Dynamics and Control of Mechanical Systems

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Lecture – 32
Case Studies

Welcome to this NPTEL lectures on Dynamics and Control of Mechanical Systems. In this week, we will look at Case Studies of Control of Mechanical Systems. My name is Ashitava Ghosal. I am a Professor in the Department of Mechanical Engineering, the Centre for Product Design and Manufacturing and also in the Robert Bosch Centre for Cyber Physical Systems, Indian Institute of Science Bangalore.

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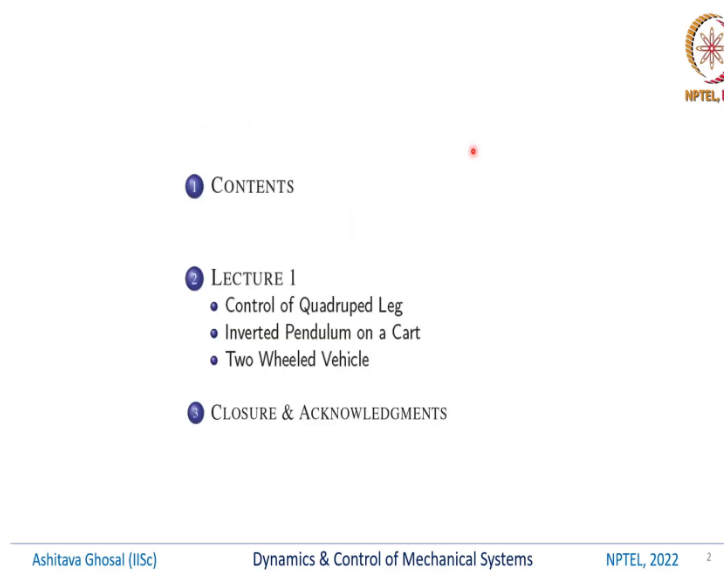

In the first lecture, we will look at three examples. We look at the control of a quadruped leg. The second example is that of an inverted pendulum on a cart and the third example is that of a two wheeled vehicle. I will end this module with closure and some acknowledgements.

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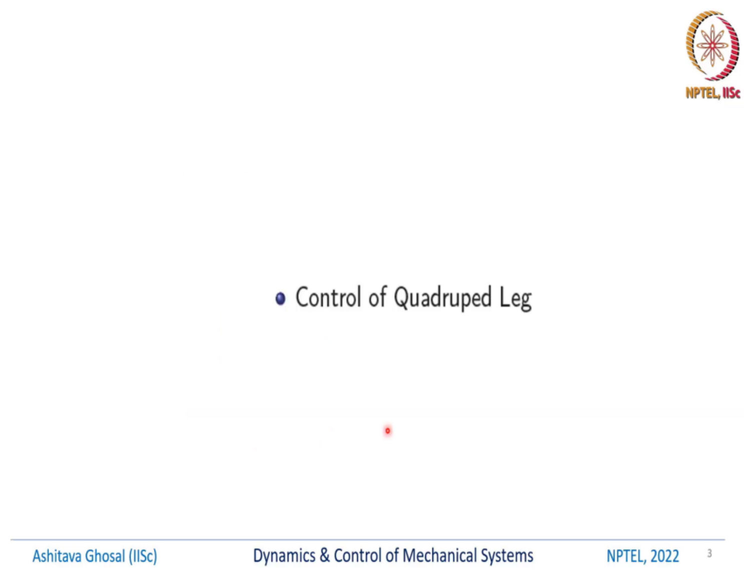

So, let us start with control of a quadruped leg.

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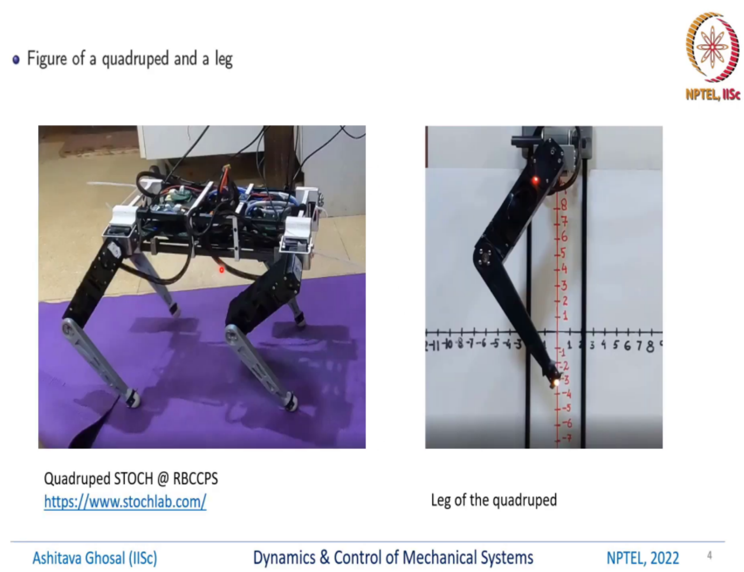

The figure on the left shows a quadruped the figure on the right shows a leg of the quadruped. A quadruped is basically a four-legged mobile robot. Each leg consists of these two links or two limbs. There are these joints here, one here, one here there could be another joint here. So, when you move these joints with the means of a motor, these legs can lift up. It can go forward and depending on how you plan the motion of each one of these legs, this quadruped can go forward, it can go backwards, it can go sideways, it can do various manoeuvres. A quadruped is a nice device, a nice mobile robot because it is a little bit more stable than a bipedal robot like human beings. We are not going to go into the details of the quadruped. Anyone who is interested in this quadruped can go to this link. This quadruped is called STOCH and the work on this quadruped has been continuing in the Robert Bosch Centre for Cyber Physical System for several years now and there are lots of videos and details about the STOCH quadruped in this website. At this lecture or we are only interested in how to model and control one of the legs of this quadruped. So, as I said, the right-hand side shows one of the legs. It consists of two links. So, there is one link here, another link here and there are these motors at this joint and at this joint. Depending on how you move these motors, the tip of this serial chain can move in various directions and various parts in the plane. We have looked at this 2R planar robot earlier this is very similar to that. In fact, it is very exactly the same as a planar 2R robot. So, what we want to show you is that this robot can go up and down. So, we want to control the motion along this red line. So, this is like the y-axis this is like the x-axis so, by controlling the current or the voltages which are going into the motor, we would like to ensure that this tip of this planar 2R goes along this vertical line. So, this is just one of the motions of this leg and I am going to show you how we can control the motion of this tip of this leg using PD control.

We have looked at PD control earlier actually, we have looked at PID control but we will use a simpler form of PID control which is the PD controller.

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- Planar 2R robot as a quadruped leg

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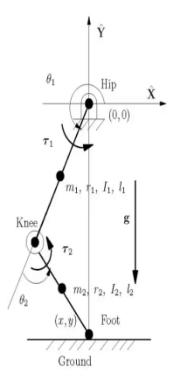

- Two non-linear second-order ODEs

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} =$$

$$\begin{bmatrix} I_1 + I_2 + m_2 l_1^2 + m_1 r_1^2 + m_2 r_2^2 + 2 m_2 l_1 r_2 c_2 & I_2 + m_2 r_2^2 + m_2 l_1 r_2 c_2 \\ I_2 + m_2 r_2^2 + m_2 l_1 r_2 c_2 & I_2 + m_2 r_2^2 \end{bmatrix} \begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix}$$

$$+ \begin{pmatrix} -m_2 l_1 r_2 s_2 (2\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2 \\ m_2 l_1 r_2 s_2 \dot{\theta}_1^2 \end{pmatrix} + \begin{pmatrix} m_2 g (l_1 c_1 + r_2 c_{12}) + m_1 g r_1 c_1 \\ m_2 r_2 g c_{12} \end{pmatrix}$$

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So, each leg of this quadruped can be modelled reasonably well, as a planar 2R robot -- 2R here stands for two rotary joints. There is one rotary joint here, there is another rotary joint here. This is actually, the hip of the robot. This is the knee of the robot. So, this second rotation at the knee is denoted by θ_2. The rotation at the hip is denoted by θ_1. There is a torque τ_1 which is acting due to a motor here. There is also another torque τ_2 which is acting due to a motor at the knee. Sometimes the motors are kept here and there could be a transmission between these two joints. We will assume that there is a τ_1 and a τ_2 which is acting at the hip and the knee. So, we have studied this system earlier in the module 5 and 6 when we looked at the equations of motion of a planar 2R serial chain.

I am just repeating the equations of motion here again. So, the equations of motion for this planar 2R robot can be given by τ_1 τ_2 --these are the two torques τ_1 and τ_2 -- this is equal to some mass matrix into $\ddot{\theta}_1$, $\ddot{\theta}_2$ plus there are these Coriolis and centripetal terms and then there is this gravity term. So, again I_1, I_2, m_1, m_2 these are the inertia and the masses of each one of these links. So, as you can see here for the second link, we will assume m_2 and I_2 as mass and inertia. The cg is located at r_2 and the total length of this link is l_2. Likewise, for link 1 it is m_1, I_1 which is mass and inertia, r_1 is the location of the cg and l_1 is this link. So, we have derived - this we have looked at this system, we have derived these equations of motion and if you do not recollect, please go back and see the module 5 and 6.

So, here, θ_2 is this angle and so on θ_1 is here. θ_1 and θ_2 appear in the gravity term. θ_2 appears in the inertia term and also in the Coriolis and centripetal term and of course we have $\ddot{\theta}_1$, $\ddot{\theta}_2$ and $\dot{\theta}_1$ into $\dot{\theta}_2$ and $\dot{\theta}_2$ square and so on.

