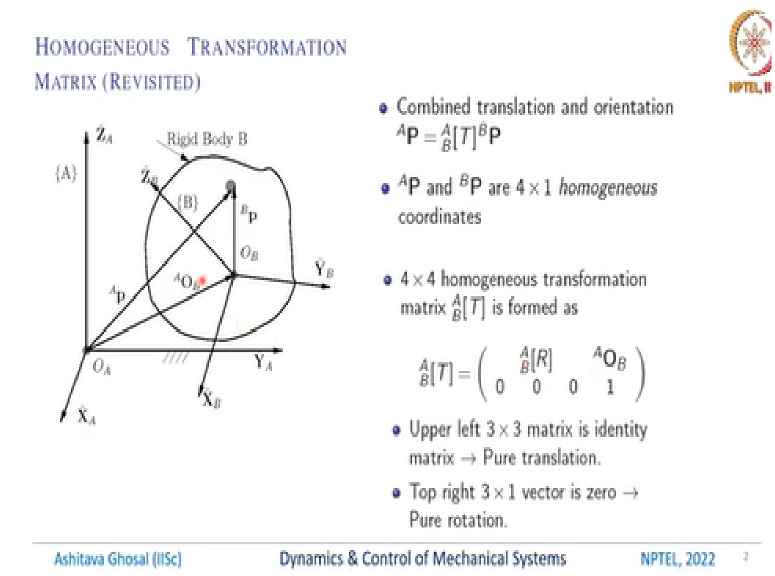
Dynamics and Control of Mechanical Systems
Prof. Ashitava Ghosal
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru

Lecture - 08
Position, Velocity and Acceleration in Multi-body Systems

(Refer Slide Time: 00:32)



So, in the next lecture we will look at position, velocity and acceleration in multi body systems. Before we go to mechanical systems with several rigid bodies let us recapitulate how we can represent the translation and orientation of a rigid body using homogeneous transformation matrices. So, this picture shows a rigid body with respect to a coordinate system A. So, $X_A\, Y_A\, Z_A$ and the origin O_A.

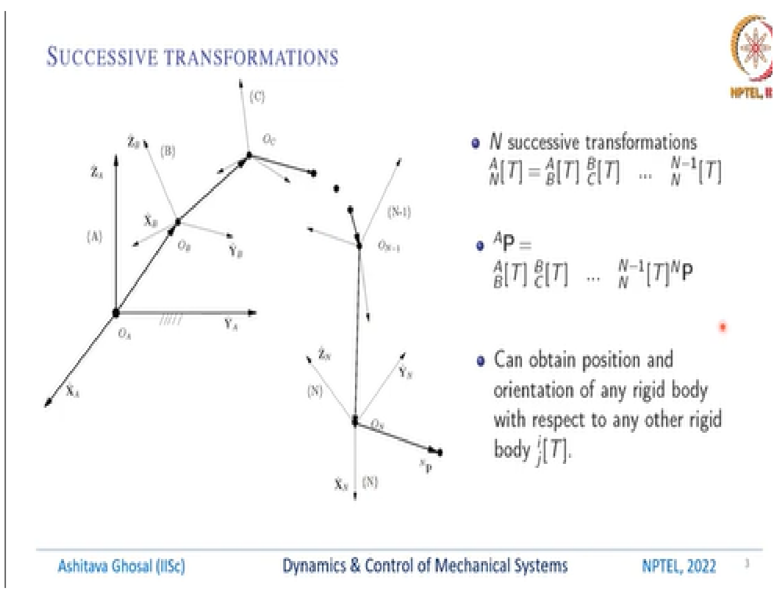
There is a coordinate system B which is attached to this moving rigid body with $X_B\, Y_B\, Z_B$ and the origin O_B. Any point on this moving rigid body in its own coordinate system is given by Bp and in the reference coordinate system is given by Ap. The origin of the B coordinate system is given by this vector AO_B. So, to represent combined translation and orientation of this rigid body B with respect to the A coordinate system.

We propose to use a 4 by 1 homogeneous coordinates in which basically we had this position vector we added a 1. So, our normal position vector is 3 by 1 we added another equation which was 1 = 1 and then we combine the position, translation as well as the orientation of this rigid body using this 4 by 4 homogeneous transformation matrix. And we would obtain an expression which is AP which is $BA[T]$ BP where AP and BP are 4 by 1 homogeneous coordinates.

So, these are nothing but the position vector which is 3 by 1 and we concatenate A one with that 3 by 1 position vector. The 4 by 4 homogeneous transformation matrix $BA[T]$ is formed from the rotation matrix which is $BA[R]$ and the position vector locating the origin of the B coordinate system. So, this part is a 3 by 3 rotation matrix and this is a 3 by 1 column vector and the last row is triple 0 0 0 1.

See the upper 3 by 3 matrix if it is identity then it is a pure translation of this rigid body if the top 3 by 1 vector is 0 then it is a pure rotation. So, this homogeneous 4 by 4 homogeneous transformation matrix represents position of the origin as well as the rotation of this rigid body with respect to the A coordinate system.

(Refer Slide Time: 03:33)



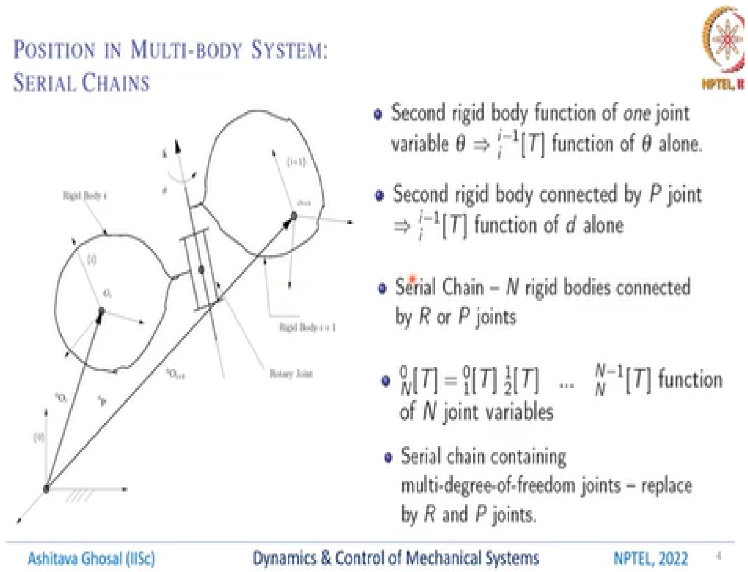
So, let us continue. Now we have this rigid body undergoing successive rotations and translations. So, we have one rigid body or another way of saying is there are several rigid bodies connected one after another. So, we have a rigid body B which is $X_B Y_B Z_B$, O_B and then this rigid body moves to C which is $X_C Y_C Z_C$ and so on or we have a sequence of rigid bodies which is reference A one fixed reference then a rigid body B then a rigid body C and so on.

All the way to our rigid body N which has the origin as O_N and $X_N Y_N Z_N$ and if we have a point on this rigid body then again, we can find the position vector of this point with respect to the nth coordinate system and then we can find the vector with respect to the fixed reference A coordinate system. In order to do that we need the transformation matrix from A all the way till N so N with respect to A the 4 by 4 homogeneous transformation matrix is nothing but a product of all these matrices.

So, we go from A to B, B to C all the way till N - 1 to N. Each of these are 4 by 4 homogeneous transformation matrices containing both rotation and translation. So, the position vector of this point P 4 by 1 homogeneous coordinates in with respect to the A coordinate system A capital P is nothing but $BA[T]$, $CB[T]$.. $NN - 1[T]$ NP where NP is nothing but this position vector N small t with a one added at the end the fourth element is one.

So, hence we can find the position and orientation of any rigid body with respect to any other rigid body. So, for example if I want to find between $ji[T]$ I need to do is multiply suitable homogeneous transformation matrices. So, if I want to find what is C with n. So, I will go from C to D, D to E and so on all the way from N - 1 to N. So, we multiply all this transformation matrices and we get from C to N.

(Refer Slide Time: 06:25)



Now let us look at the position in multibody systems and we will start with serial chains. So, in remember in serial chains we have a joint a link and a joint one end is fixed and one end is free. So, this picture shows two rigid bodies connected by a rotary joint. We will look at other kinds of joints also. So, what is the goal? We know the position vector of the ith link and we want to know what is the position vector of the i + 1 th link.

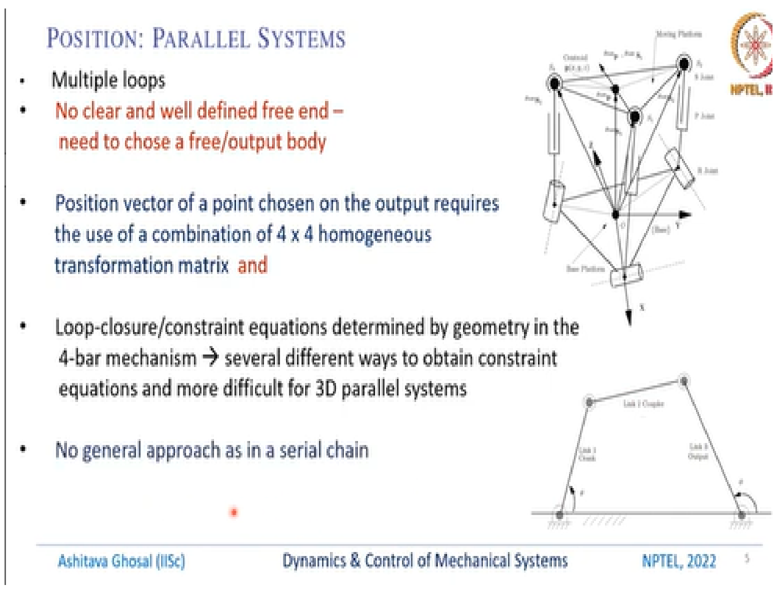
So, I know $0O_i$ and then I want to find out $0O_{i+1}$ and we assume that these two rigid bodies are connected by means of a rotary joint. Recall a rotary joint allows this i + 1 rigid body to rotate with respect to the rigid body i about an axis along k by an angle theta. So, the second rigid body is a function of one joint variable theta the rotary joint allows one degree of freedom. So, the position and orientation of this rigid body with respect to this previous rigid body can contain only one independent variable which is theta.

So, i - 1 i this transformation matrix is a function of theta alone. So, the rigid body i with respect to i - 1 will be a function of this rotary joint theta if they are connected by a rotary joint as shown in this figure. If it was a translatory joint if this joint was a prismatic joint, then $ii - 1[T]$ would be a function of the translation along this prismatic join which we denote by D. So, in a serial chain the N rigid bodies are connected by R and P joints that is what we will look at in this course most of the time.

So, hence the last link or the last rigid body N with respect to the fixed reference frame or 0 the transformation matrix is nothing but a product of $N0[T]\ 10[T] \ldots NN - 1[T]$ and this right hand side will be a function of N joint variables. So, if there are N rotary joints then each one of this transformation matrix is a function of theta. So, there are N thetas if there are some thetas and some D's then these N joint variables are broken up into thetas and D’s.

Theta is the rotation at the rotary joint, D is the translation the prismatic joint. So, in a serial chain containing multi degree of freedom joints. So, for example if there was a spherical joint, we would have to replace that spherical joint either with three rotary joints or some other combination. So, if it was a cylindrical joint which is normally not used then you would have a R and P joint.

