


**Dynamics and Control of Mechanical Systems**  
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**Lecture - 06**  
**Introduction to Multi – Body System**

Welcome to these NPTEL lectures on dynamics and control of mechanical systems. My name is Ashitava Ghosal I am a professor in the department of mechanical engineering and in the centre for product design and manufacturing and also in the Robert Bosch centre for cyber physical systems, Indian institute of science Bangalore. In this week we will look at multibody systems.

So, basically whole bunch of rigid bodies connected in some way, and we will try to study these multi-body systems.

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In this week there will be three main topics, these are in these three lectures, lecture 1, lecture 2, and lecture 3. In the first lecture we will introduce what are multi-body systems. So, basically, I will show you examples of multi-body systems in mechanical systems. In the second week we will look at how these different rigid bodies are connected and the way they are connected is through joints.

And we will look at this very important concept of degrees of freedom in a multi-body system and also the constraints in a multibody system. In the last lecture of this week, we will look at position velocity and acceleration in multibody systems.

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LECTURE 1

- Introduction

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So, lecture 1 introduction.

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RECAP

- Motion of a single rigid body
  - Position as a vector in 3D space and orientation in the form of a rotation matrix
  - Derivative of position vector of a point  $\rightarrow$  linear velocity
  - Two kinds of angular velocities obtained from skew-symmetric matrices obtained from derivative of the rotation matrix
  - Linear and angular accelerations obtained from derivatives of linear and angular velocity
- Coriolis acceleration plays an important role in many atmospheric phenomenon & in MEMS gyroscopes
- MEMS gyroscopes and accelerometers can be used to measure angular velocities and linear accelerations
- Algorithms exist to estimate orientation and position of a rigid body from MEMS gyroscopes and accelerometers.
- System of connected rigid bodies

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So, to recapitulate in the last week we looked at the motion of a single rigid body in 3D space. So, we looked at the position as a vector in 3D space and the orientation of a rigid body in the form of a rotation matrix. The derivative of the position vector of a point on the rigid body leads to this notion of linear velocity. Then we looked at the orientation of the rigid body and we looked at the rotation matrix.

And then there were these two kinds of angular velocities which were obtained from a skew symmetric matrix which in turn was obtained from the derivative of the rotation matrix. And then we looked at the linear and angular accelerations obtained from the derivatives of the linear and angular velocities. One of the interesting things was this something called Coriolis acceleration which plays a very important role in many atmospheric phenomena and in MEMS gyroscope.

So, if you recall it was something like twice  $\omega$  the rotation angular velocity of the rigid body into the relative velocity of a particle on the rigid body that gets this Coriolis acceleration. And in turn this Coriolis acceleration played an important role in MEMS gyroscopes and mems gyroscopes and accelerometers could be used to measure the angular velocities and linear accelerations of any rigid body in 3D space.

And there were also algorithms which we did not go into too much detail but which could be used to estimate the orientation and position of a rigid body from the measurements obtained from MEMS gyroscopes and accelerometers. This week we will look at a system of connected rigid bodies

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### INTRODUCTION

**Rigid Bodies connected together by joints**

Serial chains – A fixed body & a free end and No Loops  
Double pendulum, serial manipulator

Parallel/Closed-loop systems – One or more loops  
Simplest example – planar 4 bar mechanism

Tree Structure -- one fixed body & branches as in a tree  
Cobots with two arms

Hybrid – Serial chains + Loops  
Automobile, Protein structure

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So, let us look at a system of rigid bodies connected together by joints. So, a kind of system of rigid bodies these are sometimes called a serial chain. In these serial chains we have one fixed body and a free end and there are no loops, I will show you what is the loop very soon. A simple example or a well-known example of a serial chain is a double pendulum or even a serial manipulator or a serial robot.

In the next slide I will show you many pictures of such serial chains. Schematic way of looking at a serial chain is that there is something called as a fixed ground and this is sometimes also called the zeroth rigid body it is connected to another rigid body which is 1, 2 and 3 and 4 and 5 these are all different rigid bodies and they are connected in a series. So, 0 is connected to 1, 1 is connected to 2, 2 is connected to 3 and so on.

