Dynamics and Control of Mechanical Systems
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Lecture-17
Introduction and Examples of Equations of Motion

Welcome to this NPTEL lectures on dynamics and control of mechanical systems. In the last week we had looked at how to derive equations of motion and we had given many examples of the equations of motion of multi-body systems. In this week we will look at simulation of multi-body systems. My name is Ashitava Ghosal; I am a professor in the department of mechanical engineering in the center for product design and manufacturing and also in the Robert Bosch Center for Cyber Physical Systems, Indian Institute of Science, Bangalore.

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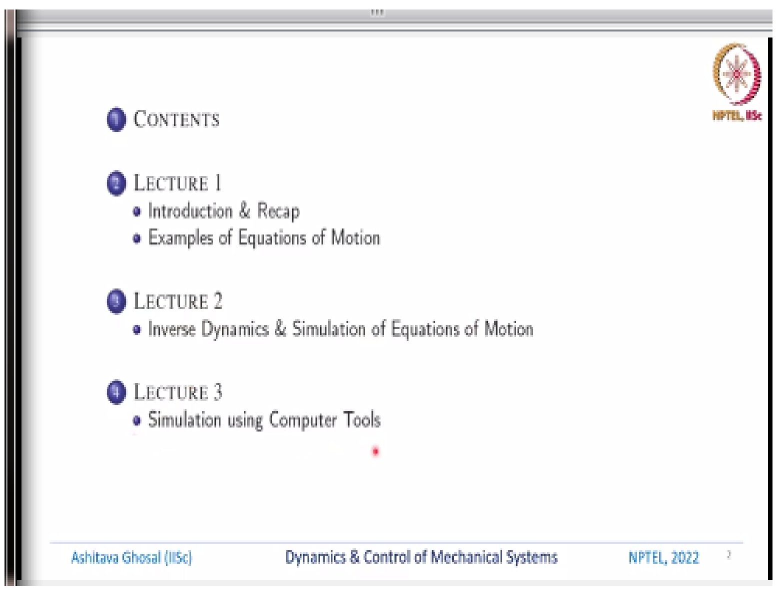

In this week there will be 3 lectures, in the first lecture we will introduce and discuss what is simulation and a little bit of recapitulation of what was done previously. Then I will show you some examples of the equation of motion, most of them are what we derived last week. In lecture 2 we will look at the inverse dynamics and simulation of equations of motion there are 2 kinds of simulation of multi-body system one is called inverse dynamics and one is the simulation of equations of motion. And in the last lecture in this week we will look at simulation using computer tools.

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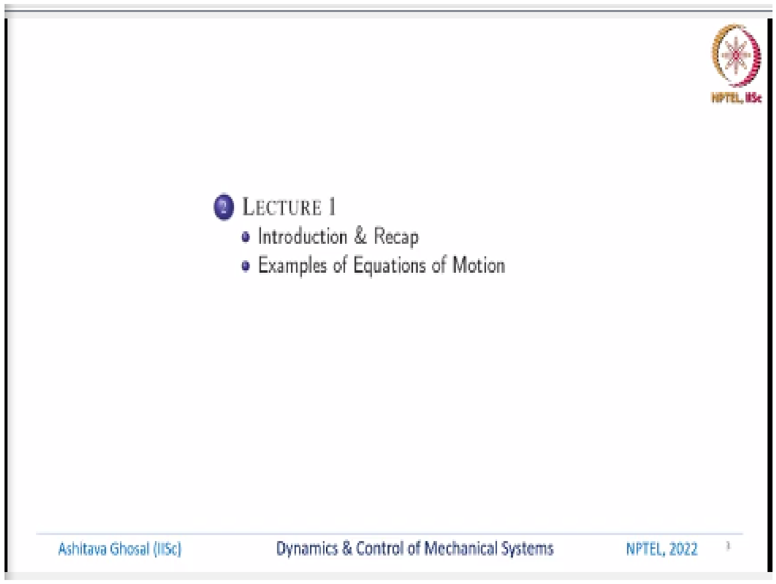

So, lecture 1, introduction and recap and examples of equations of motion.