(Refer Slide Time: 09:51)



In a parallel system or a parallel mechanism or with loops there are multiple loops as I have mentioned earlier in the structure. So, we have the serial chain we have the parallel chain and we are tree structure and so on. So, in a parallel system we have multiple loops. So, this is an example of a parallel robot this is the three degree of freedom three RPS robot. So, the RPS comes from each chain has a rotary joint, a prismatic joint and a spherical joint.

So, that there are three such chains and what you can see is I can start from some let us say one fixed rotary joint I can go to prismatic joint, I can go to a spherical joint, I can come to this spherical joint and come back to the fixed base. So, this is one loop. So, we can have another loop which is this RPS this S which is S_2 and then come back here. So, there are three possible loops. So, in a parallel system there are multiple possible loops.

And there is no clear and well defined free end of as in a serial chain. So, in a serial chain one end is fixed and one end is free. So, in this case which is the free end. So, once somebody can say that this is my free end which is reasonably good choice but somebody can say that this link is my output and this is my free end. So, we need to choose what is the free end and which is the output rigid body in parallel systems and parallel mechanisms and parallel robots.

So, the position vector of a point chosen on the output requires the use of a combination of this 4 by 4 homogeneous transformation matrix and we need to use loop closure and constraint equations determined by the geometry in the mechanism. So, for example in this mechanism if I want to find this point which is the let us say one point on the output link, I can use the 4 by 4 homogeneous transformation matrix consist corresponding to this rotary joint.

This prismatic joint and then spherical joint which will have to be considered as three rotary joints and then come to this point. So, this is one possible way to come to this. But we could have also come in some other way. So, we would need to use what are called as loop closure constraint equations to really obtain what is the output link and the output link as a function of the actuated joint variables.

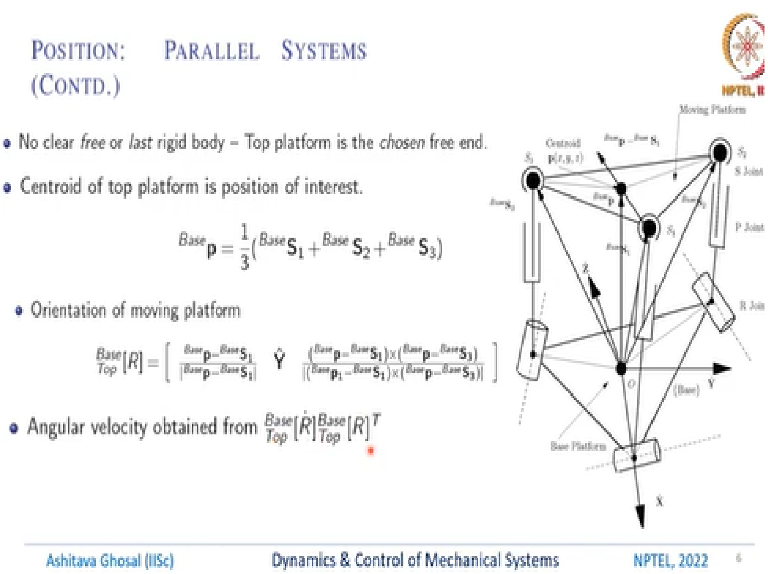
So, in this 3 RPS parallel mechanism we can show that it has 3 degrees of freedom. So, there are only three possible actuated joints. So, most of the time these prismatic joints this one translation here, one translation here and one translation here are the three actuated joints. We need to find expressions for these three rotary joints and even maybe for the three spherical joints. So, we need to use what are called as loop closure equations to obtain the expressions for these rotary joints and what is happening at the spherical joints.

And there are several ways to obtain this constraint equations and in a parallel robot it is easier but in a 3D system like this it is much harder. So, for example in this parallel robot in this simple parallel mechanism which is a 4-bar mechanism there are four links one possibility is this is the input and this is the output. So, we have basically one single loop and it is in a plane it is the simplest possible parallel system and we can say that this is my output.

Somebody else can say that the second link 2 which is sometimes called the coupler is the output. So, we need to choose what exactly is the output in any parallel mechanism and this 4 bar illustrates the point very clearly. In many kinematics books or kinematics of machinery this is the output which is the output link and this is called the crank and sometimes when you want to guide a rigid body through some positions then the coupler is chosen as the output link.

So, there are no general approaches as in the serial chain to choose which is the input and which is the output and also which is the fixed and which is the free end and how do we go from the fixed to the free end. There are no general approaches in case of parallel systems. So, we need to make sure that we need to choose which is my output link and then we do the analysis.

(Refer Slide Time: 15:12)



So, let us continue. In a parallel system we can obtain the position of the output link. If we choose this as the output link then we can find the centroid of this top moving platform as my position of interest of this output link of interest. So, how do I find the centroid of this top moving platform? This can be obtained by once we know what are the position vectors locating these three spherical joints.

So, if I know what is the position vector of S_1 S_2 and S_3 which is the three spherical joints from a fixed reference coordinate system which is X, Y and Z with origin 0. So, I want to know this vector. So, if I know this vector if I know the second spherical joint S_1 S_2 and S_3 then the centroid is nothing but one third of this sum of these three vectors. So, as you can see, I am denoting this vector as base S_1 which is the base platform is a coordinate system which is called base with X base, Y base and Z base.

And origin and this vector is from the origin of the base coordinate system to S_1 S_2 and S_3. So, if I can know these three vectors then I can find the centroid this is the formula for a centroid knowing three points of a triangle. The orientation of the top platform can also be obtained. Once I know these three locations of these spherical point joints or three points on this moving platform. So, the first X axis can be assumed to be from S_1 to the centroid.

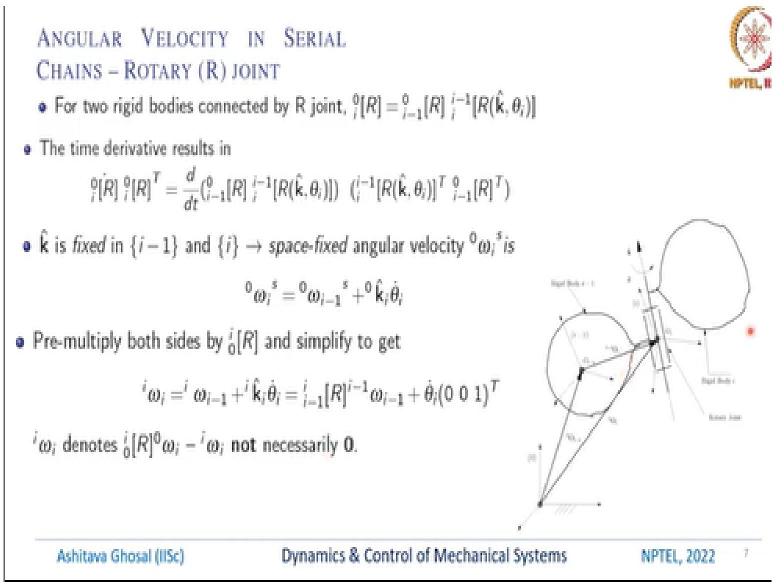
So, this vector $Basep$ - $BaseS_1$. So, $Basep$ is this vector $BaseS_1$ is this vector so the difference of these two vectors will be along this arrow along this vector. And then we can find it as a unit vector which is nothing but this vector divided by the magnitude of this vector. We can also find what is the normal to this plane. So, we can find base to P so $Basep$ - P S 1 which is this vector we can also find $Basep - BaseS_3$ which is another vector which is along this direction.

And the cross product of these two vectors will be normal to this plane. Again, we can divide by the magnitude to make it as a unit vector. So, this vector is the Z axis, this is the X axis and Y axis is the cross product of Z and X. So, remember the rotation matrix is nothing but the X Y and Z axis along the columns. So, the first column is the X axis, third column is the Z axis and the second column is the Y axis which is nothing but Z cross X.

So, we can find the orientation of the top platform with respect to the base or the moving platform with respect to the base again if I know these three locations of these three spherical joints. The angular velocity of the top platform can be obtained from $[\dot{R}]$ $[R]^T$. Again, this base and top and base and top look tells us what the angular velocity of which platform or which rigid body is we are interested in.

So, here we are interested in the angular velocity of the top platform with respect to the base or the moving platform with respect to the base platform. And once I know what the rotation matrix is, we can do $[\dot{R}]\ [R]^T$ this will be a skew symmetric matrix and I can extract the three components of the angular velocity vector.

(Refer Slide Time: 19:28)



So, let us now go back to serial chains. So, I want to find formulas for finding the angular velocity and the linear velocity of one rigid body connected to a previous rigid body by means of a rotary joint. So, first we will look at rotary joints. So, then for two rigid bodies connected by a rotary joint the rotation matrix of the ith with respect to the fixed reference 0 is nothing but 0, i - 1 into what is happening at the rotary joint.

So, remember at the rotary joint we are only allowing a rotation of theta i with along k axis. So, we can find the rotation matrix and then we can pre-multiply this rotation matrix with the rotation matrix of the ith i - 1 with rigid body. So, 0 i - 1 into i - 1 i will give me the rotation matrix of the ith rigid body with respect to the 0th rigid body. So, if you take the time derivative of this equation, I want to find out the angular velocity.

What happens to the angular velocity of the secondary rigid body with respect to the previous rigid body? So, we need to take the time derivative of this if you take the time derivative and do some simplification. You can show that this $[\dot{R}]\ [R]^T$ of the left-hand side is nothing but this $\frac{d}{dt}\ (i - 10[R]\ ii - 1[R(\hat{k},\ \theta)])\ \ (ii - 1[R(\hat{k},\ \theta)]^T\ i - 10[R]^T)$. So, we are trying to find out what is the angular velocity the space fixed angular velocity if you recall which is $[\dot{R}]\ [R]^T$

And this follows from the derivative of left hand side and this follows from the derivative of the right hand side. So, since k is fixed in i - 1 and i so k is the common axis above well about which the second rigid body is rotating with respect to the first rigid body. So, we can rewrite this equation as the $0\omega_i^s\ =\ 0\omega_{i-1}^s\ +\ 0k_i\ \dot{\theta}_i$, it is very sort of obvious.

So, I have angular velocity of the first rigid body, the second rigid body is connected with the first rigid body by means of a rotary joint. The rotary joint is allowing a rotation of the second rigid body with respect to the first. So, the angular velocity of the second will be nothing but the angular velocity of the first plus what is happening at the rotary joint. So, this is exactly this equation written in a formal mathematical form which we can use later on for analysis.

So, if you pre-multiply this equation with $0i[R]$. So, what is $0i[R]$? It is nothing but the transpose of this rotation matrix. So, $0i[R]$ is 0 with respect to i till now we have we were doing i with respect to 0. So, this is 0 with respect to R so that is nothing but the inverse of the rotation matrix which is same as the transpose. So, if you pre-multiply both sides of the equation what you can see is you will get $0i[R]$ into $0\omega_i^s$.

So, again 0 and 0 sort of cancels out in your mind and we are left with $i\omega_i$ on the left hand side. On the right hand side, we will have $i\omega_{i-1}$ and then we have $ik_i\ \dot{\theta}_i$. So, what is ik_i? So, what is the k axis in its own coordinate system depends on what is chosen as the k axis. Most of the time the; rotary joint is assumed to be along the Z axis. So, ik_i is nothing but the Z axis in its own coordinate system hence that is 0 0 1.