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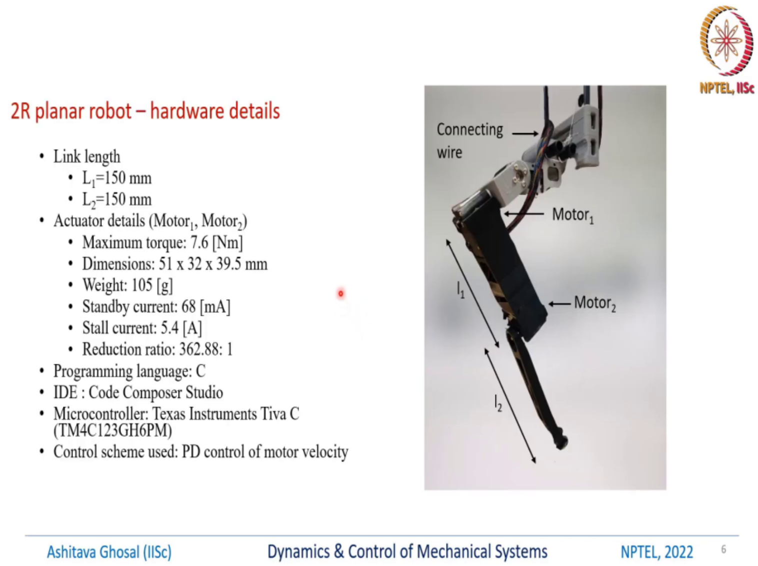

Let us look at quickly, the details of this hardware which was used for this planar 2R robot which is the leg of this quadruped. So, the link lengths L_1 and L_2 are 150 millimetres. The two motors have a maximum torque of 7.6 Newton-meter. The dimensions of these motors are 51 x 32 x 39.5 millimetres. The weight of each one of these motors has 105 grams. There is a standby current which is 68 milliamperes. There is a stall current which is 5.4 amperes and then there is a gearbox which has a reduction ratio of 362.88 to 1. So, again, remember we have seen that we cannot really connect a motor directly to the link. We need to have a gearbox and this is the gearbox. The programming language, to control these motors is C. This can be done using this code composer studio.

The microcontroller is a Texas instrument Tiva C which has these numbers, and the control scheme used to control the motion of these motors is PD control of motor velocity.

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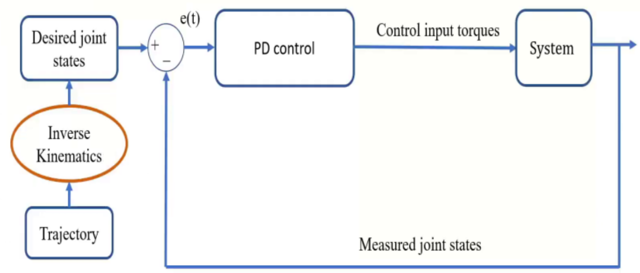
PD CONTROL



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The block diagram of the PD controller looks like this. So, this is the plant or the system which is basically two motors and two links. Then we have control input torque which are coming into this plant and the output is the rotation which are the measured joint states. And this is fed back and from a trajectory planner or what we want this leg to do - whether it should go forward or backward. All those things can be decided by something called as a trajectory planner which is basically gives you the desired joint states. And then subtraction of this from the measurement gives you the error and this error is going to this PD controller. So, it is like $K_p e + K_d \dot{e}$. We have looked at a PD controller earlier and where do we get this desired joint states from? So, we basically know what the tip of this leg or what the robot should be doing. So that is the desired trajectory in Cartesian space for this leg. We can do inverse kinematics and we can find what is the joint angles. Those of you who do not know robotics or who have not heard the term inverse kinematics -- it is basically a means to convert the *x, y* positions of the tip to the rotations at the joints of the leg.

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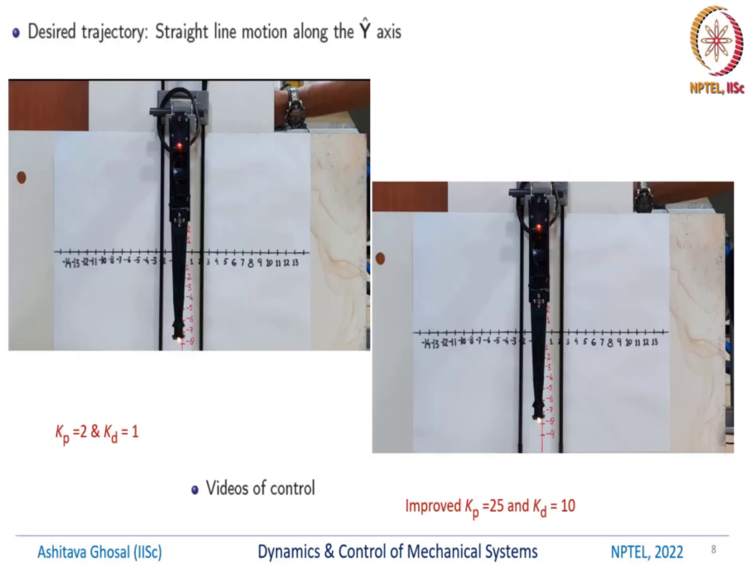

So, let us look at a desired trajectory which is a straight line along the Y-axis. As I showed you, I want the tip to move along the Y-axis. And I am going to show you two simulation results in which one of them, the proportional gain, is chosen as 2, the derivative gain is chosen as 1. **(Video Starts: 10:37)** So, as you can see, it is really, really badly designed controller. **(Video Ends: 10:50)**

It is nowhere near going along the straight line which we want, we want it along the Y-axis, so, the student and the TA which is this work was done by Mr Pramod Pal. So, we have, he has tried a lot of different ways of controlling the system. Then he tried out various gains of various values of K_p and K_d. And here is an example after some trial and error, we find $K_p = 25$ and $K_d = 10$ and this is what the motion looks like.

(Video Starts: 11:26) As you can see, this is reasonably good. This is much, much better than the previous one. In this case, at least, it is tracing a straight line. So, what I want to show you is**(Video Ends: 11:44)** this is a non-linear system the equations of motion were non-linear. We are using a standard, PD controller which is a subset of a PID controller which we have looked at in this course. And by tuning the gains, by carefully choosing the gains K_p and K_d we can ensure that this tip of this leg approximately traces a straight line. We can do better if you have more time and if you do different control schemes. So then even the small error away from the Y-axis can also be reduced.

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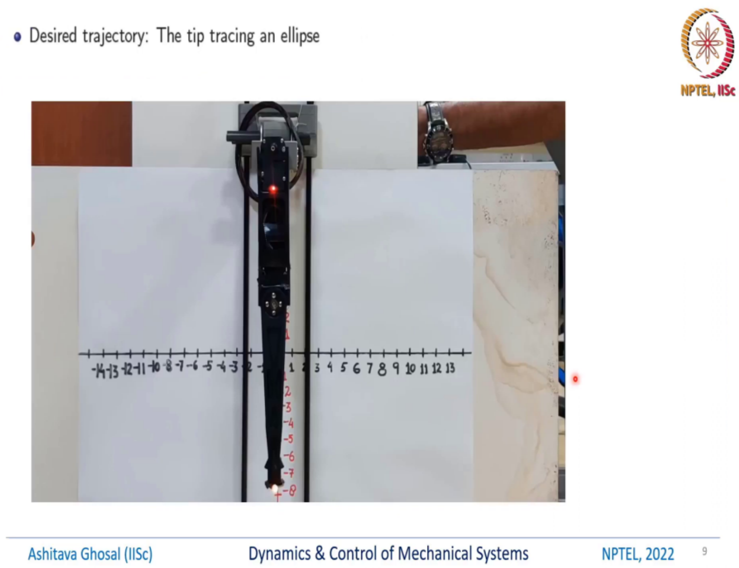

Here is another example, in which we want the tip not to move along the straight line along the Y-axis but to trace a curve in the X-Y plane. The curve chosen in this example is that of an ellipse. So, it turns out that sometimes the tip of the leg traces, some kind of an ellipse or close to an ellipse. **(Video Starts: 12:46)** Here also the gains have been chosen carefully, such that it more or less traces the ellipse which we want. **(Video Ends: 13:09)**

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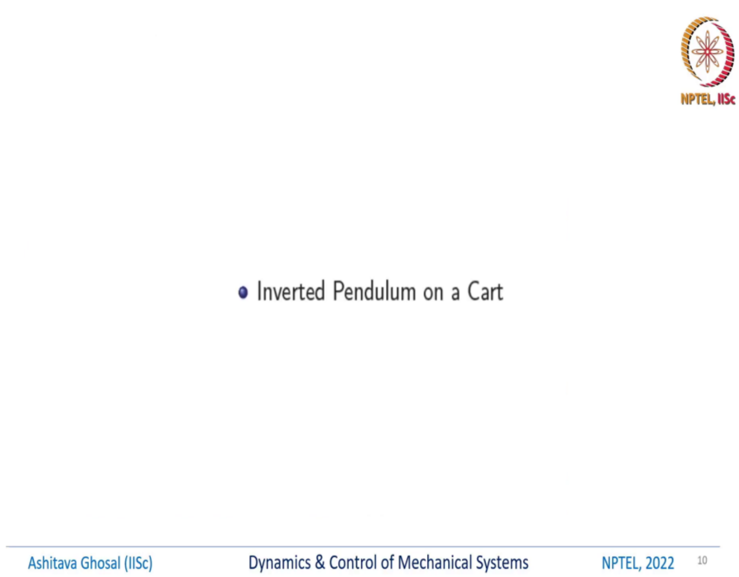

The next example is that of an inverted pendulum on a cart.

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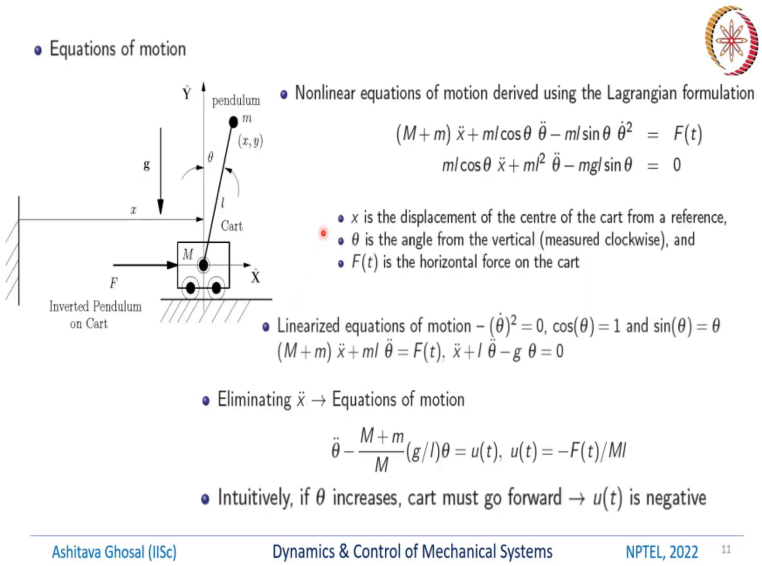

In the example of the inverted pendulum on a cart, we first derive the equations of motion. To derive the equation of motion, we first look at the variables and the system. This box here represents the cart - it has a mass capital *M*. There is an inverted pendulum which is attached to the centre of the cart. The length of this pendulum is *l*. There is a bob, mass small *m*, and the inclination of this pendulum from the vertical is denoted by θ.