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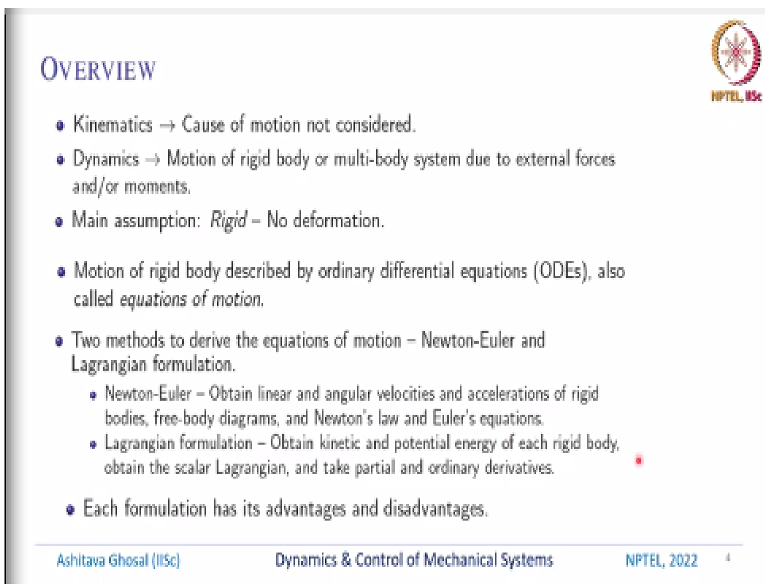

So, in overview: In kinematics we do not consider the cause of the motion; we look at the geometry of motion basically. In dynamics the motion of a rigid body or a multi-body system due to external forces and or movements are considered. The main assumption is that the multi-body system contains only rigid bodies; there is no deformation in the bodies which make up the multi-body system.

The motion of the rigid body are described by ordinary differential equations, also called equations of motion. So, if the elements of the multi-body system are rigid then we will get ordinary differential equations. There are 2 method methods to derive the equations of motion which we had looked at last week. One of them is called as the Newton-Euler formulation and the other one is the Lagrangian formulation.

In the Newton-Euler we obtained linear and angular velocities and acceleration of rigid bodies of all the rigid bodies in the multi-body system. We obtain the free-body diagrams and then we use Newton's law and Euler's equation to derive the equations of motion and in between we had to eliminate the constraint forces. In the Lagrangian formulation on the other hand we obtain the kinetic and potential energy of each rigid body obtains the scalar Lagrangian.

And then we take a set of partial and ordinary time derivatives and then we collect all the terms in a particular manner and we get the equations of motion. Each of these 2 formulations Newton-Euler and Lagrangian they have their own advantages and disadvantages. So, we had looked at some of these advantages and disadvantages earlier. So, basically in the Newton-Euler we need to compute both position velocity and acceleration.

And in Lagrangian formulation we can just stop at velocity because it is an energy based formulation and we need to only use kinetic and potential energy. One of the disadvantages of Lagrangian formulation is that the constrained forces at the joints which do not do work, require some effort to obtain what is the nature of the constrained forces and what do these constraint forces imply.

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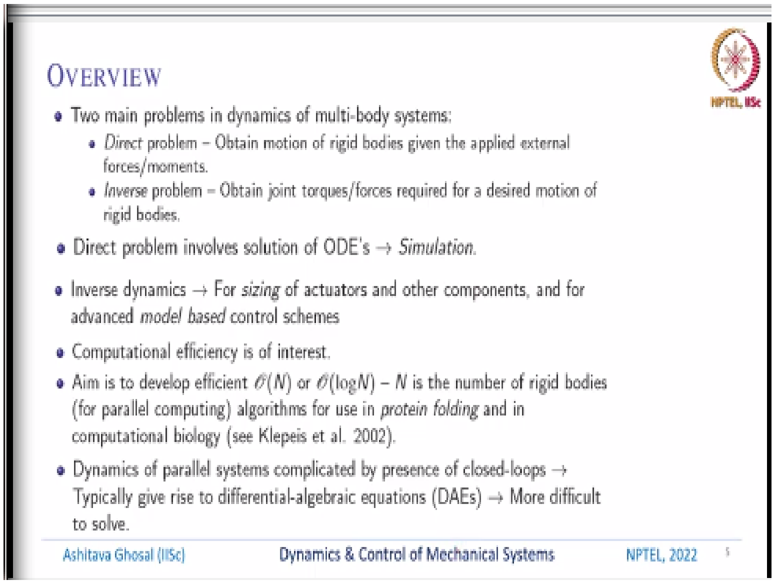

So, let us continue with overview. There are 2 main problems in dynamics of multi-body systems; the first is called the direct problem. The direct problem we obtain the motion of the rigid bodies which make up the multi-body system given the applied external forces and moments. And in the inverse problem we obtain the joint torques and forces required for a desired motion of the rigid bodies. So, what do we have?

We have the equations of motion, they are basically ordinary differential equations which says that the torques or the forces are given by some mass times acceleration and so on. So, in the direct problem the torque and the forces are given to you as a function of time and then we need to solve the equations of motion to obtain q as a function of time. Whereas, in the inverse problem we are given the right hand side, we are given q as a function of time, $\dot{q}$ as a function of time and maybe $\ddot{q}$ also as a function of time.

And then we need to compute the external forces and torques which are required to obtain those s q and $\dot{q}$ as a function of time. So, obviously the direct problem involves the solution of the ODE's. So, we have $f = m\,a$ I am giving you f we need to find what is a which is straight forward but then you have to integrate those equations of motion to obtain velocity and position.