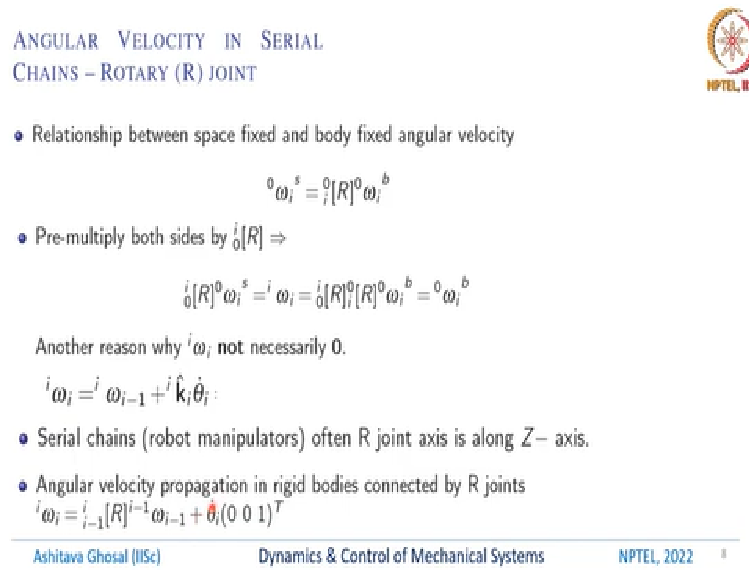
So, this term will become $\dot{\theta}_i$, 0 0 1, column vector. Whereas this term $i\omega_{i-1}$ can be rewritten as $i - 1i[R]\, i - 1\omega_{i-1}$. So, again simple matrix multiplication. So, if you pre multiply this $i - 1\omega_{i-1}$ with this rotation matrix $i - 1i[R]$. So, basically i - 1 with respect to the ith coordinate system opposite. Remember we are going from i - 1 to i but this is telling you what is the $i - 1i[R]$.

So, we will get this. So, one important thing in this equation is that this $i\omega_i$ is not necessarily 0. What is $i\omega_i$? It is nothing but this angular velocity of the rigid body i with respect to the zero-coordinate system but expressed in the ith coordinate system. So, we know what is $0\omega_i$ that is nothing but this $[\dot{R}]\ [R]^T$and we want to pre multiply by $0i[R]$ and that is what we are denoting as $i\omega_i$, it is not necessarily 0.

We will take a look at these next few slides to give a more interesting interpretation of what is $i\omega_i$ and why it cannot be zero all the time. So, in this picture as a summary what do we have we have this rigid body i we have another rigid body i - 1. So, the angular velocity of this rigid body i with respect to i - 1 is nothing but the angular velocity of this rigid body i - 1 + the rotation at this rotary joint.

So, these are two vectors we just add them in the same coordinate system and we get the angular velocity of this rigid body i with respect to i - 1 when it is connected by a rotary joint.

(Refer Slide Time: 26:24)



Now let us look at what is exactly this $i\omega_i$ in a little bit more detail. We know from a previous week that the space fixed angular velocity vector which is $0\omega_i^s$ with the superscript s is related to the body fixed angular velocity vector $0\omega_i$ with the superscript b as in this form. So, this is $\dot{[R]}\ [R]^T$ this is $[R]^T\ \dot{[R]}$. If you go back and see your previous lectures this is $[R]^T\ \dot{[R]}$.

So, we can pre-multiply by a rotation matrix and we can get the space fixed angular velocity vector. So, now let us pre-multiply both sides by $0i[R]$. So, again basically $0i[R]$ is nothing but 0 with respect to i, the previous fixed coordinate system with respect to the moving coordinate system. So, it is in some sense inverse of $i0[R]$. So, let us pre multiply both sides of the equation by this matrix. So, I have $0i[R]$ into $0\omega_i$ this will give me $i\omega_i$.

And what is happening to the right hand side? I have $0i[R]$ which is this and multiplying $i0[R]$ which is as before. So, now the product of these two matrices is nothing but identity. Because this is zero coordinate system with respect to i and this is i coordinate system with respect to 0. Again, in our mind if you think of this as 2 matrices you can cancel these zeroes and we have $ii[R]$. So, there is no rotation.

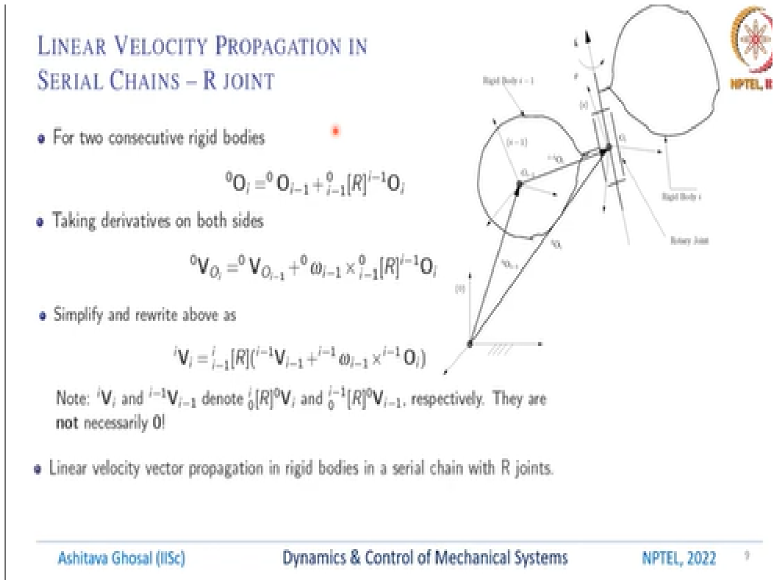
So, this is an identity matrix and hence we are left with only this term which is $0\omega_i^b$. So, what this simple algebra is showing you that this quantity which is $i\omega_i$ is nothing but this body fixed angular velocity vector and hence $i\omega_i$ is not necessarily 0. Because the body fixed angular velocity vector need not be zero all the time. So, finally what we are left with is that $i\omega_i$ is same as $i\omega_{i-1} + ik_i\dot{\theta}_i$.

So, in serial chain as I have mentioned last slide that the rotate R joint axis is typically along the z axis and hence $i\omega_i$ can be written as $i - 1i[R]\, i - 1\omega_{i-1} + \dot{\theta}_i(0\ 0\ 1)^T$. So, what is the nice feature of this formula? We can start with let us say i = 1. So, when you put i = 1 in this formula we have what is this $0\omega_0$. So, typically what is 0? That is the fixed or the reference coordinate system which is not typically rotating unless it is on some complicated system where the base itself is moving.

So, then this we can find the right hand side we know what is happening to the at the first joint which is i = 1 remember θ_1 is the first joint. So, this is about the Z axis. So, this formula will tell you what is the angular velocity of the first link with respect to the previous link. Then we can put i = 2 then the right hand side is $1\omega_1$ which we have obtained from the previous step and then this will give me $2\omega_2$.

So, I can basically go from the fixed base all the way to the free end every time and we add this what is happening at the rotary joint and I can get the angular velocity each of these links in a serial chain. This is a very useful formula these are called angular velocity propagation formulas in rigid bodies connected by R joints.

(Refer Slide Time: 31:06)



We can also find what is happening to the linear velocities of each rigid body with respect to the previous rigid body. And this comes from this basic expression for the position vector of a point on the second rigid body with respect to the first rigid body. So, we can see that this origin $0O_i$ is nothing but the $0O_{i-1} + i - 10[R]\ i - 1O_i$. So, that it is all consistent.

So, here is an example. So, what we have is the $0O_i$ which is the vector which is locating the origin of the ith coordinate system. It is nothing but the vector which is from origin of the fixed or the reference coordinate system to the origin of the i - 1 coordinate system or the i - 1 rigid body and then the vector from the i - 1 origin to the ith origin. So, this vector is nothing but this plus this but we cannot simply add two vectors in two different coordinate system.

See this is $i - 1O_i$. whereas this is $0O_i$. and this is $i - 1O_i$. So, we have to pre-multiply by a rotation matrix. So, from this expression I can take the derivatives of both sides and I get the velocity of the origin of the ith coordinate system or the ith rigid body is nothing but the $0V_{O_i} = 0V_{O_{i-1}} + 0\omega_{i-1} \times i - 10[R]\ i - 1O_i$. We can show that this will lead to the derivative of the second term will lead to something like $\omega \times R$.

And again, we can say that this we need to do it properly. So, that the omega is in the 0th coordinate system and then this R is also in the 0th coordinate system in the fixed reference or zero coordinate system. So, this is a well-known formula. We are now you know we are putting all kinds of superscripts and subscripts just to make it very formal that if you have whole bunch of rigid bodies then we can relate the linear velocity of the ith rigid body in terms of the i - 1 rigid body.

Both of them are connected by a rotary joint as nothing but the linear velocity of the origin of the i - 1 rigid body into some omega times R. And again, we can pre-multiply by a rotation matrix $0i[R]$. So, basically i with respect to 0 and then we will get a term which $iV_i = i - 1i[R]\ (i - 1V_{i-1} + i - 1\omega_{i-1} \times i - 1O_i)$. So, just pre multiply the previous equation with i to 0 rotation matrix and then rearrange and we will get this.

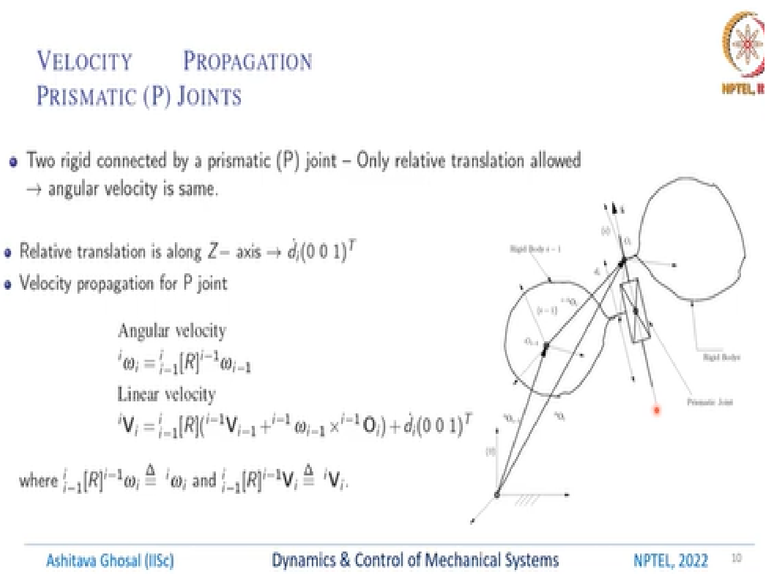
So, this is also a propagation formula for the linear velocity vector. So, if I know what is the linear velocity of the origin of the i - 1 rigid body or the coordinate system origin of the i - 1 coordinate system and then I know that the second rigid body is you know connected to the first rigid body by means of a rotary joint then we can do this. So, just add this velocity with $\omega \times R$ and again this $iV_i\ and\ i - 1V_{i-1}$ are not necessarily 0.

So, what is iV_i? It is nothing but the linear velocity of the origin of the ith rigid body with respect to zero but pre-multiplied by a rotation matrix. And what does this pre-multiply by a rotation matrix means we are expressing it in another coordinate system. And the same story is what is happening to $i - 1V_{i-1}$. It is nothing but the linear velocity of the i - 1 rigid body or the origin of the i - 1 rigid body and then pre-multiply by a rotation matrix.

So, these quantities are not necessarily 0. Remember we had discussed this much earlier that the derivative of a position vector will give the linear velocity. But I can express this linear velocity in some other coordinate system. So, this is exactly what we are doing here. We are finding the derivative in the 0th coordinate system but we are expressing it in the ith coordinate system or for i - 1 rigid body we are finding the derivative in the 0th coordinate system but we are expressing it in the i - 1 coordinate system.