And at the end there is one which is free, and, in the beginning, there is something which is fixed to the ground. On the other hand, we can also have what are called as parallel or closed loop systems. So, this basically has one or more loops, and I will show you an example of a parallel or a mechanism in the next slide actual physical parallel mechanism but the simplest example is that of a planar 4 bar mechanism.

Many of you would have seen a four-bar mechanism in your undergraduate. Schematically a parallel mechanism or a closed loop system can be shown in this manner. So, basically, we have a fixed ground which is this 0 and you can see that there are more than one places which has 0 and then 0 is connected to 1 then there is a rigid body which has different points. And so, 2 this rigid body is connected to 4 and 4 again is connected to ground.

Similarly, 1 is connected to 3 and 3 is again connected to ground. So, as a result what you can see is we can have loops so I can go from 0 to 1 to 3 and to 0 likewise, it can have 0 to 3 to 2 to 4 and 0. So, in this example there are three loops the third loop is 0 1 2 4 and 0. So, we are starting from a fixed ground and we are again coming back to the same fixed ground. So, this is called as a loop.

So, in parallel mechanisms or closed loop systems we have these loops we can also have something which is called this tree structure, which is basically there is one fixed body and there are branches as in a tree. So, this is like a very well-known example is that of a Cobot we will see what is a Cobot with two arms. So, a human being we have a fixed body but we have two arms. So, this is also sometimes called as a tree structure.

So, here is an example of a tree structure schematically so we have a fixed ground so then we have 0, 1, 2, 3, 4 but at 4 then you can have 2 branches. Similarly, from the 0, I can have another branch which is 0, 7, 11 and again 7 is connected to 8, 9 and then there are three

branches so there are many free ends. So, in the serial chain there was one fixed ground and one free end.

In the parallel or closed loop systems there are many fixed grounds and there are many loops. Here there are no loops but there are starting from a fixed ground, I can have several branches and then there are this something which is called hybrid systems in which there are serial chains as well as loops. So, here is an example and there are many such examples in existence.

An automobile has many serial chains but it also has loops so it is a combination of both serial chains and loops. And so, there is something called as a protein, now it is made up of a whole bunch of amino acids and they are connected together but there are also loops. So, schematically a hybrid system looks like this there is a fixed ground which is again 0 and then there is one branch here which is like a tree structure.

This is very similar to the previous one. There is another branch which goes like this which is also very similar to this part. But between 3 and 9 are connected with another link. So, in this case there is a loop but there are also serial chains, so which is why it is called as a hybrid structure.

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**INTRODUCTION - EXAMPLES**

**Serial Chains**

Azimuth-Elevation Sun tracker

Planar 2R robot

PUMA 560 Robot

Laparoscopic surgery tool with extra joint

Hyper-redundant Robot

2x speed

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So, here are some examples let me show you a few videos of examples. So, these are all serial chains, so this example is that of a planar 2R robot.

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So, what you can see is there are two links they are connected by one this is the fixed part then there is one more joint here and then this is one free end which can go up and down in this example. This is a well-known planar two degree of freedom robot it is also called as a 2R robot. This is another example of a serial chain this is a laparoscopic surgery tool. So, this was also made in IISC.

So, basically what happens in laparoscopy is that you make some holes in your stomach region. And you push this instrument through those holes and at the end of this tool there are these various kinds of grippers or scissors and other things. So, what you can do is you can operate this grippers and scissors standing outside. So, a surgeon can rotate the scissor which is what he is showing then it can do grasping and it can also bend so like your human hand.

**(Video Ends: 10:35)**

So, this laparoscopic surgery tool was made at IISC and the novel feature of this tool is it has an extra joint so it looks like a hand. So, the last wrist part we can move.

**(Video Starts: 11:00)**

This is another very nice example of a serial chain this is a hyper redundant robot. So, those are some of you would have seen a robot a robot has some number of degrees of freedom. So, typically 6 if you have more than 6 degrees of freedom in 3D space or if you have more than 3 degrees of freedom in a plane these are called hyper redundant robot. Basically, this robot can reach a point in 3D space or in a plane in infinitely many ways.