To locate the center of the cart we use this variable *x* which is from a reference here and to locate the mass or this inverted pendulum, we obtain the angle from the vertical which is θ. There is gravity which is acting this way which is this *g* vector and then there is a force along the X-axis which is acting on the cart. So, we can derive the equations of motion following the Lagrangian formulation.

So, basically, we need to find the kinetic energy of the cart which is like (//2) $M\dot{x}^2$. Then we need to find the kinetic energy of this pendulum which will have terms with both is θ and *x* and their derivatives, and then we can find the potential energy. And we can find the Lagrangian and then following the principles and the steps which were shown earlier when we looked at module 5 and 6 about the equations of motion using the Lagrangian formulation.

We can derive the nonlinear equations of motion from the Lagrangian. It turns out that the equations of motion can be written as $(M + m)\ \ddot{x} + ml \cos\theta\ \ddot{\theta} - ml \sin\theta\ \dot{\theta}^2$ and this is equal to $F(t)$ – $F(t)$ is the force as a function of time. The other nonlinear equation is
$ml \cos\theta\ \ddot{x} + ml^2 \ddot{\theta} - mgl \sin\theta = 0$.

In these two nonlinear ordinary differential equations, x as I mentioned, is the displacement of the centre of the cart from a reference θ is the angle from the vertical which is measured clockwise. So, θ is increasing when it is measured clockwise and $F(t)$ is the horizontal force on the cart. These two non-linear ordinary differential equations can be linearized.

So, I have shown you earlier we can do some partial derivatives and then we can linearize about an operating point and so on, when we looked at state space formulation. However, we do not have to do these complicated things for this simple example. In this simple example, we can obtain the linearized equations of motion by simply substituting $\dot{\theta}^2$ is 0, cos θ is 1 and sin θ is θ. If you make the substitutions in these nonlinear ordinary differential equations, we will get these two linearized equations of motion. They are given by $(M + m)\ \ddot{x} + ml\ \ddot{\theta} = F(t)$ and
$\ddot{x} + l\ \ddot{\theta} - g\ \theta = 0$. These are two linearized equations of motion obtained from the original non-linear ODEs. These two equations can also be used to eliminate $\ddot{x}$.

You can solve for $\ddot{x}$ from one of the equation and substitute in the second equation and we will get the following single ordinary differential equation. Which is $\ddot{\theta} - (M + m)/ M\ (g/ l)\ \theta = u\ (t)$ -- $u\ (t)$ is similar to force but actually, $u\ (t) = F\ (t)\ /\ M\ l$ - if you work it out. This is in the form of a linear ordinary differential equation, and we can analyze and design controllers for this kind of system. Intuitively, you can see if theta is increasing that means the mass is falling to the right. We know that F should be in the same direction when it is falling. This way the cart should move forward. Anybody who has tried to balance a stick on the palm, and then you can see that if the stick is falling away from you, then you have to move the hand away from you, and opposite. So, in some sense, this -$F\ (t)$ divided by $M\ l$ makes sense.

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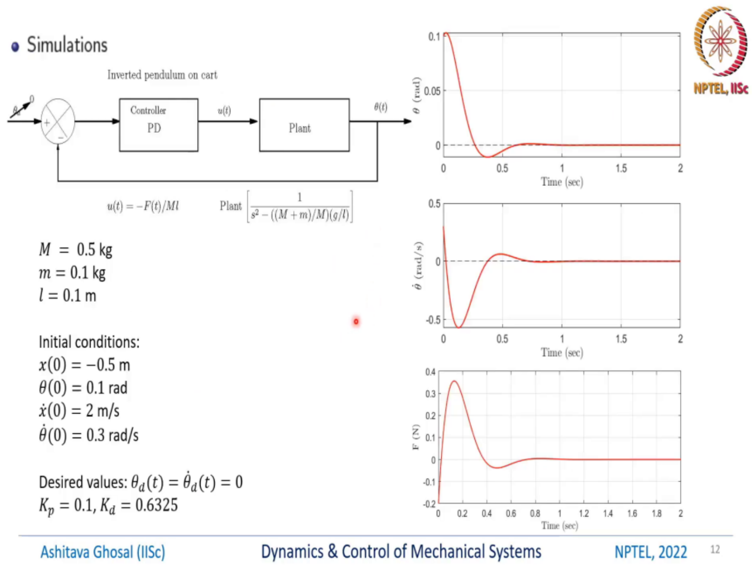

The equations of motion that we have obtained for the inverted pendulum on a cart can be simulated. This block diagram shows here the inverted pendulum on a cart. This is the plant which is the inverted pendulum on a cart, and the transfer function of the plant is given by 1 divided by $s^2 - (M + m)/M\,(g/l)$. So, it is slightly different from an inverted pendulum because in the inverted pendulum there was only (g/l).

Then we have this controller which is the PD controller. The output of the controller is $u\,(t)$ in our case $u\,(t)$ is - $F\,(t)/\,Ml$ and we want θ_d to be 0. We measure θ, we feed it back and then we put it into the controller and the output is $u\,(t)$. So, for simulations we have chosen $M = 0.5$ kg, m is 0.1 kg and l is 0.1 meters and some initial conditions. So, x (0) is - 0.5 meters, $\theta(0)$ is 0.1 radians, $\dot{x}(0)$ is 2 meters per second, $\dot{\theta}(0)$ is 0.3 radians per second. We use these initial conditions in the original ordinary differential equations and then we solve for it. We were not using this in root locus or some other way of using - but you know using inverse Laplace transform. We directly solve the differential equations because then we can take into account all these initial conditions.

The desired value θ_d is 0, $\dot{\theta}_d$ is 0 and we have chosen by trial and error K_p is 0.1 and K_d is 0.6325. You can see the plot of θ as a function of time -- it starts from 0.1 and then it quickly goes down, it overshoots little bit but settles at 0. Similarly, $\dot{\theta}$ starts from 0.3 radians per second, it undershoots it goes to less than - 0.5 radians per second but then after one oscillation it settles down to 0 radians per second, as we want. And the force is given by this plot, where initially the force is negative. The important thing is initial $\dot{x}$ is positive but the force is in the same direction as the velocity to maintain $\theta = 0$. This is the usefulness of this example and in the next slide we will look at an implementation of this cart on a pendulum and show you a video.

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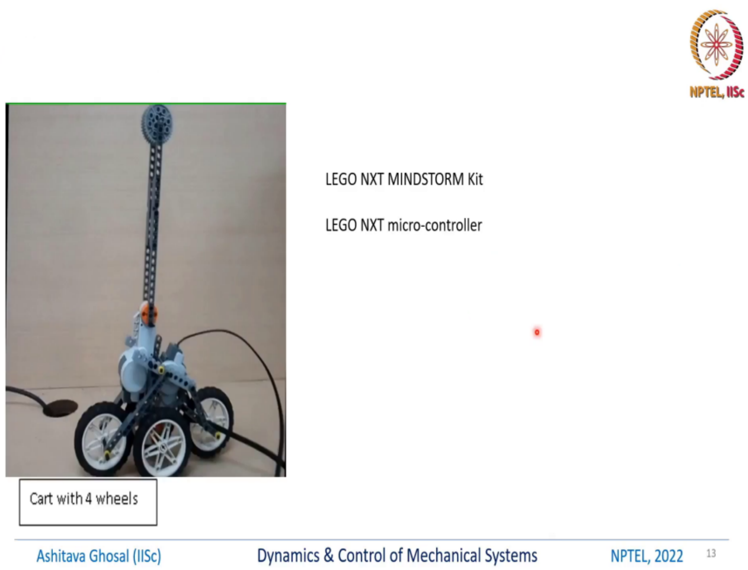

So, this inverted pendulum was built in the lab. In this case, in order to move it on the horizontal plane, we made a cart with four wheels. Four wheels are slightly easier to move so that it does not tilt on the sides. There are these four wheels which are driven by some motors this inverted pendulum and cart was built using a LEGO, NXT MINDSTORM kit. These are some very nice kits which are available on which you can do various quick prototyping and you can even test some simple controllers. This LEGO, NXT kit, also comes with a microcontroller and everything is here. There is also a battery, and you can send commands to this microcontroller either by means of a cable or even by Bluetooth. We built this kit and then I will show you some videos of what this kind of pendulum on a cart can do.

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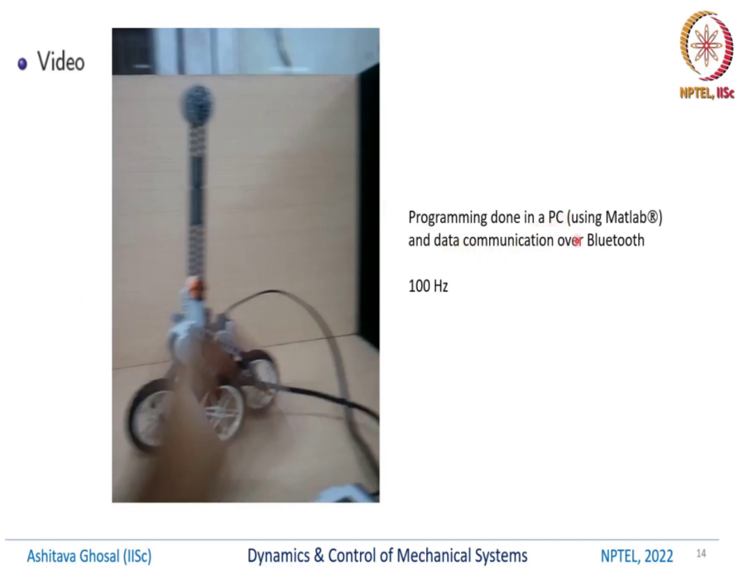

The programming, as I said, was done in a PC using Matlab and the data communication was over Bluetooth. We used about 100 hertz as a controller frequency. **(Video Starts: 23:27) (Video Ends: 23:37)** As you give a small input perturbation to the pendulum, you can see that the cart is moving and the most important feature in this quick prototype is that you can see that the cart moves in the direction in which you are moving the pendulum.