This direct problem also is sometimes called simulation, so we simulate the system given the external forces and moments and show how the multi-body system is moving. The inverse problem is useful for sizing of actuators and other components and also for advanced model based control schemes. So, what do we mean by sizing of actuators? So, suppose I give you a multi-body system and I tell you that these are some of the typical q as a function of time, $\dot{q}$ as a function of time and $\ddot{q}$ as a function of time.

I want the multi-body systems to do. So, I can put it in the equations of motion and then I can compute the torque required to achieve those q as a function of time and $\dot{q}$ and $\ddot{q}$. So, hence by doing these computations or doing these work I can tell you how much is the torque required. And which basically means that I can tell you what is the motors or actuators that you require such that you can achieve the desired motions.

The same idea is also used in model based control schemes which we will not go into too much in this course. Anybody who is interested in model based control schemes they can look at some courses in robotics. In both these problems direct and inverse we are interested in computational efficiency. Basically the aim is that we want to solve these 2 problems in a way such that it is scales with the number of links.

So, if I have some effort required for let us say n links, so let us say 5 links and if the number of links becomes 10 then the effort should only double, it should not become square or cube. So, there are algorithms which are available for multi-body system which are O N, so that is basically they are computational complexity is linear in the number of links, sometimes it is important to have algorithms which are log N complexity, well, N is the number of rigid bodies.

So, this is are possible if you do parallel computing which we are not going to discuss in this course. And sometimes this parallel computing is required if you have very large number of rigid bodies. So, there is a problem called protein folding which is very important in many biology and computational biology areas. So, you have large number of amino acids and proteins in inside the protein and these amino acids can be modeled as rigid bodies and they are connected together.

And in a protein you can have something like 100 or even up to 500 such rigid bodies. So, we cannot really worry about if you have an algorithm which is not very good, so we need very efficient algorithms to solve the motion of such long chains as in a protein. Anybody who is interested in computational biology you can see this book; this is a very nice book, Klepeis et al., published in 2002.

The dynamics of a parallel system, however, is much more complicated and the complication is basically due to the presence of closed loops. The main complication or the main difference between a serial chain and a parallel system is that in a serial chain we have only ODE's, we have only ordinary differential equations. However, in a parallel system or a parallel chain we have ordinary differential equations arising from the Lagrangian formulation or any other formulation.

But we also have loop closure constraint equations which are algebraic. So, what we have is what are called as differential algebraic equations, and these are much more difficult to solve.

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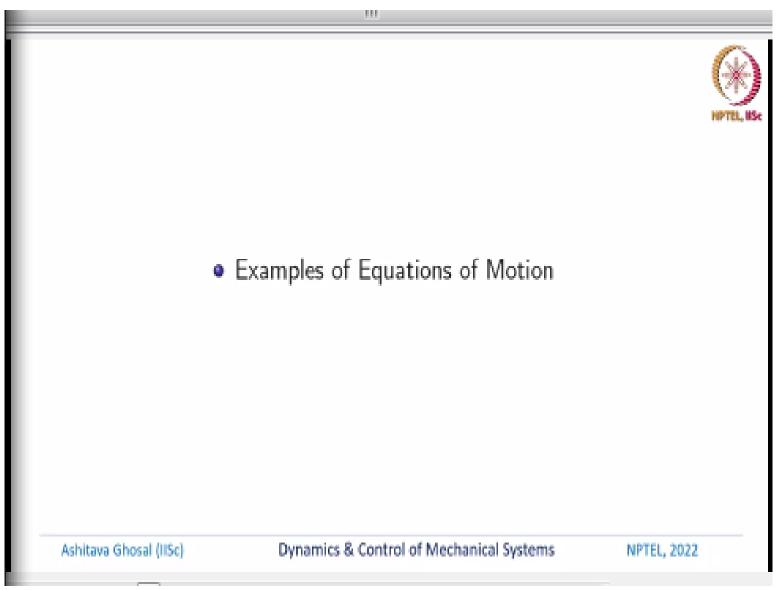

In the rest of this lecture, we will look at examples of equations of motion of multi-body, rigid body systems.