So, this is just a way to show that we can go from fixed base which is normally 0 velocity then we can keep on adding these $\omega \times R$ terms and this $i - 1V_i$ terms to obtain the linear velocity of all rigid bodies in this serial chain. And in this case, we are assuming all the serial rigid bodies all the rigid bodies in the serial chain are connected by R joints. So, this formula is very well known and it is used in many serial chains. It is nothing but the linear velocity vector propagation in rigid bodies in a serial chain with R joints.

(Refer Slide Time: 37:33)



Now let us look at if the rigid bodies are connected by prismatic joints. Prismatic joints are the one which allows relative translation between the two connected rigid bodies, only relative translation is allowed. So, if only relative translation is allowed then the angular velocities of these two rigid bodies will be same, there is no rotation happening to the at the joint. So, the second rigid body and the first rigid body will have the same angular velocity.

So, here is a figure. So, I have a rigid body i and have a rigid body i - 1 they are connected by this prismatic joint which allows translation along this joint axis and the variable which represents translation is d_i. And again, we have all these different axes and the origin of the i – 1th rigid body and the origin of the ith rigid body. So, if the relative translation is along the Z axis meaning we choose that the k axis is the Z axis which is very commonly used in robotics.

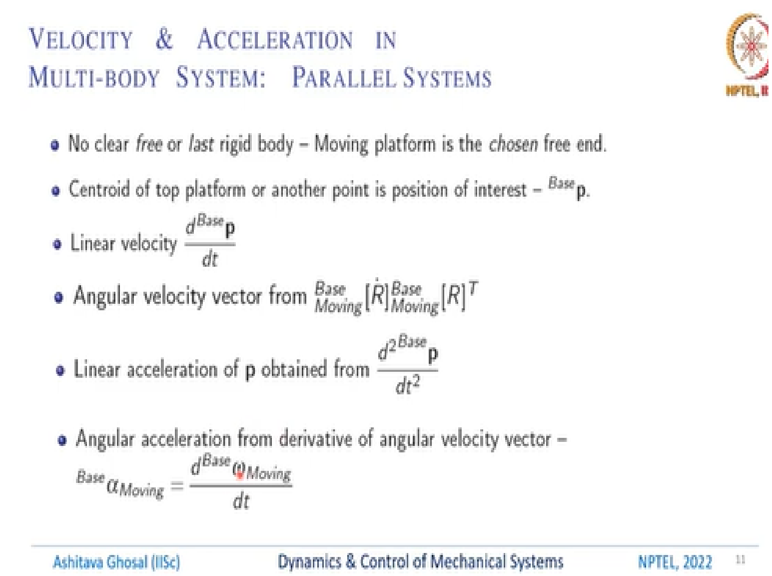
Or in many multi-body systems that the joint axis is chosen as the Z axis. So, what is the relative translation? It is $\dot{d}_i$ along 0 0 1. So, d_i is the translation. So, the derivative is denoted by $\dot{d}_i$. And what is the velocity propagation for the prismatic joint? As I said the angular velocitiespk are same. So, the angular velocity of this i - 1 is the same as the angular velocity of i and again we rewrite in this form that $i\omega_i$ is nothing but $i - 1\omega_{i-1}$.

But we have to write both these left and right hand side in the same coordinate system. So, we pre-multiply by rotation matrix $i - 1i[R]$. So, remember what is $i\omega_i$, it is nothing but the body fixed angular velocity vector of the rigid body i in the 0th coordinate system remember I had proven this earlier. Likewise, the linear velocity of the ith rigid body can be written in terms of the linear velocity of the is i - 1 rigid body.

So, what will it be? It will be nothing but this velocity vector will be equal to this velocity of the i - 1 + some $\omega \times R$. But now we have a translation along the Z axis because the second rigid body can translate with respect to the first rigid body along this direction along this k which is chosen as Z which is 0 0 1 for this example. So, we have $i - 1V_{i-1}$ some $\omega \times R$ and then this $\dot{d}_i$ and we would like to write in this recursive.

Or this propagation form which is iV_i some matrix multiplication into($i - 1V_{i-1}$ + $i - 1\omega_{i-1} \times \quad i - 1O_i$) + $\dot{d}_i(0\ 0\ 1)^T$. And again, this $i - 1i[R]\, i - 1\omega_i$ is nothing but $i\omega_i$ which is the body fixed angular velocity vector of rigid body i. So, it is $0\omega_i$ with the superscript b and the same story is with iV_i they are not both zeros. So, iV_i is nothing but the linear velocity of rigid body but written in a different coordinate system.

(Refer Slide Time: 41:32)

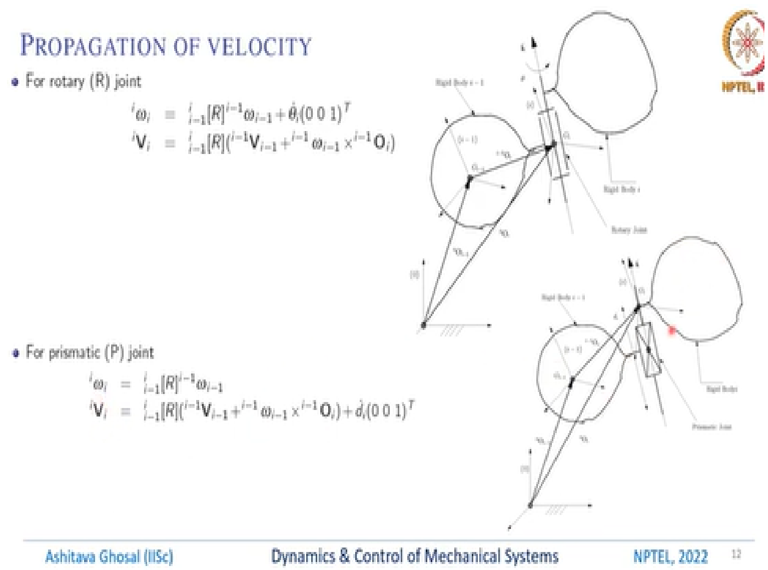


So, let us continue. Let us look at velocity and acceleration in multibody systems and we want to look at parallel systems. So, to repeat I have said these many times that there is no clear free or last rigid body in a parallel system. So, most of the; time the moving platform is chosen as the free end. The centroid of the top platform or any other point of interest can be obtained with respect to the fixed or the base coordinate system by a vector.

The linear velocity of this centroid is nothing but the derivative of this with respect to time the angular velocity of this moving coordinate system is nothing but $[\dot{R}]\ [R]^T$. So, this is the space fixed angular velocity vector. The linear acceleration of a point on this moving platform in particular if it is the centroid then it is nothing but the second derivative of this position vector. So, we can find the second derivative and then that will be the acceleration of this point.

The angular acceleration can be obtained from the derivative of the angular velocity vector. Again, I have said it earlier going from rotation matrix to angular velocity requires you to compute this $[\dot{R}]\ [R]^T$ then up find the skew symmetric matrix and then extract from that skew symmetric matrix the X, Y and Z components. Whereas if you want to go from angular velocity to angular acceleration all you need to do is just take the derivative because this is now a vector, we do not have to worry about this $[\dot{R}]\ [R]^T$ and so on.

(Refer Slide Time: 43:30)



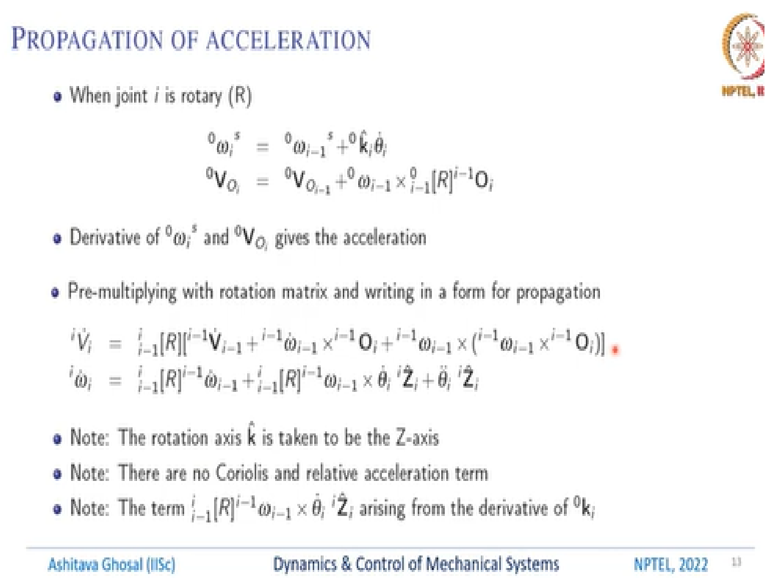
So, let us look at propagation of velocity in summary. So, for an R joint the angular velocity of the ith link in terms of the i – 1th link is nothing but the $i - 1i[R]\; i - 1\omega_{i-1} + \dot{\theta}_i (0\ 0\ 1)^T$. Assuming again that the rotation axis is the Z axis and again all this pre-multiplication by a rotation matrix is to write it in a suitable convenient form. Similarly, the linear velocity of this ith rigid body is nothing but the $i - 1i[R]\; (\; i - 1V_{i-1} + \; i - 1\omega_{i-1} \times \; i - 1O_i\;)$.

Again, all these matrices and other things are to make it consistent and write it in a suitable form in a desired form. If you have a prismatic joint then the linear velocity of this prismatic joint will be the linear velocity of the previous rigid body. So, linear velocity of the ith rigid body will be nothing but the linear velocity of the i - 1 with rigid body + some omega across R. But we now have to add the translation at this prismatic joint.

And how about the angular velocity? The angular velocity of these two rigid bodies are same, because the prismatic joint does not allow any relative rotation, it only allows relative translation. So, this is what is written in these two equations. So, $i\omega_i$ is nothing but $i - 1i[R]\; i - 1\omega_{i-1}$ to make it consistent and again $i\omega_i$ is nothing but the body fixed angular velocity vector of rigid body i written in the ith coordinate system.

And similarly, the linear velocity of this rigid body i described in the ith coordinate system is nothing but $i-1i[R]\left(i-1V_{i-1}+\ i-1\omega_{i-1}\times\ i-10_i\right)+\ \dot{d}_i(0\ 0\ 1)^T$.

(Refer Slide Time: 45:59)



The propagation of acceleration is a little bit more interesting and we start again with the linear and angular acceleration of rigid body i with respect to the 0 coordinate system. So, as I said earlier the angular velocity of rigid body i with respect to the reference coordinate system of the 0 coordinate system is nothing but the $0\omega_i^s = 0\omega_{i-1}^s + 0k_i\dot{\theta}_i$, S stands for space fixed plus the rotations at the joint.

This is when it is connected by a rotary joint. The linear velocity at the origin of the ith coordinate system is nothing but the $0V_{O_{i-1}} + \ 0\omega_{i-1}\times\ i-10[R]\ \ i-1O_i$. So, this $\omega\times R$ has to be done properly because this vector $i-1O_i$. you cannot take the cross product with a vector which is $0\omega_{i-1}$. So, we pre multiply so that these two together is a vector in the zeroth coordinate system.

So, the derivative of these two will give the acceleration because these are vectors. Pre multiplying the derivative rotation matrix and writing in the form for propagation we can show that the derivative of $\dot{V}_i$ with respect again written in the ith coordinate system is again something which is very similar in concept. It is nothing but the $^{i-1}_{i}[R]\left(^{i-1}\dot{V}_{i-1} + {}^{i-1}\dot{\omega}_{i-1} \times {}^{i-1}O_{i} + {}^{i-1}\omega_{i-1}\times({}^{i-1}\omega_{i-1} \times {}^{i-1}O_{i}\right)$.