This is not a very sophisticated controller or a sophisticated device. There are many such devices which can be made where actual control and the motion is more precise. But the basic idea is I can show that the inverted pendulum the basic idea is that it will control the inverted pendulum by moving this cart forward and backward.

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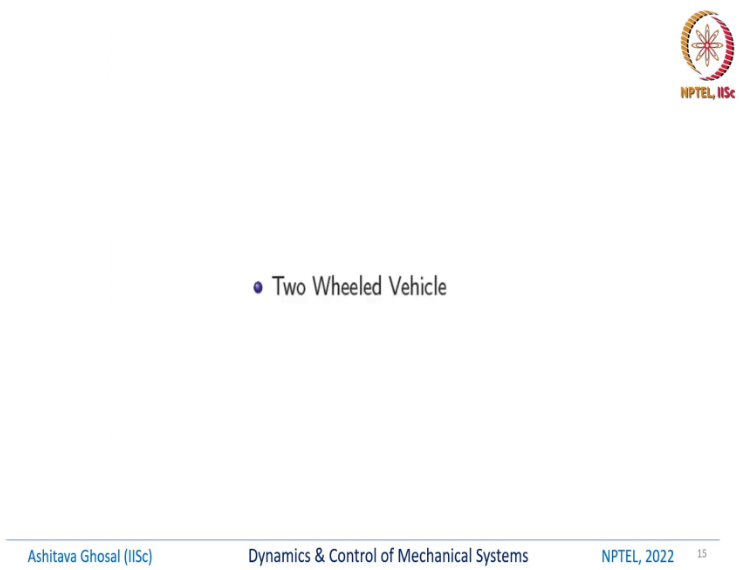

The last example is that of a two wheeled vehicle.

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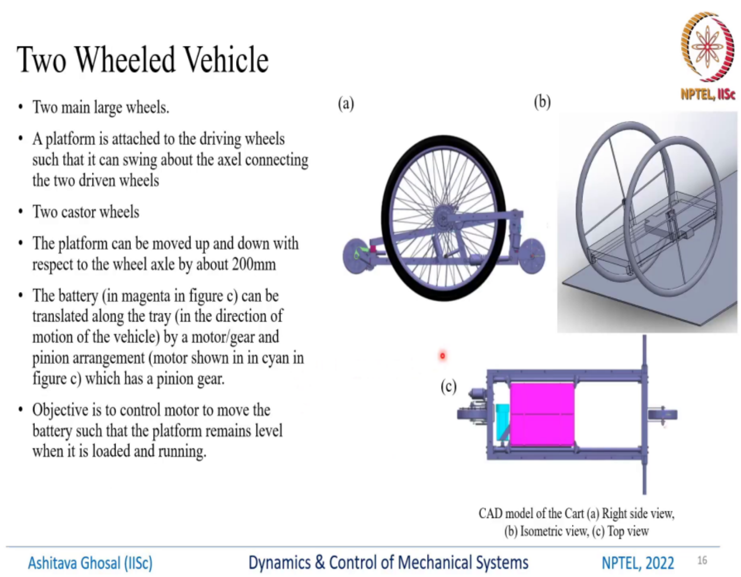

This figure here shows the two wheeled vehicle. Basically, this vehicle has two large wheels like this 1 and 2. There is a tray in between mounted on this axis between the two wheels and on the tray there is a huge weight which is basically nothing but the motor and the battery. So, the platform is attached to the driving wheels such that it can swing about the axle connecting to the two wheels. So, this whole platform can tilt up and down. There are also two other castor wheels which are not shown in this figure. The platform can be moved up and down with respect to the wheel axis by 200 millimetres. So, this whole platform can also be raised up and down and this figure here shows this is the battery and there is a motor here. And this motor is connected to a gearbox and a pinion arrangement such that this battery can be moved in this platform.

The battery can move in this direction up and down. The objective is to control the motor to move the battery, such that the platform remains level when it is loaded and running. So, this platform can go forward, it can go backward but then there could be some disturbances on the road. There could be a slope or there could be some pothole or something such that this platform can tilt. And the basic idea is that we will move this battery in this tray, such that this tray again becomes horizontal. This tray, for example, could be carrying some load and as this two wheeled vehicles is moving on a road and due to some tilting or external disturbances, this whole platform could be tilting and while it is tilting, this tilt will be sensed by means of some sensor and we will move this motor such that the tilt is made 0. So, again the platform becomes horizontal.

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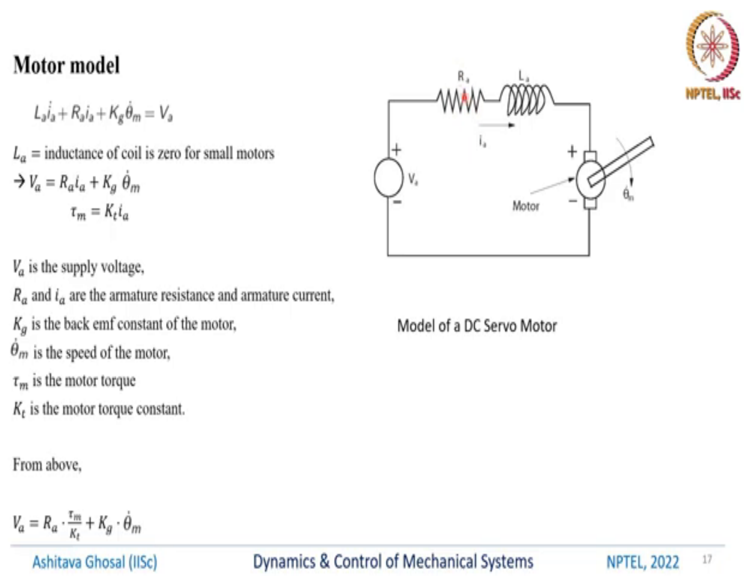

Let us start with a simple model. We have seen that the model of a DC server motor which is given by this we have seen this earlier. There is an applied voltage. There is a coil with the resistance R_a. There is also an inductor and this current is flowing through this coil and the permanent magnet which is the rotor, will start to spin when you apply some current. And due to the motion of this rotor, there will be a back emf which is generated, this back emf is given by some constant into $\dot{\theta}_m$. We can write the differential equation which is nothing but the voltage drop along the resistor, the inductor and due to back emf is equal to the applied voltage -- which is this equation. Most of the time in small motors this inductor is very small, so, we can drop this term $L_a \dot{i}_a$. So, we have voltage applied, is nothing but the drop in the resistor,

$R_a i_a$ plus whatever is the back emf.

We can solve for $\dot{i}_a$ from this equation – $\dot{i}_a$ will be given by $V_a - K_g \dot{\theta}/R_a$. Once we have the current which is flowing through this armature, *a* here stands for actually armature, then we can obtain the motor torque. The motor torque is given by $K_t i_a$. So, when you apply some voltage, we will get some current flowing through and due to the current, there will be a motor torque which is developed, and also the back emf which is proportional to the speed of the rotor so, these are the terms which are explained here. V_a is supply voltage or applied voltage,
R_a and i_a are the armature resistance and armature current, K_g is the back emf constant of the motor $\dot{\theta}_m$ is the speed of the motor, τ_m is the motor torque, K_t is the motor torque constant, and then as I said, V_a can be written as $R_a (\tau_m / K_t) + K_g \dot{\theta}_m$.

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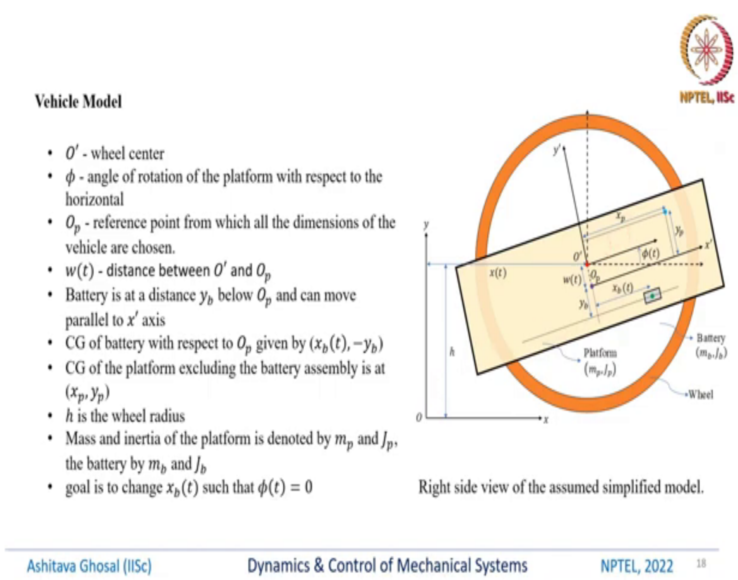

We can also have a model of the vehicle so, basically what we have is a wheel then there is this tray and then we can obtain what is the wheel centre which is *O*', f is the angle of rotation of the platform with respect to the horizontal, so, this is, this angle. O_p is a reference point which we choose from which all dimensions of the vehicle are chosen. So, this is some reference point. w(t) is the distance between O' and O_p here.

The battery is at a distance y_b so, this is the battery this is at a distance y_b and O_p can move in this direction along this x' axis. So, the CG of the battery with respect to O_p is given by x_b (t) which is this distance and – y_b which is the so, it is below the reference O_p. The CG of the platform excluding the battery assembly is at x_p , y_p and h is the wheel radius.