So, this robot has 1, 2, 3, 4, 5, 6, 7, 8 links and 8 joints. So, and it is trying to trace a circle so what you can see is there is a whole bunch of mathematical modelling and algorithms and control and everything going on behind. Such that you move these links and so the end of this robot traces this circle. So, at the end of this robot we have attached a pen, and this is on the floor of the lab and it is trying to trace a circle.



**(Video Ends: 12:08)**

So, this is a nice example of a serial chain. This is another example of a serial chain. This is a well-known puma 560 robot; it has 6 degrees of freedom it has 6 rigid bodies 1, 2, 3, 4 and then set up there are some rigid bodies here. They are all attached using joints and as you can see this is one end is fixed and the other end is free.

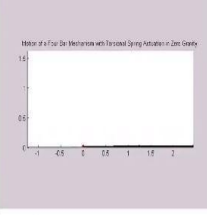
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INTRODUCTION – EXAMPLES


Parallel/Closed-loop Systems


3-RPS Parallel Manipulator



Planar 4 bar Mechanism



3-UPU Parallel Manipulator



Gough-Stewart Platform

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Let us continue with some examples of a parallel or closed loop system. So, this example here which I am going to show you this is a very well-known 3 RPS parallel manipulator. Just to give a little bit of a background, this parallel manipulator is used to track the sun. So, there is a way to collect solar energy and harness solar energy in which basically you put a mirror in and the mirror is rotated or moved in such a way that the incident beam from the sun is focused onto a receiver.

So, in this picture there are two such mirrors one of them is actually a serial device there is one motor here and there is another motor here one after another. And if I rotate these two motors in a particular way, I can track the motion of the sun and then the incident beam will be reflected onto this receiver. I can do the same thing with the serial parallel robot part or parallel system. In a parallel robot we have these three actuators or three joints and they form loops.

**(Video Starts: 13:51)**

So, this video shows how these two different heliostats also called heliostats moves, so as to focus the incident beam onto this receiver. And remember the sun moves in the east-west direction and a little bit in the north-south direction. So, the motion of these mirrors are is not straightforward it is not about one axis, it must rotate about two different axes. So, as you can see both the mirrors are trying their best to focus.

So, there are always some errors in manufacturing but nevertheless you can see both the beams are at almost at the same place. So, by moving this mirror throughout the day

according to some algorithms you can see now it is looking very different than in the morning we can still focus the beam to this receiver.

**(Video Ends: 14:58)**

And in an actual solar system maybe you will have some kind of a system here which will convert this thermal energy into electrical energy.

**(Video Starts: 15:13)**

This is a picture of a four-bar mechanism. Again, as I said four bar mechanisms are very well known.

**(Video Ends: 15:15)**

So, there is one rigid body this is another rigid body this is the third rigid body and in closed loop mechanism systems this base or the 0 which was the fixed link is also counted. So, there are four rigid bodies here and as you could see it moved in a particular way. This is the image of another parallel mechanism in which there is again this mirror and then there are this one rigid body which is the base which is the fixed.

This is the moving one and then there are these links which can extend. So, again there are loop so I can go from fixed to this link to this link and come back to this again fixed. There is also a very famous and very well-known parallel mechanism it is called as a Gough Stewart platform. So, in this picture this is a Gough Stewart platform which is used for vibration isolation.


But in many applications basically there are one moving platform and one fixed platform. And then this moving platform can be made to move along x y z and rotate about x y z by actuating these actuators. There are some motors here in inside these links which can extend these links and hence stock platform can move along the six degrees of freedom or along the six directions.

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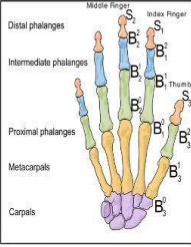


INTRODUCTION – EXAMPLES


Tree Structure



Baxter Robots  
robots.ieee.org



Anatomy of a human hand  
<https://en.wikipedia.org/wiki/hand>



Video of flexible gripper

Mahapatra et al. -- <https://arxiv.org/abs/2003.04593>

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So, a tree structure as I said consists of one fixed body and then there are these arms. So, this is a well-known robot which is called as a Baxter robot this is basically used in many factories. So, there are two arms so just like a human being this robot has two arms and it can manipulate an object it can coordinate and it can pick up objects heavy object with both the arms and so on. This is another example of a tree structure our hands, with these five fingers.