So, omega dot i - 1 is nothing but the $^{i-1}_{i}[R]\ {}^{i-1}\dot{\omega}_{i-1} + {}^{i-1}_{i}[R]\ \ {}^{i-1}\omega_{i-1} \times \dot{\theta}_i\ {}^{i}\hat{Z}_i + \ddot{\theta}_i\ {}^{i}\hat{Z}_i$. So, most of you would have seen this expression that the acceleration when you have two rigid bodies connected by a rotary joint is the acceleration of the previous rigid body then omega cross alpha cross R and then omega cross $\omega\times R$. So, this is like the centripetal acceleration.

The angular acceleration when we take the derivative of this is nothing but the angular acceleration of the previous rigid body and then what is happening at the joint which is $\ddot{\theta}_i$ along Z axis. But more interestingly we also have another term this will appear if you do properly what it tells you is that we have a term which is ${}^{i-1}\omega_{i-1} \times \dot{\theta}_i\ {}^{i}\hat{Z}_i$. So, this is normally not seen when you are considering a single rigid body.

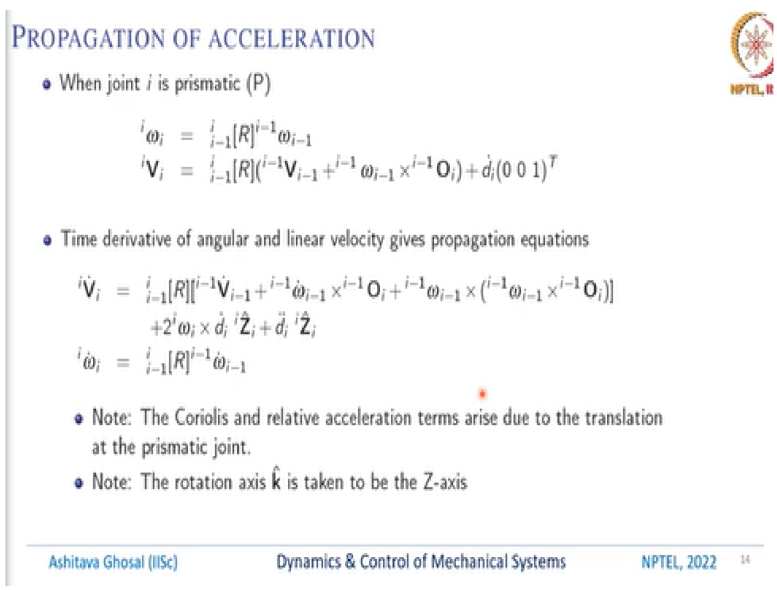
But if you have a whole bunch of rigid bodies which are connected by rotary joint. What is happening at $\dot{\theta}_i$ my along the Z axis this is a vector and you can take the cross product of this ${}^{i-1}\omega_{i-1}$ into this vector. This will also contribute to the acceleration of the ith rigid body and again we need to pre multiply by some rotation matrix to write it in this correct form in a propagation form which is suitable for propagation.

So, this term is normally not seen if you are considering only a single rigid body but this will appear if you do take these derivatives properly. So, as I said the rotation axis is taken as the Z axis most of the time note that there is no Coriolis and relative acceleration term when you have the two rigid wedge is connected by a rotary joint. So, Coriolis acceleration happens when there is a relative velocity of a point on the ith rigid body.

So, a point on the ith rigid body is moving in its own coordinate system with some velocity or with some acceleration. So, then we get these two terms. Remember in the acceleration expression there were five terms here there are only three terms which are the two terms which are missing, the Coriolis term is missing and the relative acceleration term is missing. Because they are not there when you have two rigid bodies connected by a rotary joint.

And I have as I said little while back this is a interesting term. This term which is $i - 1\omega_{i-1} \times \dot{\theta}_i \, i\hat{Z}_i$ arises from the derivative of this k axis. So, if you have several rigid bodies the k axis of 1 is not the same everywhere, the k axis is also rotating and this is what gives this term.

(Refer Slide Time: 51:36)



So, the propagation of acceleration when two rigid bodies are connected in a serial chain with a prismatic joint is very straightforward. We start with the propagation of angular velocity and linear velocity. So, these expressions have been given earlier. The angular velocity of the ith rigid body is same as the angular velocity of the i - 1th rigid body. The linear velocity of the ith rigid body which means the linear velocity of the origin of the coordinate system attached to this ith rigid body is nothing but the linear velocity of the origin of the previous rigid body.

Plus, some $\omega \times R$ and this is the translation at the prismatic joint. If you take the derivative of both these equations so let us start with the simple one. If I take the derivative of the angular velocity equation, we can see that the angular acceleration is nothing but the same as the previous one. So, $i\omega_i$ dot is nothing but $i - 1i[R]\ i - 1\dot{\omega}_{i-1}$. So, that it is in a propagation form suitable for propagation.

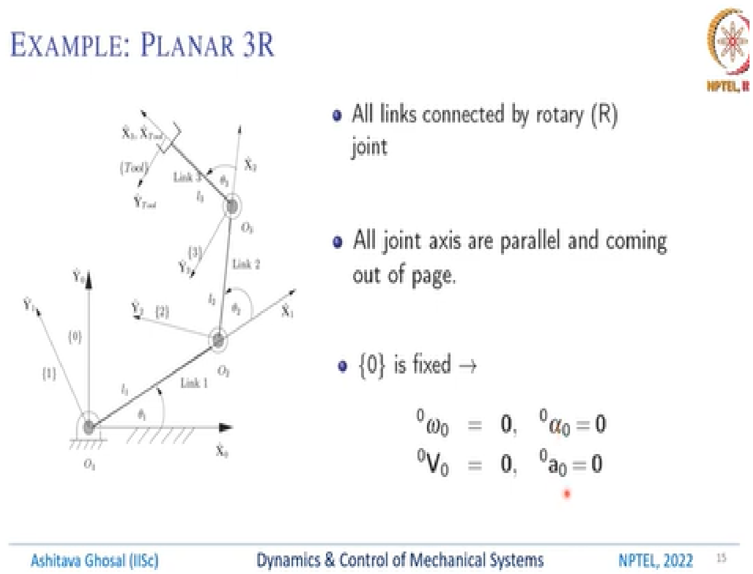
But let us now take a look at this derivative of the linear velocity. So, we have three terms here one which is the velocity of the origin then some $\omega \times R$ and then this is $\dot{d}_i$, what is happening at the prismatic joint. Now in the second rigid body there is some relative motion in its own coordinate system because of this d_i. So, hence we will have Coriolis term and then we will also have the relative acceleration term which is what shows up here.

So, we have some $2\ i\omega_i \times \dot{d}_i\ i\hat{Z}_i$. Both of these a relative is along the z axis and $2\ i\omega_i \times \dot{d}_i\ i\hat{Z}_i$. Of course, we have the other terms which are the centripetal acceleration term then alpha times R and the acceleration of the origin. So, when you have two rigid bodies connected by a prismatic joints, we have all the five terms which are seen in the acceleration of a rigid body in 3D space.

So, in this case it should be noted that the Coriolis and relative acceleration term arises due to the translation at the prismatic joint. So, you can think of this expression that the prismatic joint allows the relative motion of the second rigid body with respect to the first rigid body. So, it is similar to the ant example which we did for a plane. So, we have a rotating coordinate system or some moving coordinate system and then this ant is moving inside in that coordinate system itself.

So, in this case the ith rigid body is moving with respect to the i - 1 its rigid body due to the joint. And again, in this example or in many other examples we will take the k axis as the Z axis. If the k axis is some other axis these expressions will be much more complicated.

(Refer Slide Time: 55:20)



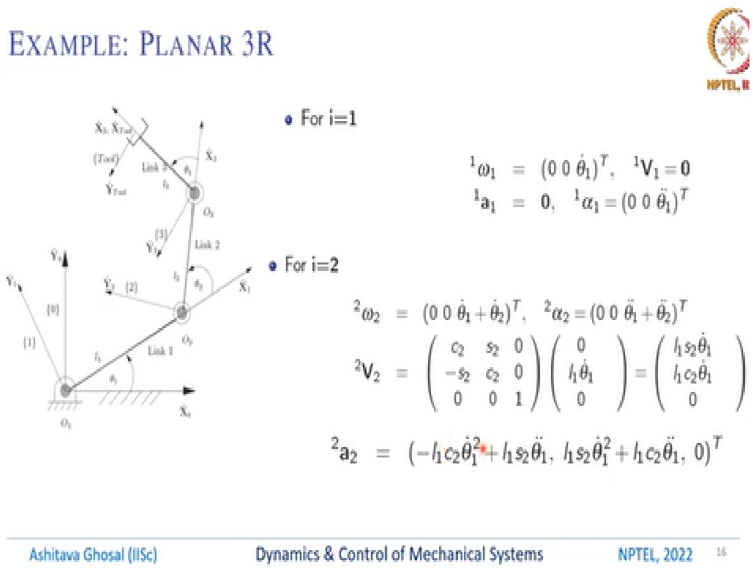
So, let us look at the example of a planar 3 degree of freedom robot. Because this is very simple and we can work it out on the slides and then you can work it out yourself. So, what do we have here, we have three links link one, link two and this is the last link. So, we have a fixed coordinate system which is X_0, Y_0 with the origin here. This is labelled as 0 coordinate system then we have this link 1 which is rotating with respect to the fixed X axis by θ_1 and the X and the Y axis are shown here.

Then we have the second link which is rotating with respect to the link 1 by another θ_2 and this is the X and the Y axis and the origin is here. And then we have the third link which is rotating with respect to the second link by θ_3 and this is the X and the Y axis for the third link. So, this is X_3, Y_3 this is X_2, Y_2 and this is the origin and this is X_1, Y_1 and this is the origin. So, this is an example in a plane.

So, all the Z axis at every coordinate system is coming out of the page and they are parallel to each other. So, all links are connected by rotary joints, all joint axis are parallel and coming out of the page. These are the Z axis. So, if 0 is fixed then the angular velocity of the 0 with respect to 0 is 0. So, these are obvious. So, the bases or the fixed reference coordinate system is not rotating and not translating.

So, the angular velocity and the linear velocity, the angular acceleration and the linear acceleration there they should be all zero.

(Refer Slide Time: 57:28)



Now we look at this i = 1 which is nothing but this link. So, for this link we have 1 omega 1 will be equal to something which is happening at the 0th link. So, $0\omega_0$ plus what is happening at the first rotary joint. The first rotary joint is 0 0 $\dot{\theta}_1$, it is a column vector and this rotary joint is along the Z axis. So, we can obtain what is $1\omega_1$ by substituting in this propagation formula and then you will get $1\omega_1$ is nothing but 0 0 $\dot{\theta}_1$.

Now about the first velocity, the origin of the first link and the 0th link is at the same place. So, you can intuitively see that the origin of the first link O_1 is not moving with respect to O_0. So, hence this is 0, you can also check it mathematically by going back to the propagation equation. And there you can see you will have one term which is linear velocity of i - 1 and $\omega \times R$. So, the linear velocity of the 0th link is 0 and $\omega \times R$ is also 0 because R is 0.