The mass and inertia of the platform is denoted by m_p and J_p, the mass of the battery is denoted by m_b and J_b, and the goal is to change x_b such that f (t) is 0, as the wheel is moving forward.

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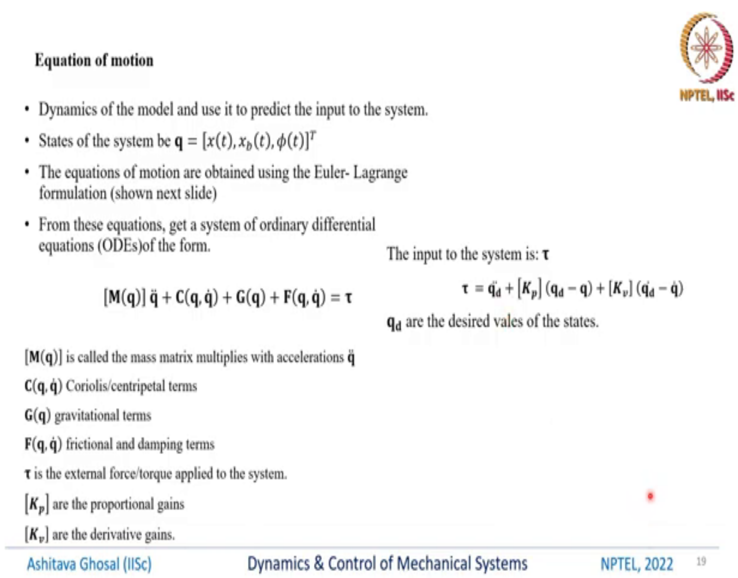

We can obtain the equations of motion, we can obtain in terms of the q's which are $x(t)$, $x(b)$ and f (t). We use the Euler-Lagrange formulation -- I will show you little bit later. What are the equations of motion which we obtained? But we have looked at Euler-Lagrange formulation earlier. Basically, we find the kinetic energy of the system, we find the potential energy of the system. We have obtained the Lagrangian and then we do all those derivatives. So, from these equations we get a set of ordinary differential equations of this form and this is the standard form which we have seen earlier for any serial chain -- which is some mass matrix into $\ddot{q}$ + $C\,(q,\,\dot{q})$ is the Coriolis centripetal term, $G\ (q)$ is the gravity term $F(\ (q,\,\dot{q})$ is kind of a friction and damping term, and τ is the external torque which is supplied from the motor. And we can obtain this torque in terms of a PD control. So, this is $Kp\ (q_d - q)$, so, q_d is the desired state of the system, q is the measured state of the system and similarly the derivative part is $\dot{q}_d$ $Kv\ (\dot{q}_d - \dot{q})$. And these derivative K_v and K_p proportional gains can be chosen, we can design a controller such that it makes sure that this f (t) is 0 all the time.

And I had briefly mentioned when we had discussed controllers that sometimes we also add what is called the $\ddot{q}_d$. So, this is the desired acceleration which is often available when you are dealing with robots and various other devices where you do a trajectory planning -- a nice smooth trajectory plan in which there is a desired q_d , there is a desired $\dot{q}_d$ and also a desired $\ddot{q}_d$ which we can compute.

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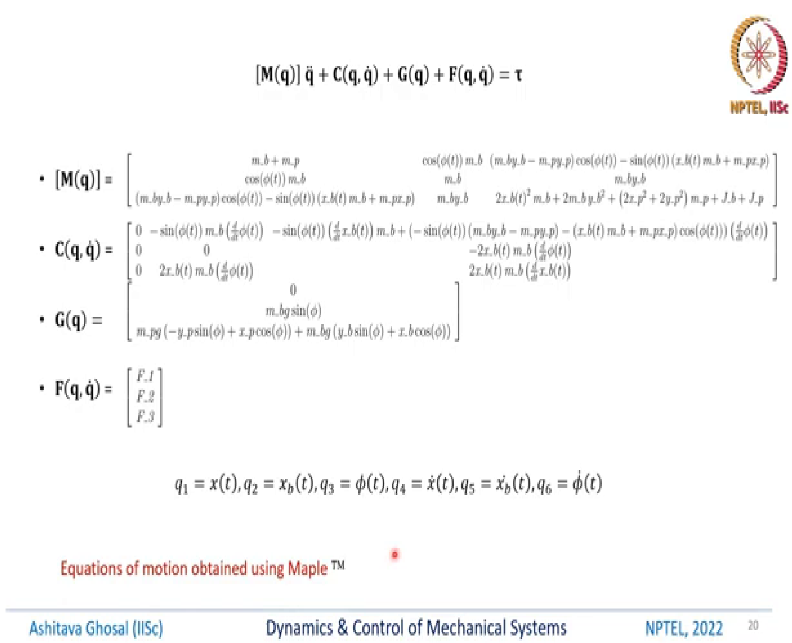

The equations of motion is given by $M(q)\ddot{q} + C + G + F$ -- so, this is the inertia term, this is the Coriolis centripetal term, this is the gravity term and this is the friction term and this is equal to the external torque. I am not going to show you all the gory details, but we can find the equations of motion from the Lagrangian formulation, and in this case the mass matrix is 3 by 3. It will contain mass of the battery, mass of the platform so that is m_b and m_p, some angles cos (f (t)) and then we have all these various terms. So, this is the and the location of the mass with the platform, which is x_p, y_p and the inertia which is J_b of the battery and J of the platform. So, this is obtained from the kinetic energy. We find the Lagrangian, do all the derivatives which we have done in the module on dynamics and then we find the mass matrix.

Likewise, we can find the Coriolis term and the gravity term and this friction term will be something which you will add in an ad hoc manner. So, the mass matrix is 3 x 3, the Coriolis term is also like 3 x 3 and $G(q)$ is a 3 x 1 vector and this friction is a 3 x 1 vector. So, we can rewrite these equations of motion in the state space form -- in the state space form we will have six states.

We will have q_1 which is x(t), q_2 which is the position of the battery, q_3 which is the angle, q_4 is the derivative of q_1, q_5 is the derivative of x_b and q_6 is $\dot{\phi}$. These equations of motion were obtained using Maple and I had shown you, in one of the previous modules, how we can use Maple to derive equations of motion. Very similar, something to that has been done for this example also.

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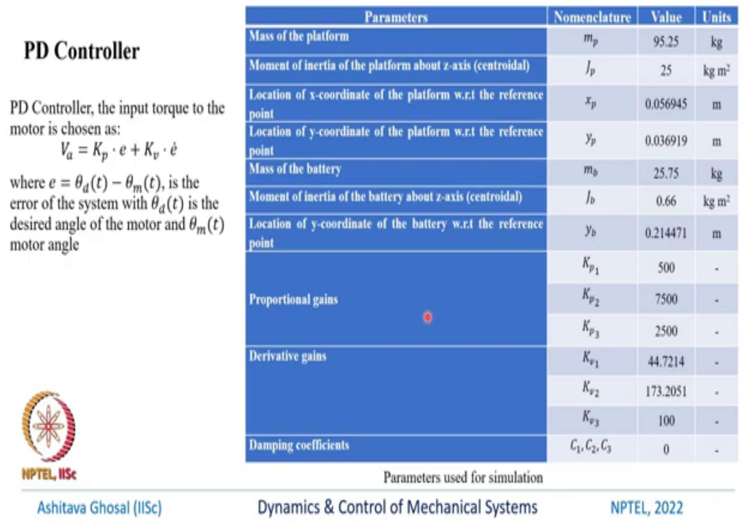

Parameters	Nomenclature	Value	Units
Mass of the platform	m_p	95.25	kg
Moment of inertia of the platform about z-axis (centroidal)	J_p	25	kg m^2
Location of x-coordinate of the platform w.r.t the reference point	x_p	0.056945	m
Location of y-coordinate of the platform w.r.t the reference point	y_p	0.036919	m
Mass of the battery	m_b	25.75	kg
Moment of inertia of the battery about z-axis (centroidal)	J_b	0.66	kg m^2
Location of y-coordinate of the battery w.r.t the reference point	y_b	0.214471	m
Proportional gains	K_{p_1}	500	-
	K_{p_2}	7500	-
	K_{p_3}	2500	-
Derivative gains	K_{v_1}	44.7214	-
	K_{v_2}	173.2051	-
	K_{v_3}	100	-
Damping coefficients	C_1, C_2, C_3	0	-

Parameters used for simulation

Now, let us look at a PD controller for the two-wheeled vehicle. In a PD controller, the input torque to the motor is chosen as $K_p\, e + K_d\, \dot{e}$. So, this is the applied voltage to the motor is proportional to the error and to the rate of change of error, where error is defined as $\theta_d - \theta_m$. So, for this two-wheeled vehicle, we have all these different parameters so, for example the mass of the platform m_p is 95.25 kilograms, moment of inertia of the platform about the z-axis centroidal moment of inertia J_p is 25 kg meter square. The location of the x-coordinate of the platform with reference to the reference point is at 0.0569 meters, the location of the y-coordinate is 0.0369 meters, the mass of the battery is assumed to be 25.75 kilograms. The moment of inertia of the battery about the Z-axis again the centroidal moment of inertia is 0.66 kg meter square. The location of the y-coordinate of the battery with respect to the reference point is at 0.2144 and we do various experiments and do lot of simulations and we come to a set of proportional gains and derivative gains. So, K_{p1} is 500, K_{p2} is 7500, K_{p3} is 2500 whereas K_{v1} is 44.7214, K_{v2} is 173.2051, K_{v3} is 100 and we assume that the damping coefficients are 0. Remember we had three variables, τ and these are the proportional, and derivative gains for those three differential equations and the three variables which go into this model of the state equations.