So, there is a fixed base which is somewhere here and then there are these three. So, there are these bones which come out, so this is one finger this is another finger this is another finger and so on and each finger in turn consists of links and joints. So, as I showed you from 0, I can have 1, 2, 3, and 4 and then some another 5 6 7 8 like that. So, these are different rigid bodies which are connected to the same base to the same 0. Here is the example of a sort of like a tree structure which we have built in at IISc.

**(Video Starts: 18:10)**

So, what you can see is there are these three fingers. Each finger in turn has some subtleties there are these discs and there are these cables which are going through it and by actuating these cables I can move these fingers as you can see by moving these fingers, I can grasp this cube and then I can rotate the cube and I can do various kinds of manipulation.

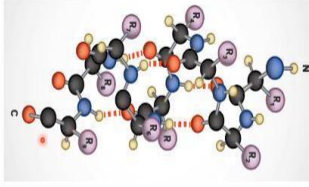
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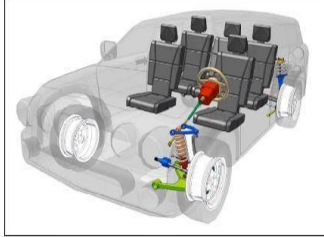
INTRODUCTION – EXAMPLES

Hybrid Systems

Serial chains & Loops



<https://themedicalbiochemistrypage.org/protein-structure-and-analysis/>



ADAMS/Car model

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So, as I said there is the last kind of systems which are hybrid systems. So, a very well-known example is that of a protein. So, protein consists of various amino acids which in turn have some carbon and nitrogen and hydrogen and various other elements and these elements are connected by bonds. So, in the case of a protein you do not have joints but there are these bonds and then there is a whole bunch of amino acids which are connected together.

And not only they can be in a serial arrangement, but they can be loops. So, for example this dotted line is one part which is connected to the other part so if there is a loop which is formed and there is also a serial structure. So, this is an example of a hybrid system. Similarly, in a car I can have a serial chain from the steering wheel then there are these some arrangements and then there are some things called steering mechanisms.

And so, this is one such serial chain but then there are some other serial chains, and these are all connected together. So, this is a model of a car it just shown in this there are these software packages called ADAMS and it is taken from that, so in a car there are many serial chains as well as there are loops which form.

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## INTRODUCTION



- This course will deal ONLY with dynamics of rigid serial and parallel/closed-loop mechanisms
- For tree-structure and hybrid systems – advanced course in multi-body systems/ robotics
- Basic ideas, concepts and simple examples will be worked out
- Systems with many rigid bodies and complex structures – Software tools such as ADAMS, Simscape available
- Short video on how to use basic features of ADAMS and Simscape will be shown  
Links to ADAMS, Simscape –  
<https://github.com/roboticslabiisc/NPTEL-Dynamics-and-Control-of-Mechanical-Systems-2022>

So, in this course we will only deal with dynamics of rigid serial and parallel closed loop mechanisms. So, we are not going to look at tree structure and we are not going to look at hybrid systems you know they you have you need to study some advanced course in multi-body system and robotics if you are interested in these topics. We will also look at reasonably basic ideas and concepts and most of the time some of these concepts will be shown through simple examples.

It is not that the concepts are only valid for those examples, but it is easier to bring out the concept using simple examples, in sometimes even planar examples. If you have a system with many rigid bodies and complex structures like this car or this protein there are software tools which exist and there are several such software tools commercial software tools one of them is known as ADAMS, another one which is Simscape.

So, this ADAMS is available as a student version if you are a student and if you apply to this company, they will give you a student version. Similarly, Simscape comes with MATLAB. So, in NPTEL courses MATLAB is available so maybe Simscape is also available. So, what you can what I will show you is that you can use both these software tools to basically study a set of rigid bodies connected by joints.

We can look at their motion we can look at their velocity, acceleration various things. So, there are two short videos on how to use the features of ADAMS and Simscapes and we will show you or we will give you a link to these videos. So, these videos will be available to all the students in NPTEL.