Likewise, the acceleration $1a_1$ is 0 and $1\alpha_1$ is $\ddot{\theta}_1$. For i = 2, we substitute back i = 2 in the propagation equations and we will get $2\omega_2$ as(0 0 $\dot{\theta}_1 + \dot{\theta}_2$), $1\alpha_1$ will be (0 0 $\ddot{\theta}_1 + \ddot{\theta}_2$). The

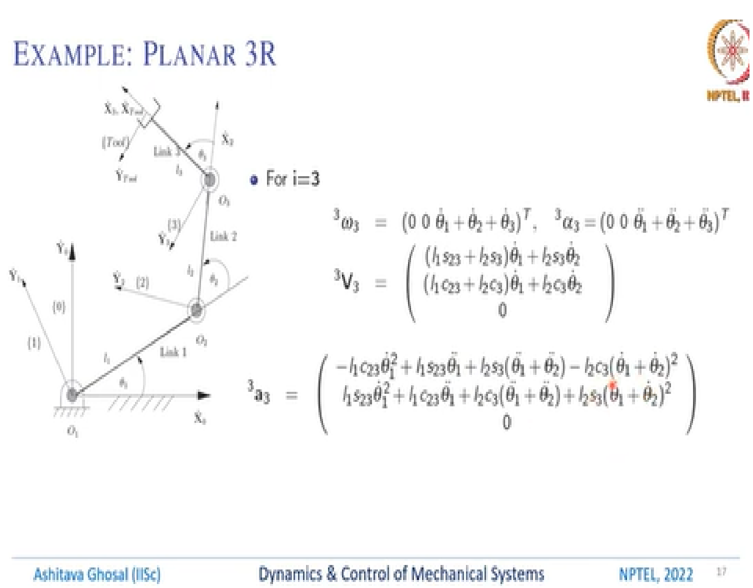
$2V_2$, what does $2V_2$ means? It is the velocity of the second link which is the velocity of the origin of the second link expressed in the second coordinate system.

So, we first find out $0V_2$ and then we multiply pre-multiply by rotation matrix $02[R]$. So, $02[R]$ or it looks like this and then if you expand it, you will get $l_1 s_2 \dot{\theta}_1$, $l_1 c_2 \dot{\theta}_1$ and 0. So, you can sort of see that it looks, why? Because $\dot{\theta}_1$ is what is happening here. The velocity of this will be related to $l_1 \dot{\theta}_1$ and this sin θ_1, sin θ_2 and cos θ_2 will appear if you do it consistently.

The acceleration on the other hand will have the derivative of this with respect to the 0 coordinate system and then rewritten in the second coordinate system you will have all these terms. So, you will have $-l_1 c_2 \dot{\theta}_1^2 + l_1 s_2 \ddot{\theta}_1$, $l_1 s_2 \dot{\theta}_1^2 + l_1 c_2 \ddot{\theta}_1$, 0. So, recall this $\dot{\theta}_1^2$ is similar to the centripetal acceleration, $\ddot{\theta}_1$ is the tangential acceleration.

There is no Coriolis term in this and there is no relative acceleration because there is no relative motion happening.

(Refer Slide Time: 1:01:03)



For i = 3 which is what is happening at this origin of the third link. That is the linear velocity of the origin of the third link again we can go back and put substitute in the formula. So, $3V_3$ will be related to $2V_2 + \omega \times R$. We go back and substitute and you will get this expression which is nothing but $(l_1 s_{23} + l_2 s_3)\dot{\theta}_1$ and something into $\dot{\theta}_2$. And the Y component will have the cosine of the angles.

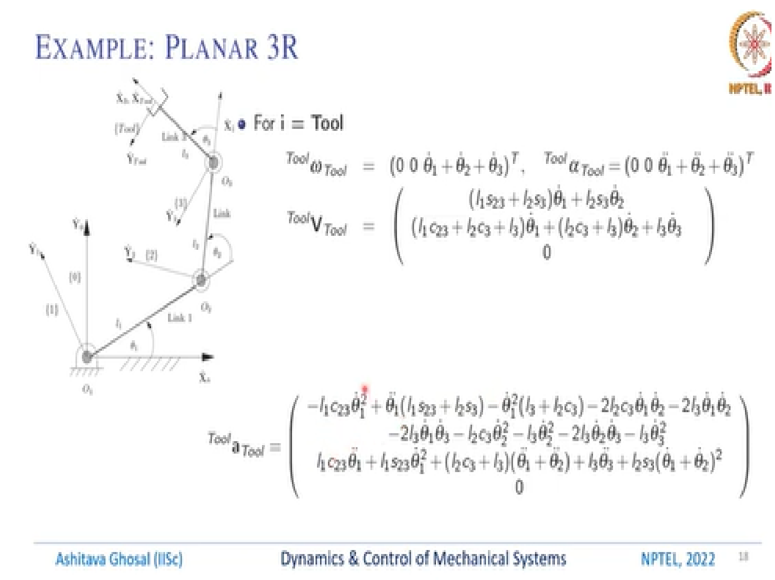
The angular velocity is nothing but $\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$. So, this is $\dot{\theta}_1$, this is $\dot{\theta}_2$, this is $\dot{\theta}_3$. So, this link will have the angular velocity of this plus this plus this and it is sort of obvious because all the angular velocity vectors are coming out of the page. So, if they are all connected by rotary joints the last one will be the sum of all the previous ones. Likewise, the angular acceleration will be $\ddot{\theta}_1 \ddot{\theta}_2 \ddot{\theta}_3$ as its Z component.

The X and Y components are both 0 for both angular velocity and angular acceleration. The acceleration of this third link, the origin of the third link is much more complicated. So, now you can see that you will get some $\dot{\theta}_1^2$, you will also get some $\ddot{\theta}_1$, you will also get some $\ddot{\theta}_2$ and you will also get some $\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2$. So, these are the centripetal terms.

This $\dot{\theta}_1^2$ and $\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2$ these are on the other hand they are the tangential acceleration term. So, there is a velocity which is happening $\dot{\theta}_1$ and then there is $\dot{\theta}_2$because of this $\dot{\theta}_1^2$ and $\dot{\theta}_2^2$ there will be some centripetal acceleration term. The Y component will have similar things but now you have sin ($\theta_2 + \theta_3$) cos($\theta_2 + \theta_3$) here it was sin ($\theta_2 + \theta_3$) and so on.

So, there is some difference between sine and cos and of course there are some changes in the plus and minus.

(Refer Slide Time: 1:03:48)

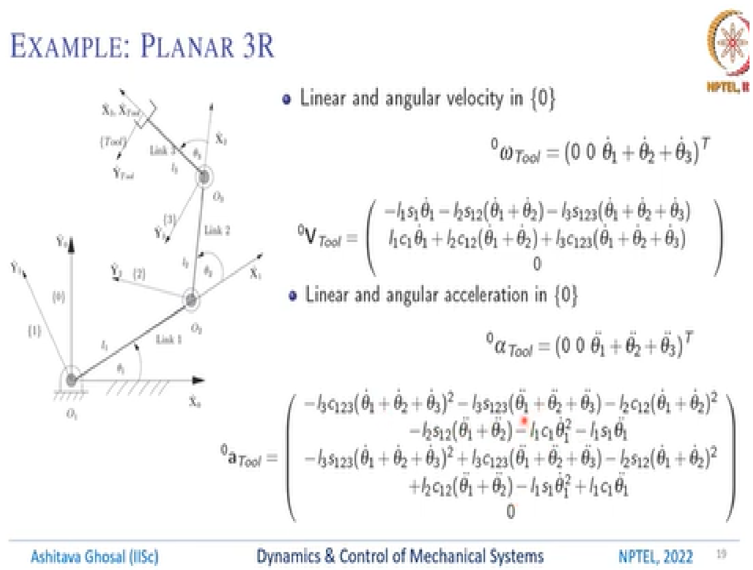


Let us continue. Let us find the angular velocity and the linear velocity of the end effector or the last of the free end of this robot planar three degree of freedom robot. So, this is the serial chain first link, fixed link one joint another link, another joint another link another joint and all the way to a free end. This free end is called the vertex the tool coordinate system. So, basically this is where you would carry the tool which the robot will operate.

So, does not matter in these mechanical systems we consider it to be the free end. So, for the tool coordinate system the angular velocity will be nothing but $\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$. The angular acceleration will be the second derivative of $\ddot{\theta}_1$ $\ddot{\theta}_2$ and $\ddot{\theta}_3$. The velocity of this tool coordinate system will contain $\dot{\theta}_1$ $\dot{\theta}_2$ and also θ_3 and all these angles in its own coordinate system.

The acceleration is much more complicated it is actually long expression. But we can find it just by using the propagation equations which I have shown earlier. So, in this case again we have some centripetal term then we have some tangential acceleration term and then we also have these terms which are $\dot{\theta}_2$,$\dot{\theta}_3$. So, we have $\ddot{\theta}_1$ then we have some $\dot{\theta}_2$, $\dot{\theta}_3$ which is also like a centripetal term and then we have various other terms. So, we can find these terms just by applying the expressions for propagation of acceleration.

(Refer Slide Time: 1:05:53)



Finally let us look at what is happening to the free end in terms of the fixed of the reference coordinate system which is 0. So, the angular velocity of the free end or the tool coordinate system is nothing but the angular velocity of the three joints. So, $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ all along the Z axis. The linear velocity of this is nothing but the expressions which are this. This one you can sort of see which makes sense.

Why is why does it make sense? So, let us look at the X coordinate of this point. So, the X coordinate of this point will be $l_1 \cos\theta_1$ this is the projection then $l_2\ \cos(\theta_1\ +\ \theta_2)$because the angle of this link with respect to the horizontal will be $\theta_1 + \theta_2$. Then the angle for this link will be $(\theta_1\ +\ \theta_2 +\ \theta_3)$. So, the X coordinate will be projection of this which is $l_1 \cos\theta_1\ +\ l_2 \cos \cos\left(\theta_1\ +\ \theta_2\right) l_3 \cos(\theta_1\ +\ \theta_2 +\ \theta_3)$.

So, remember they are all cosines. So, now if I take the derivative in order to get the X component of the velocity we will have $-\ l_1 s_1 \dot{\theta}_1\ -\ l_2 s_{12}(\dot{\theta}_1\ +\ \dot{\theta}_2)\ -\ l_3 s_{123}, s_{123}$ means sin of $(\theta_1 + \theta_2 + \theta_3)$ $(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$. Likewise, the Y component will be sin of θ_1 into l_1 then $\sin sin\left(\theta_1\ +\ \theta_2\right) l_1 s_1\ +\ l_2 s_{12}$ and so on.

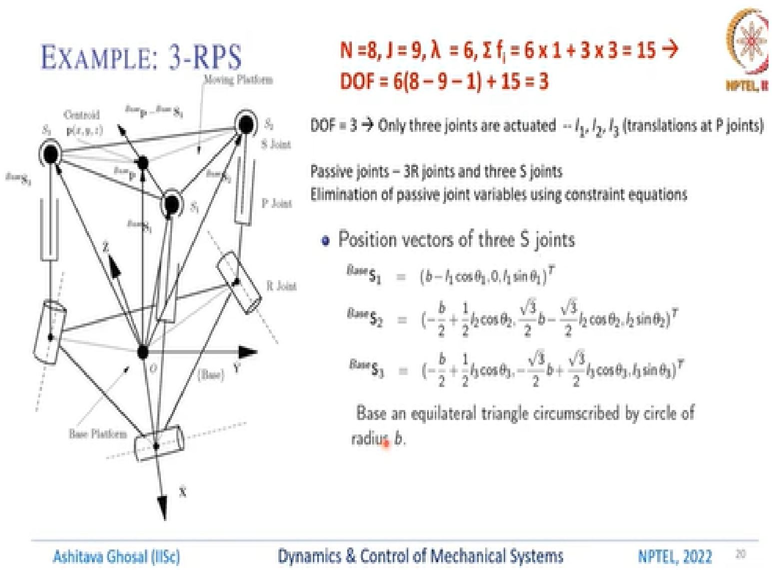
And then when you take the derivative you will have cosine $-\ l_1 c_1 \dot{\theta}_1\ -\ l_2 c_{12}(\dot{\theta}_1\ +\ \dot{\theta}_2)\ -\ l_3\, c_{123}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3)$.. So, if you think a little bit and look at your basic trigonometry from high school this makes sense. So, the velocity vector of this contains $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ and also it contains all the sins and cosines. The linear and angular acceleration of this last scope free end with respect to the 0, the acceleration is very simple it is nothing but $\ddot{\theta}_1$ $\ddot{\theta}_2$and $\ddot{\theta}_3$.