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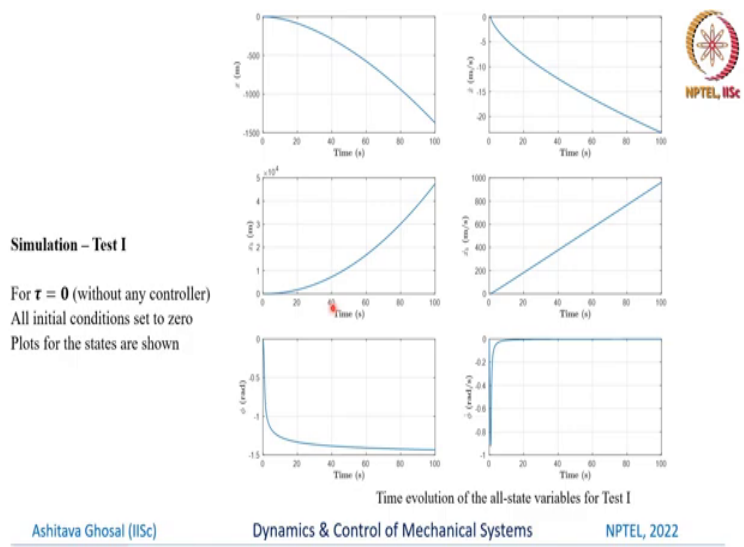

So, once we have set up the equations of motion and the PD controller, we can do several simulations. So, here is one simulation in this figure so, the first simulation is we put our set $\tau = 0$. So, there is actually no torque and there is also no controller. So, all the initial conditions are set to 0 and we plot the states which are x, x_b and f. So, as you can see without a torque or without any controller, the x will continue to increase, $\dot{x}$ will also increase, x_b will also increase with time. And the angle f, which is the tilt angle of the platform, will also increase with time -- it will go to some value which is like 1.4 approximately. The rate of change of $\dot{\phi}$ will also start from some value and maybe go to 0 because it sort of settles down at the end. But this is not what we want, we want to maintain f as 0. We want the platform at always horizontal, so, hence we need to use some controller.

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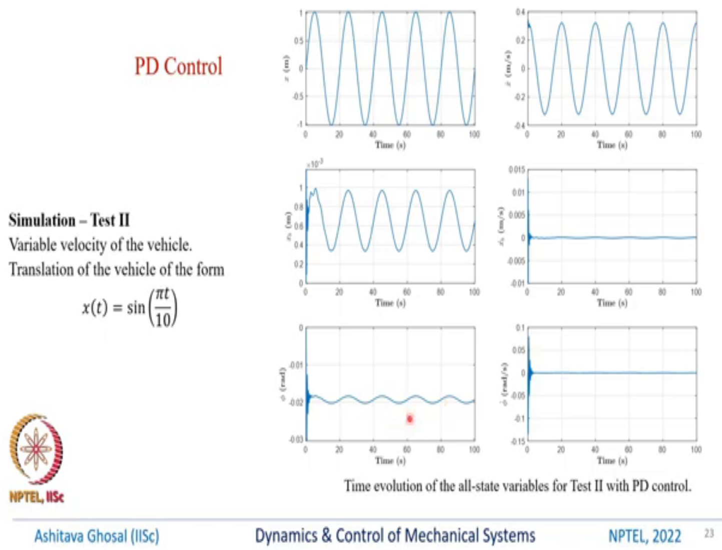

If you do PD control then we want to give some translation of the vehicle, some arbitrarily chosen translation of the vehicle of the form $x(t)$ is sin ($\pi\ t/\ 10$). So, $x(t)$ is this nice sinusoidal function, $\dot{x}$ is this and what you can see is that the motion of the battery now can be determined. So, this is x_b this is $\dot{x}_b$ and this is f. So, the platform tilt angle f is not going off to a large number. It is staying very close to 0.02 and $\dot{\phi}$ is also very staying very close to 0. So, it is not really 0 but it is oscillating about some number. So, this we can play around and then find out what will be the good controller gains, such that f remains 0.

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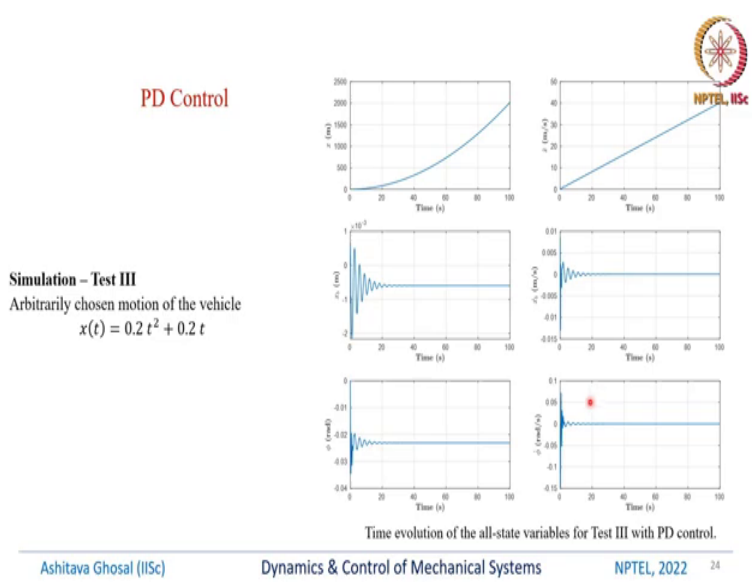

This is another test; in instead of going forward and backward, suppose the whole wheel is going consistently forward. So, $x(t)$ is $0.2\, t^2 + 0.2\, t$ -- these are numbers which were chosen arbitrarily for simulations. As you can see x will increase because it is like t^2. So, it will be parabolic, $\dot{x}$ is linear and then we can find out from simulation what is happening to the position of the battery.

The derivative of the position of the battery similarly what is happening to f and $\dot{\phi}$. So, as you can see here again something is being controlled both f and x_b they are not going off to large numbers which is staying at around 0.025.

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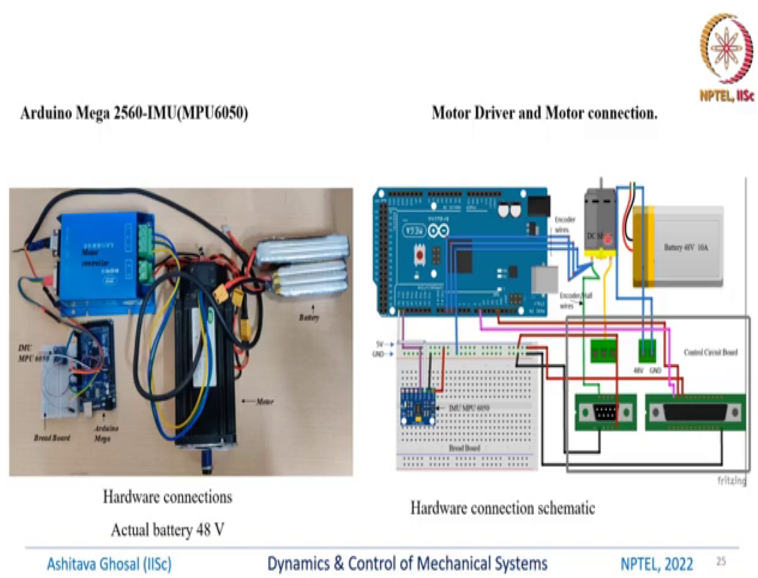

So, after doing some simulations, we also tried to make a prototype. The controller consists of Arduino Mega 2560, there is also an IMU which is used to see the tilt of the platform. So, we have this IMU which is connected to some motor controller, the motor whenever you buy a motor nowadays it comes with it is own controller, and then there is a battery this is just for simulation or just to show that there is a battery. The actual battery is a 48volt battery and then this is this motor. So, we have all these boxes which we can connect according to some layout. So, basically, we have this Arduino, then we have some bread board which contains this IMU to sense the rotation of the platform and then there are these connections to the battery and to the DC motor.

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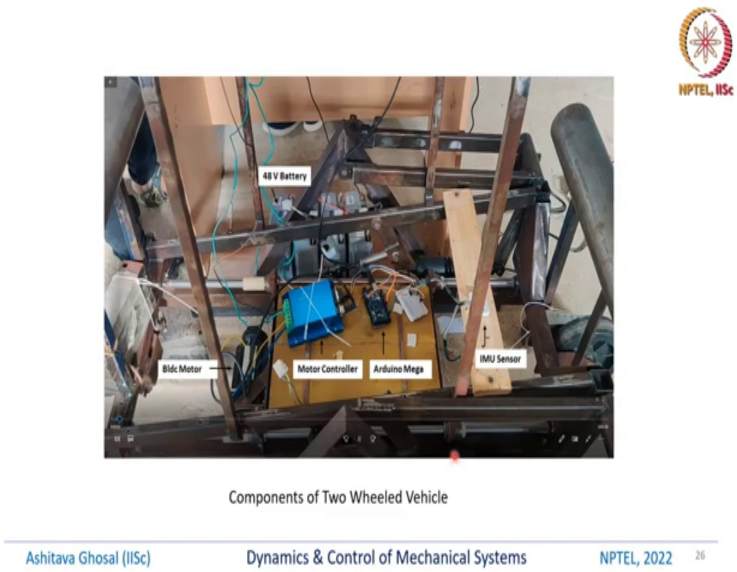

Here is the hardware setup. There is a 48 volts battery then there is a motor controller then there is this Arduino board then there is this BLDC motor and then there is a sensor IMU sensor which you will see how this platform is tilting. So, it is like a self-balancing system - that is what we want. So, if it is tilting forward it should go/move the battery such that the tilt is counteracted.

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I am going to show you a video which is done by one of the TAs of this course which is Yogesh. So, basically what he is going to show you is that we have this one box in which we are going to put the IMU and we are going to tilt this box. And as you tilt the box you can notice, what is happening to this output of the motor shaft. So, basically if it is tilted in one direction this small link which is attached to the motor shaft will move in one direction so as to counteract the tilt. He will show you some very simple similar hardware experiments which show that this combination of this IMU and the motor controller will ensure that the motor is rotating in the right direction. This is just a very initial experiment of controlling a motor using an IMU.

(Video Starts: 44:22) So now, it is in 0 degree now, I am rotating the platform by 15 degree in one direction, 15 in one direction. So, 360 again bringing to 0 correct and then again 15 in another direction and now, again back to 0, 0 is fine **(Video Ends: 45:19)**.