The acceleration on the other hand is much more complicated. So, what should it contain? We can sort of intuitively guess that it will have some Coriolis term sorry not Coriolis term some centripetal term. It will also have some tangential acceleration and we can see that there will be some sin and some cosine of the angles. So, and that is what exactly you can see. So, you will have $\ddot{\theta}_1$ $\ddot{\theta}_2$and $\ddot{\theta}_3$ into sum l_3.

And similarly$l_2(\ddot{\theta}_1 + \ddot{\theta}_2)$and $l_1\ \ddot{\theta}_3$. So, these three terms are the tangential acceleration this is like alpha times R. And then we have three other terms which are $\left(\dot{\theta}_1 + \dot{\theta}_2 +\ \dot{\theta}_3\right)^2$ this is the centripetal term. This is another centripetal term which is $\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2$and then there is this $\left(\dot{\theta}_1\right)^2$

In the case of the Y component again you will have the similar terms but now instead of cosine θ_{123} you will have sin θ_{123} . So, wherever there is a sign here for the tangential acceleration you will have a cosine. So, it sort of makes sense and you can show that this is indeed correct and true.

(Refer Slide Time: 1:10:25)



Unlike the serial chain which we have discussed if you have a mechanism with a parallel configuration or with closed loops it is not so easy to obtain the linear and angular velocities or the acceleration of all the links and this is primarily because of closed loops and it is also because there are many more joints than the degrees of freedom. So, we will take an example of a three RPS parallel system.

The three RPS mechanism looks as follows. So, we have a top moving platform which is this. There is a fixed base platform and between the top moving platform and the fixed base there is a rotary joint a sliding joint and a spherical joint. So, the three RPS means that there are three of these serial chains and each serial chain contains a rotary joint, a prismatic joint and a sliding joint. And in order to analyse or model or solve this mechanism we assign a fixed coordinate system which is called the base coordinate system.

This is attached to the fixed base platform and then we have these vectors which can locate from a fixed origin which is O and this is the X, Y and Z axis. The X axis is chosen such that it is going through one of the joints. So, we will find out what are the degrees of freedom of this mechanism and also the basic idea is that we want to locate these three spherical joints by these three vectors. So, there is a $BaseS_1$ $BaseS_2$ $BaseS_3$ which are nothing but the vectors from O to the spherical joints S_1, S_2 and S_3.

We also have a moving platform. So, hence we have chosen a point which is the centroid of this moving platform. The moving platform has been chosen to be a triangle and in particular an equilateral triangle for simplicity and this point is the centroid. And we will locate this point using a vector $Basep$ with coordinates X, Y, Z. We would also need the orientation of this moving platform and that we will see can be obtained by means of some vectors which we will define on this top platform.

So, for example one such vector is from the spherical joint to the centroid. So, this is P - $BaseS_1$ because this vector is P, this vector is $BaseS_1$. So, this vector along this direction is P - $BaseS_1$. So, if you look at this three RPS parallel system so the number of links is 8. So, they have 1, 2 so one moving one fixed base and then there are two links in each of these chains. So, there is one link here and one link here. So, that is 3 into 2, 6 + 7 + 1, 8, the number of joints is 9.

Because we have 3 rotary joints, 3 prismatic joint and 3 spherical joints. So, j is 9, lambda in this case is 6 because this whole mechanism operates in 3D space. We can also find the degrees of freedom of each one of these joints. So, the 6 1 degree of freedom which is the rotary and the; prismatic joints so that is 6 into 1. Each spherical joint has 3 degrees of freedom so that is 3 into 3 total is 15.

So, the degree of freedom can be obtained as lambda into $N - j - 1$ + the summation of all the degrees of freedom at the joints. So, if you substitute there so you will get 3 degrees of freedom. So, what this means is that there are only three joints which can be actuated. So, we will consider these sliding joints or this prismatic joints as the actuated joints and the variables which is the sliding member is l_1, l_2, l_3. So, these are the translations of the prismatic joints.

So, if you consider these as the actuator joints so the three rotary joints are passive joints and these three spherical joints are also passive joints. So, we do not have any motors which are driving this rotary or the spherical joints. So, in order to solve the kinematics of this mechanism, we have to find equations which will eliminate the passive joints and basically, we need to derive constraint equations. So, for example in the 4 bar we had seen some constraint equations earlier.

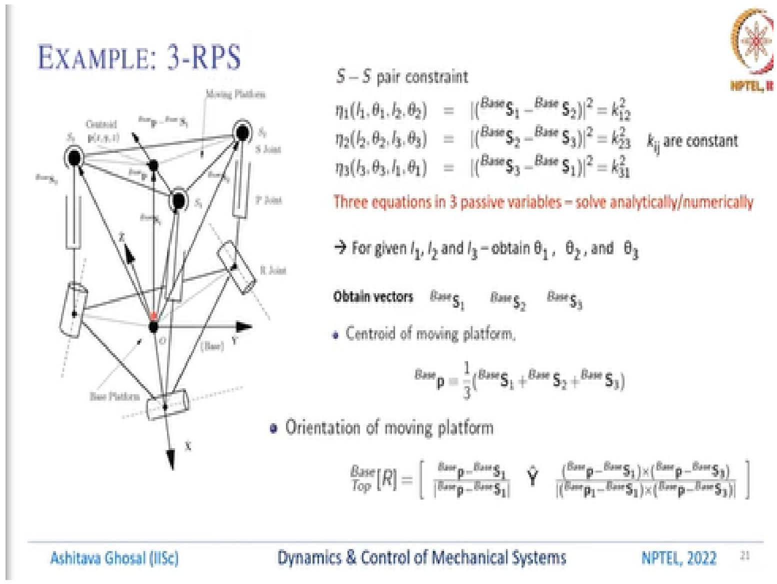
So, how do we derive this constraint equation. So, as I said we will first find the position vector of these three spherical joints S_1, S_2 and S_3 these are denoted by $BaseS_1$, $BaseS_2$ and $BaseS_3$. So, with a little bit of effort you can see that the position vector is given by $(b - l_1 \cos\theta_1, 0, l_1 \sin\theta_1)$ is the rotation at the rotary joint 1. This is b, the side of this the length of from O to this rotary joint is b. So, basically it is a centroid.

It is an equilateral triangle and the radius of the circle subscribing this equilateral triangle is b. So, this will be b, this is l_1 going this way, this angle is θ_1. So, hence it will $(b - l_1 \cos\theta_1, 0, l_1 \sin\theta_1)$ the Y coordinate will be 0 and the Z coordinate is $l_1 \sin\theta_1$So, the Z coordinate is going along this vertical direction. Likewise, with a little bit of more effort we can see that the $BaseS_2$ is given by $-\frac{b}{2} + \frac{l_2}{2}\cos\theta_2$.

And then we have a Y coordinate in this case which $is\frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_2\cos\theta_2$, $l_2\sin\theta_2$. So, b is the radius of this circumscribing circle of this base and a will be the radius of the circumscribing circle of the top platform. So, we will see what this a and b numbers are later on in a numerical example. Finally, we can also find $BaseS_3$ which is $-\frac{b}{2} + \frac{l_3}{2}\cos\theta_3$. - $\frac{\sqrt{3}}{2}b - \frac{\sqrt{3}}{2}l_3\cos\theta_3$, $l_3\sin\theta_3$.

So, this is basically simple vector algebra, we can find vectors and then we can see the components of each one of these vectors. So, as I said the base is an equilateral triangle circumscribed by a circle of radius b.

(Refer Slide Time: 1:18:01)



So, once we find out these vectors which locate this spherical joint S_1, S_2 and S_3 we can find the length between $BaseS_1$ and $BaseS_2$. So, this is one of the sides of this top equilateral triangle and this distance is constant. So, this is also sometimes called as the S S pair constraint which we had looked at earlier. So, this magnitude of $\left(BaseS_1 - BaseS_2\right)^2$ is denoted as k_{12}^2 and as you can see the vector from 0 to $BaseS_1$ will contain l_1 and also θ_1.

The vector from 0 to $BaseS_2$ will contains l_2 and θ_2. So, this constraint equation will be a function of l_1 l_2 θ_1 θ_2. Likewise, we can also say that the distance between S_2 and S_3 is constant which is k_{23}^2. So, this is $\left(BaseS_2 - BaseS_3\right)^2$ is k_{23}^2 and similarly S_3 the distance between S_1 and S_3 is also constant. So, this is $\left(BaseS_3 - BaseS_1\right)^2$ is k_{31}^2.

And similar to the first equation the second equation will only contain l_2, l_3 $\theta_2\theta_3$ and the third equation will contains l_1, l_3 θ_3 θ_1. Again, go back to this definition of what is $BaseS_1$ this contains l_1, l_1 which is this way and θ_1 what is $BaseS_3$ this will contain θ_3 and l_3. So, this distance which is the difference between this point and this point will contain only l_1, l_3, θ_1, θ_3.

So, what do we have here, we have three equations and in these three equations you can see that l_1, l_2, l_3 are actuated joint variables. So, we would be given the values of the actuated joint variables. We have three degrees of freedom so three of those joints will be known. So, in this case we have assumed that the prismatic joint variables are known. So, we have three equations and there are three passive joint variables which are what θ_1, $\theta_2$and θ_3.

So, we should be able to solve for θ_1, $\theta_2$and θ_3 analytically or numerically. So, what is the problem so we are given l_1, l_2, l_3 we have to first find θ_1 $\theta_2$and θ_3. So, once we obtain θ_1 $\theta_2\theta_3$ for a given l_1, l_2, l_3 we can then specify or find these vectors base to S_1, 0 to s_2 and 0 to s_3. So, we know these three vectors. So, the centroid of this top moving platform is one third the sum of these vectors.

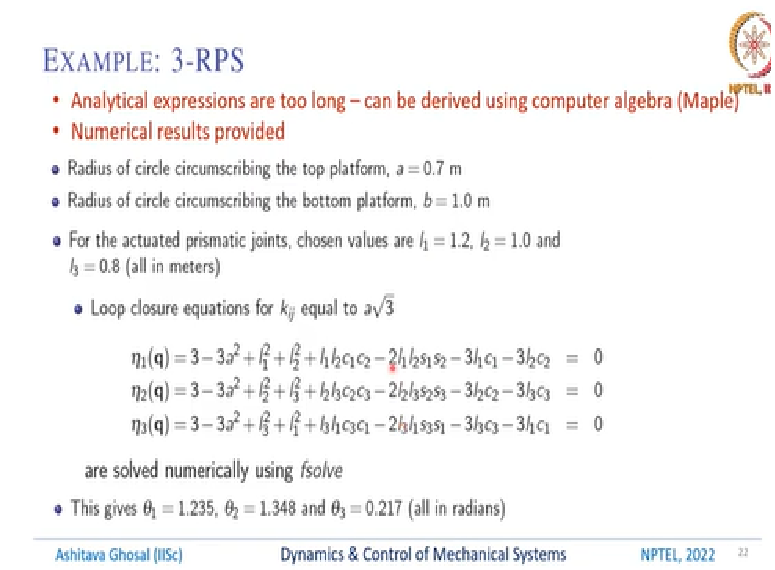
And that is because we have assumed this triangle to be equilateral triangle. If it is not an equilateral triangle then there will be some relationship between the three vectors or the three points. The orientation of the moving platform can also be found once we know θ_1,θ_2 , θ_3 for a chosen l_1, l_2, l_3. Again, we have these vectors you know from S_1 to P. So, what is S_1 to P? We know P because this from the previous step we know what is this vector.