Basically, what he has showed you is that we can control the rotation of this motor shaft by this IMU and he was also towards the end showing that if you disturb it -- you put some disturbance noise -- by knocking on this board, even then it is still more or less working.

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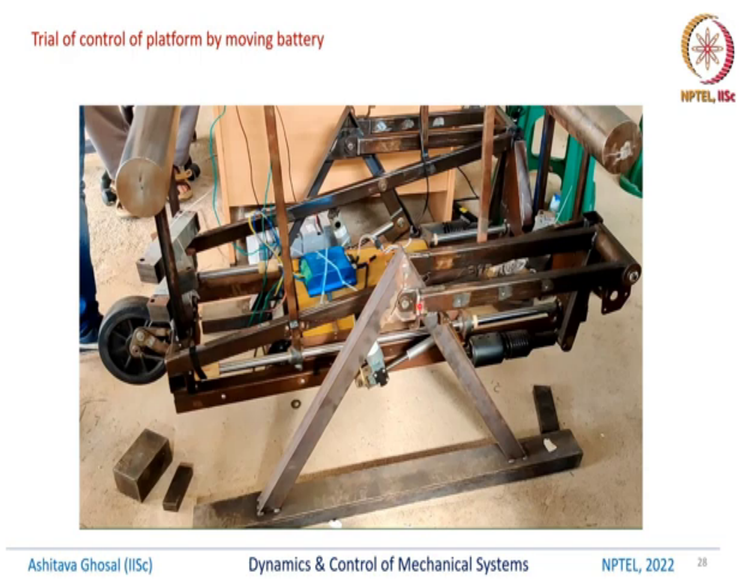

We now, implemented this control scheme which was tested on the simple board with the IMU on an actual hardware **(Video Starts: 45:56)** and this is what you can see. So, this is the tilt platform if the platform is tilting, the motor **(Video Ends: 46:04)** sorry as the vehicle is tilting the platform is going back and forth so as to counteract the tilt of the platform. Right now, there are no wheels because this is still in trial, and instead of the wheels it has mounted on a fixed thing. But you can think of imagine that this whole thing is on a wheel, two-wheels and as the two-wheels are rolling forward and if there is some disturbance to the platform this controller will move the battery so, as to make this platform horizontal again.

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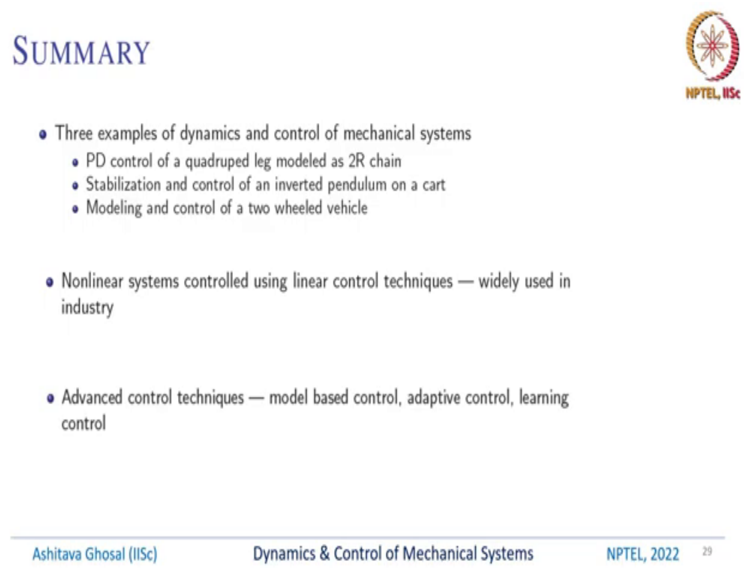

In summary, I have showed you three examples of dynamics and control of mechanical systems. The first was a PD control of a quadruped leg of this quadruped robot called STOCH, in which the leg was modelled as a 2R chain. The second example was stabilization and control of an inverted pendulum on a cart, and the third example was that modelling and control of a two-wheeled vehicle.

In all these examples basically, they were non-linear systems -- the equations of motions were non-linear. However, what we can see is that even for non-linear systems linear control techniques work more or less. You have to do lot of tuning, you have to do lot of experimentations to set the gains. But after all this effort we can control the original non-linear system with linear control techniques, and hence this is widely used in industry.

There are also advanced control techniques in which model-based control, adaptive control and learning control is used. In this course we are not going into those advanced techniques.

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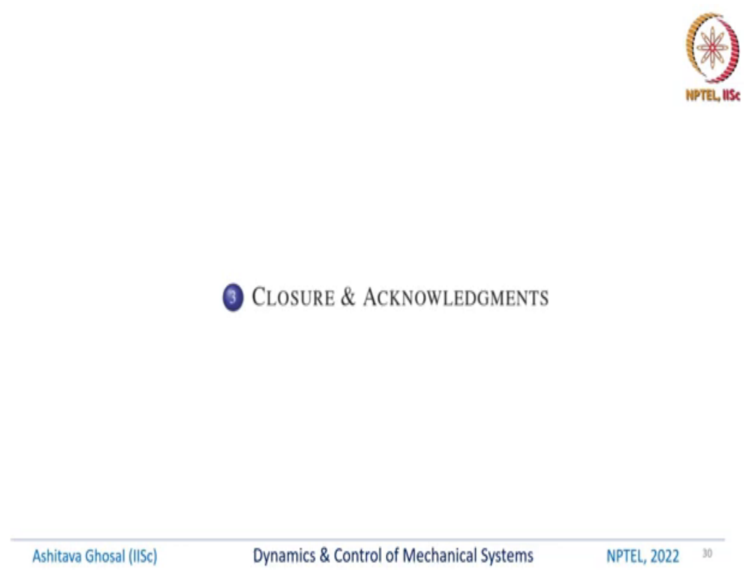

So, let us conclude this course with some closure comments and acknowledgments.

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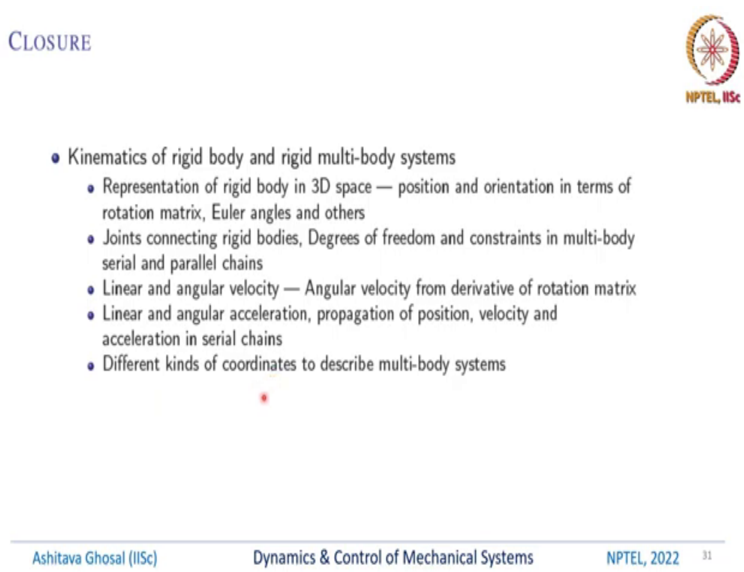

In this course, although it is called dynamics and control of mechanical systems but kinematics also plays a very important role. So, we have started with kinematics of rigid bodies and multibody systems. Basically, we looked at the representation of a rigid body in 3D space. We looked at the position and orientation in terms of rotation matrices, Euler angles, Euler parameters, quaternions and many other ways. And we showed, how each of this representation can be converted from one to the other.

The second important concept in kinematics was that of joints, connecting rigid bodies and this notion of degrees of freedom and constraints in a multibody serial and parallel chains. If you have several rigid bodies connected by joints you could find what is called as the degree of freedom of the system, and degree of freedom was intimately related to the number of actuators or number of independent actuators which you can use.

Then I showed you how we can obtain linear and angular velocity of a rigid body, the angular velocity was, in particular, obtained from the derivative of a rotation matrix. This is a new concept, most of you would not have used the concept of a derivative of a rotation matrix. But I showed you how $\dot{[R]}\ [R]^T$ will give one of the angular velocities, where $[R]$ is the rotation matrix. Then I showed you what is linear and angular accelerations and propagation of position velocity and acceleration in serial chains. Finally in kinematics, I showed you what are the different kinds of coordinates which can be used to describe multi-body systems.

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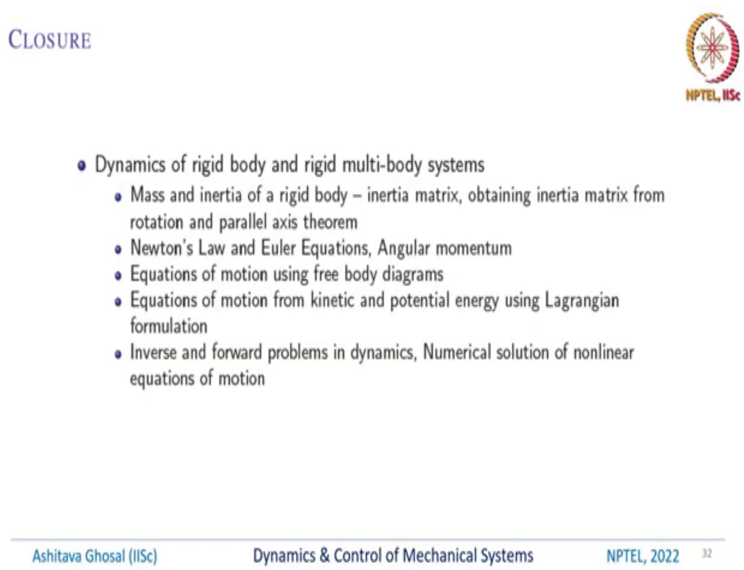

In dynamics of rigid body and multi-body systems, the first important concept was that of mass and inertia of a rigid body. The inertia was represented or given by an inertia matrix, a positive definite matrix, and we could obtain the inertia matrix from rotation and parallel axis theorem if the axis were different. Then came this Newton's law and Euler's equation so, this is like $F = m\,a$ and this is like $\tau = [\,I\,]\,\alpha + \omega \times [\,I\,]\,\omega$.

Then we had this concept of angular momentum and equations of motion were obtained using free body diagrams and Newton's Law and Euler's equation. The equation of motion could also be derived from kinetic and potential energy using the Lagrangian formulation and there are two important problems in dynamics which is inverse and forward problems. The forward problem basically was simulation of the equations of motion. And we could numerically simulate the equations of motion using tools like Matlab. So, most of the time you cannot solve the equations of motion analytically and we have to resort to numerical simulations.

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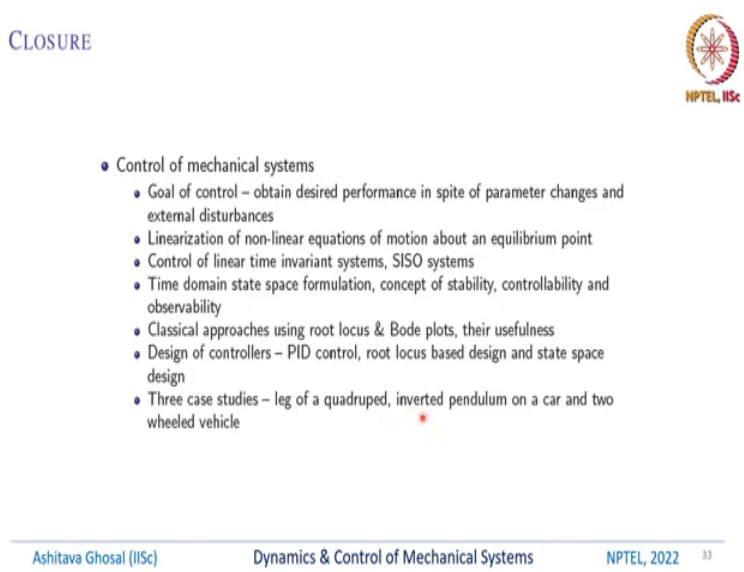

The last part of the course was in control of mechanical systems. We started with what is the goal of control. So, basically, we would like to obtain a desired performance from a system in spite of parameter changes and external disturbances. I showed you how feedback control allows you to achieve this goal. I also showed you how we can linearize a non-linear system basically, we derived nonlinear equations of motion but then if you want to control using linear control theory, we could first linearize these nonlinear equations of motion about an equilibrium point. And then I showed you control of linear time in variance systems and more importantly, we more or less considered only SISO system - single input single output systems. Why only linear time invariant systems? Because that is the simplest kind of control. It is also a foundation for other advanced non-linear control techniques. So, the control was done initially using state space formulation in time domain and that involved these three important concepts of stability, controllability and observability. I also showed you how we can use classical approaches such as root locus and bode plots and why we need to consider classical approaches.

So, basically state space control involves some integration or a convolution operation, whereas classical approaches using Laplace transforms can be done much more easily. We could easily see what is the effect of the external disturbance on the output, or quantify the effect of the internal parameter changes on the output of course, only for a single input single output system. Then we looked at designers or design of controllers. So, basically, I showed you what is PID control, root locus based design and state space design. And finally in this lecture or this week I have showed you three case studies. Basically how to control the leg of a quadruped, how to control an inverted pendulum on a cart and two-wheeled vehicle.

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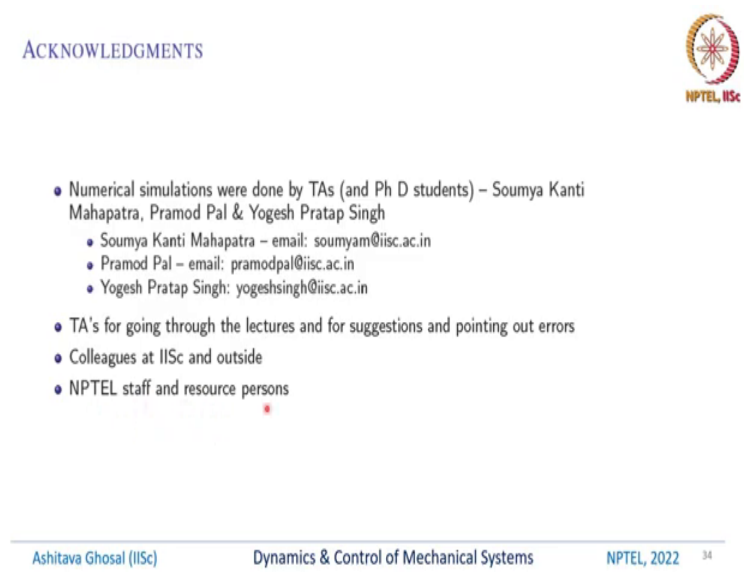

There are many persons who has helped me prepare content for this course, the numerical simulations were done by the TAs and they are also the PhD students working with me. So, they are Soumya Kanti Mahapatra, Pramod Pal and Yogesh Pratap Singh. Soumya Kanti Mahapatra's email and Pramod Pal's email and Yogesh Pratap email are given here. The TAs also went through all the lectures -- found out all the mistakes and they suggested how to improve the content. I have also discussed extensively with colleagues at IISc and from outside to hopefully make these lectures more understandable. And I would also like to thank the NPTEL staff and resource persons.

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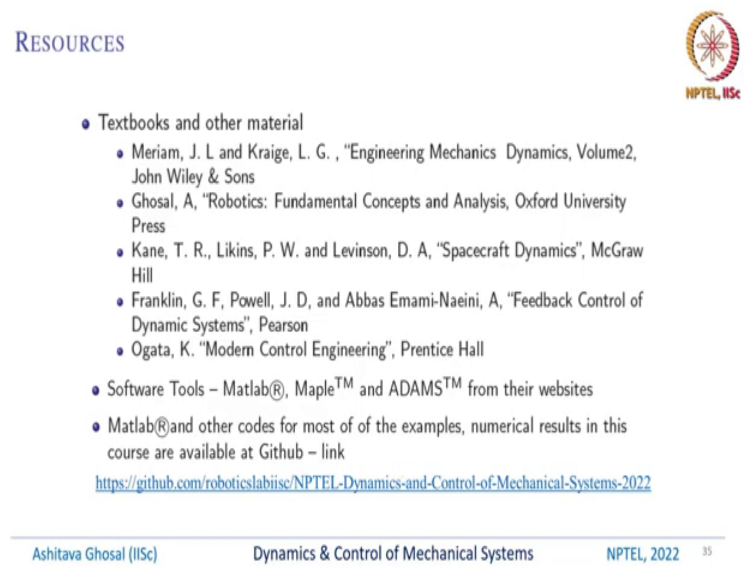

There are many textbooks on Dynamics and Control of Mechanical System which are available. Here is a list of some which I have used in this course so, first is this Meriam, J.L and Kraige, Engineering Mechanics Dynamics, volume 2. Then there is a book on robotics, Fundamental Concepts and Analysis by Oxford University Press. Then there is this very old and nice book on Spacecraft Dynamics. So, many of these problems in gravity so, for example that extended body in space which not only will be attracted towards the Earth but also rotate is from this book. Then there is this very well-known book on Feedback Control of Dynamic Systems by Franklin, Powell and Abbas Emami-Naeini, a large number of examples in this course are from this book. And then there is this; very well-known book on Modern Control Engineering by Ogata, some of the concepts are explained in this book very well.

If there is some interest by anybody to go much deeper, I would urge you to take a look at this Franklin, Powell and Abbas Emami-Naeini or Ogata for control examples. There is a lot of use for software tools such as Matlab, Maple and ADAMS. In fact Matlab is available to all the participants in this NPTEL course, and I would urge you that you take a look at Matlab it is a very, very useful and powerful software tool, to do both dynamics and control and kinematics and everything.

The Matlab and other codes for most of the examples and numerical results in this course are available in this Github link. So, these have been prepared by the TAs there are many videos, there are many animations and there are many codes for example in the kinematics, dynamics and control of mechanical systems.

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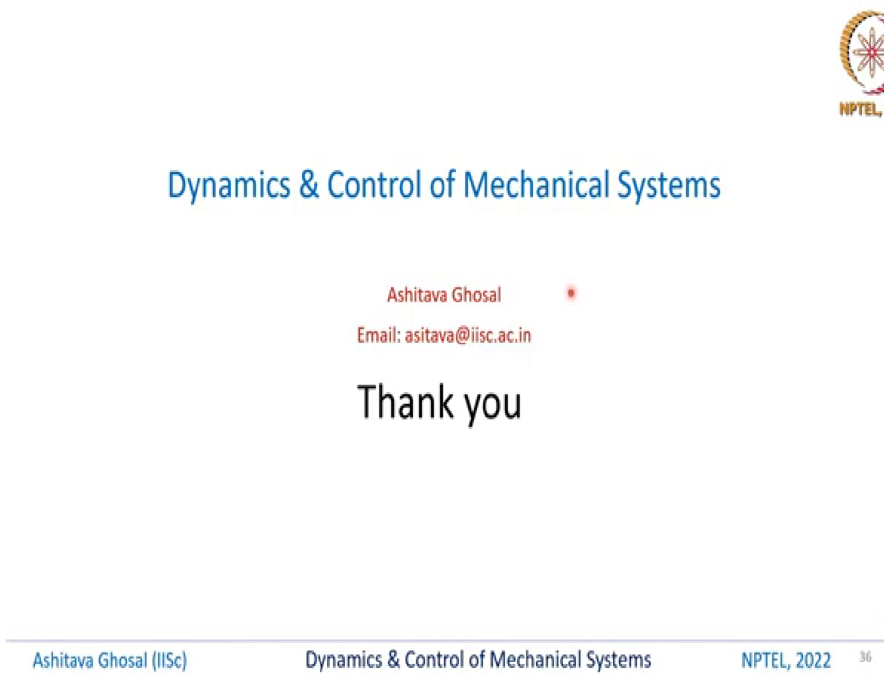

So, with this, we come to the end of this course, I am Ashitava Ghosal this is my email id if you would like to contact me, please send email. Thank you for sitting through this course, I hope you have learnt something new and enjoyed this course. Thank you again.