We also know $BaseS_1$ so this is this direction. We can also find similarly the P to 3. So, we can find this vector direction from S_3 to P. So, we have two vectors in a plane so the cross product of those two vectors will be along the Z direction which is what is shown here. So, the X axis is along this vector as shown here. The cross product of this with this vector is the Z axis and that is shown here and the Y axis is the cross product of Z cross X.

And why do we divide by the magnitude? Because we need unit vectors. So, this vector between $Basep$ - $BaseS_1$ is not an unit vector. So, when you divide by the magnitude you will get a unit vector. Similarly, we divide by the magnitude of the cross product and we get a unit vector along the Z direction and Y is Z cross X. So, we have defined the X axis of the moving platform, the Z axis of the moving platform and the Y axis of the moving platform.

And hence we know the rotation matrix. Because again if you go back and remember any rotation matrix is the moved or the X_B in A coordinate system, Y_B in A coordinate system and Z_B in A coordinate system. In this case we have basically X_B with respect to the base coordinate system, Y_B with respect to the base coordinate system and Z_B with respect to the base coordinate system.

(Refer Slide Time: 1:23:29)



So, the position vectors of the centroid of the top moving platform and the rotation matrix look giving the orientation of the top moving platform with respect to the base. They are very long and complicated in fact if I start writing them, they will not fit into this page. So, the analytical expressions are too long but they can be derived. They can be derived using computer algebra systems. So, we have used maple which we will discuss later of how to derive these expressions.

Instead, in this slide I am going to show you numerical results. So, to obtain numerical results we first choose some of these parameters. So, the radius of the circle circumscribing the top platform a is chosen as 0.7 meters. The radius of the circles are circumscribing the bottom platform b is chosen as 1 meter. And this is a three degree of freedom mechanism so there are three actuated joints and these were chosen to be the sliders P_1 prismatic joints.

So, we will assume that the slight sliding at the prismatic joints are denoted by l_1 $l_2 l_3$ and they are like 1.2, 1.0 and 0.8 meters. So, once we have those loops closure constraint equations remember we had three constraints equation θ_1 which was a function of l_1 l_2 θ_1 and θ_2. So, again using maple we can simplify that equation and we will get a reasonably complicated expression like this.

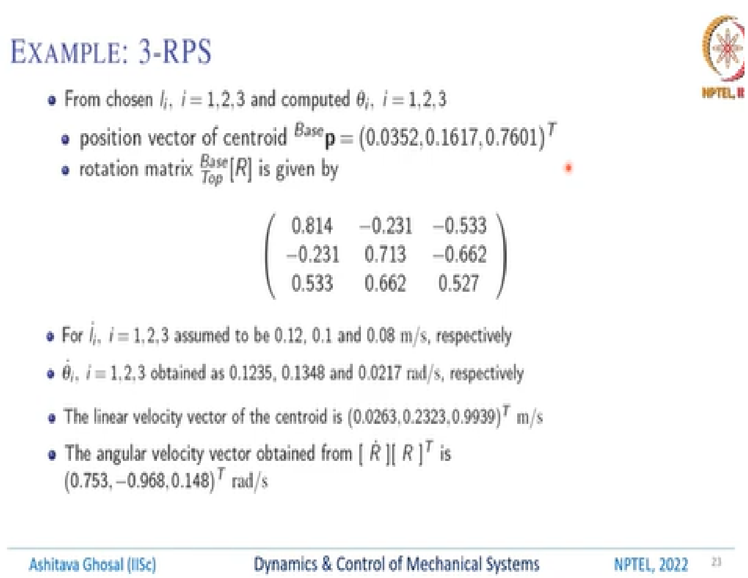
So, the first equation is $3 - 3a^2 + l_1^2 + l_2^2$ and then l_1 $and\, l_2 \cos\theta_1 \cos\theta_2 - 2 l_1 l_2 \sin\theta_1 \sin\theta_2 - 3 l_1 c_1 - 3 l_2 c_2$ this is equal to 0. So, as expected the first equation contains only l_1 $l_2 \theta_1$ and θ_2. So, c_1 here means $\cos\theta_1$ c_2 here means cos θ_2, s_1 is sin θ_1, s_2 is sin $\theta_2$and so on. The second equation which was between 2 and 3 the spherical joint 2 and 3 will contains $l_2 l_3$ $\theta_2$and θ_3 and it sort of looks very similar.

And it is $3 - 3a^2 + l_2^2 + l_3^2 + l_2 l_3 c_2 c_3 - 2 l_2 l_3 s_2 s_3 - 3 l_2 c_2 - 3 l_3 c_3 = 0$. And the third equation is the distance between S_1 and S_3 which is constant. And we are going to assume that it is circumscribed by a circle of radius a. So, hence since it is an equilateral triangle the sides are of length a root 3. So, the third equation is $3 - 3a^2 + l_1^2 + l_3^2 + l_3 l_1 c_3 c_1 - 2 l_3 l_1 s_3 s_1 - 3 l_3 c_3 - 3 l_1 c_1 = 0$.

So, these are three nonlinear transcendental equations. So, we have sin theta, cos theta and product of sin and cos and so on. So, this we cannot solve very easily but again we can solve it numerically and MATLAB gives us solver of algebraic equations like this something called f solve. So, in f solve we substitute these three equations and give some initial guesses and then we can find the values of θ_1, $\theta_2$and θ_3.

So, in these equations we substitute l_1 as 1.2, l_2 as 1.0, l_3 as 0.8 and then we give it to f solve and this will give you θ_1 as 1.235, θ_2 as 1.348 and θ_3 is 0.217 in radians. So, this is numerical solution because the analytical expressions are much too complicated. For this particular parallel mechanism analytical expressions are also available.

(Refer Slide Time: 1:28:17)



So, once we have found out θ_1, θ_2, θ_3 for the chosen l_1, l_2, l_3 we can go back and substitute the values of l_1, l_2, l_3 and corresponding values of θ_1, θ_2, θ_3 in the vectors $BaseS_1$ + $BaseS_2$ and $BaseS_3$. And then we take the one third of all some of these three and we get the position vector of the centroid and that we can obtain in this expression the numerical values are 0.0352, 0.1617, 0.7601 in meters.

The rotation matrix can also be found out as I had explained the X axis is between S_1 and P, the Z axis is the normal to the top platform and Y axis is Z cross X. So, we will get a 3 by 3 rotation matrix, the first column is the X axis, the last column is the Z axis and this is the Y axis. So, we will get 0.814, - 0.231, 0.533 and likewise for the Z axis is - 0.533, - 0.662, 0.527. So, these are given up to third place of decimal and this is the Y axis.

We can also go and find the velocity of the centroid. So, for that we need to take the derivative of the position vector which is given by this. Of course, we have to take the derivative of the analytical form of the position vector. And when you take the derivative, you will get $\dot{l}_1$, $\dot{l}_2$, $\dot{l}_3$ and $\dot{\theta}_1$, $\theta_2$dot, $\dot{\theta}_3$ they will contain all these terms and also of course l_1, l_2, l_3 and cos θ_1, sin θ_2 and all these trigonometric functions.

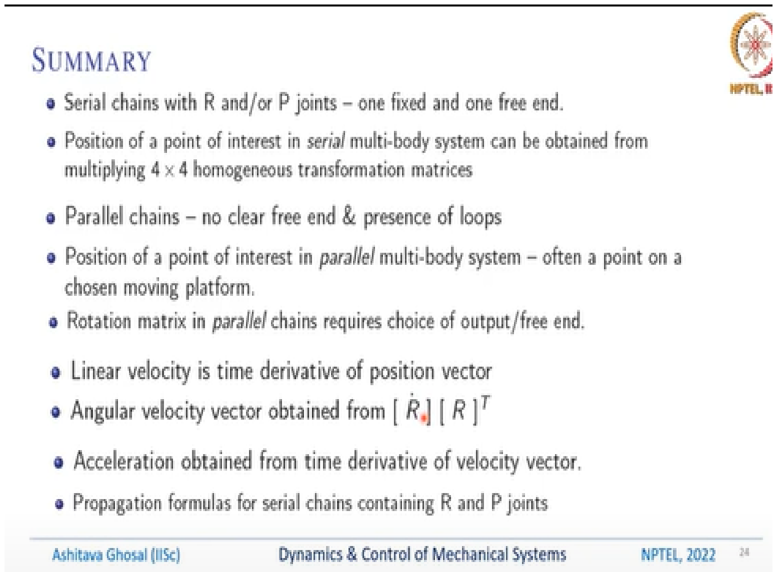
So, for obtaining the velocity of the centroid we have chosen $\dot{l}_1$ as 0.12, $\dot{l}_2$ 0.1 and $\dot{l}_3$ at 0.08 meters per second. The $\dot{\theta}$ can be also obtained from again the loop closure equations or the derivative of the loop closure equations and they are given by $\dot{\theta}_1$ is 0.1235, $\dot{\theta}_2$is 0.1348 and $\dot{\theta}_3$ is 0.0217 radians per second. So, once I have these values for the l dots and $\dot{\theta}s$ the linear velocity vector of the centroid which will contain l dot, $\dot{\theta}$ and also l and theta can be obtained.

And we can substitute and we will get 0.0263, 0.2323 and 0.9939 meters per second. The angular velocity vector can also be obtained from obtaining $\dot{R}\,R^T$. We have seen how we can obtain the angular velocity from the derivative of the rotation matrix and obtaining the skew symmetric matrix and then extracting the three components in the skew symmetric matrix which is exactly what has been done.

And you will get 0.753 as the X component so this is ω_x, - 0.968 is ω_y and 0.148 radians per second this is the z component. So, as you can see in the case of parallel system the work is much more than in a linear chain. In a linear chain we can go from base to the end effector or to the last link and obtain linear and angular velocities and even accelerations. Here we have to use the loop closure constraint equations.

We have to find the values of the passive joint variables in terms of the active joint variables. So, lot of numerical work has to be done and that is why analysis of parallel chains is much more complex than a serial chain.

(Refer Slide Time: 1:32:53)



So, in summary the serial chain with R and P joints one fixed and one free end. The position of a point of interest in a serial multi-body system can be obtained from multiplying 4 by 4 homogeneous transformation matrices. Parallel chains there has no clear free end and presence of loops. The position of a point of interest in a parallel multi-body system often is a point chosen on the moving platform.

The rotation matrix in parallel chain requires choice of output and free end. The linear velocity is the time derivative of the position vector. Angular velocity is obtained from $\dot{R}R^T$ and accelerations are obtained from time derivative of the velocity vector. And I describe to you how we can obtain propagation formulas for serial chains containing R and P joints. For parallel systems there is no propagation formula.