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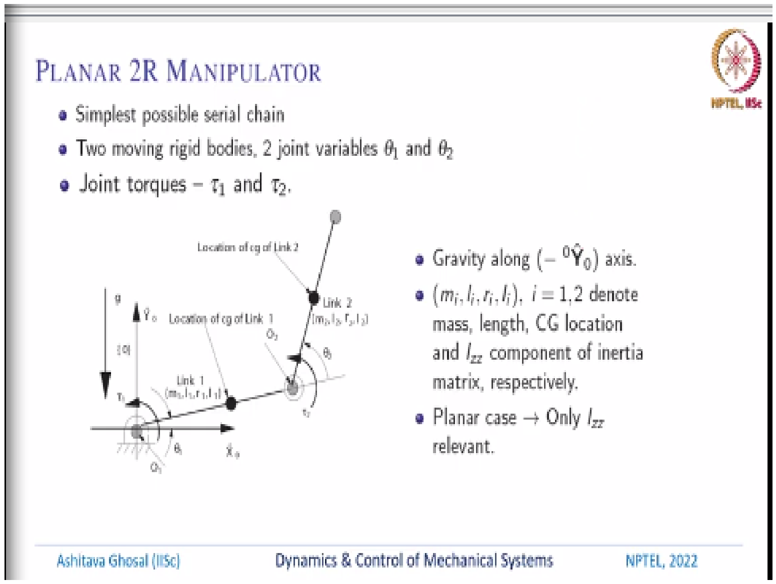

The first example is that of a planar 2R manipulator, this is also sometimes called as a double pendulum. So, it consists of 2 links, so this is link 1 from here to here and this is linked 2, the gravity is acting along the fixed y-axis opposite to the fixed y-axis and this is the x axis the z axis is coming out of the page. The link 1 can be described by a mass m_1, length l_1 from here to here, the location of the CG is at this point it is r_1 along the length and this is just for simplification.

And the z component of the moment of inertia is denoted by I_1, this link can rotate about the z axis by an angle θ_1. The second link is linked 2, it can be described or represented by a mass m_2, the length l_2, this location of the CG is at a distance r_2 from this origin and the moment of inertia about the z axis for this link is I_2. The first link can rotate about the z axis by θ_1, the second link has a relative rotation angle θ_2.

So, it is the simplest possible serial chain as I said there are 2 moving bodies, 2 joint variables θ_1 and θ_2. There are 2 joints torques which are acting at the first and the second joint respectively; these are τ_1 and τ_2. As I said the gravity is along the -Y_0 axis, m_i, l_i, r_i and I_i, i = 1, 2 denote the mass length CG location and the I_{zz} component of the inertia matrix respectively.

The motion of this 2R manipulator or the double pendulum is in a plane. So, hence only the I_{zz} component of the inertia is relevant.

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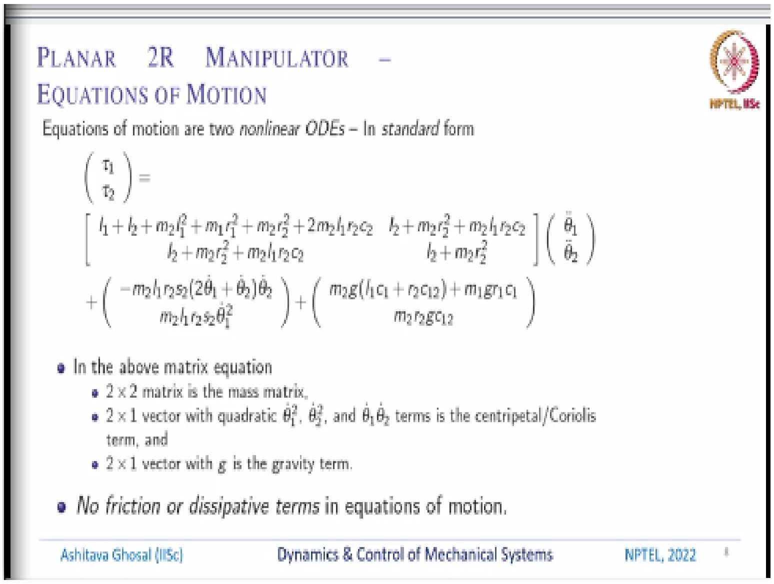

So, let us continue for the planar 2R manipulator, we can obtain the equations of motion. The equations of motion are 2 non-linear ODE's, in the standard form they are given us some τ_1 , τ_2 = sum matrix into $\ddot{\theta}_1$ into $\ddot{\theta}_2$, it is a 2 by 2 matrix plus a vector which contains $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_2^{\,2}$, 2 $(\dot{\theta}_1 + \dot{\theta}_2)$, $\dot{\theta}_1^{\,2}$.

And another term which contains the gravity, so it is like $m_2 r_2 g c_{12}$, c_{12}, here means cos(θ_1 + θ_2). So, in the above equations this is a 2 by 2 mass matrix, it is a symmetric positive definite matrix. So, this mass matrix is multiplying $\ddot{\theta}_1$ into $\ddot{\theta}_2$. Then this 2 by 1 vector contains quadratic terms $\dot{\theta}_1^{\,2}$, $\dot{\theta}_2^{\,2}$ and $\dot{\theta}_1\dot{\theta}_2$ these terms represent the centripetal and coriolis terms and then we have this gravity term.

So, these equations were derived in the last week and I have also showed you how we can derive these equations in an error-free manner using Maple. As mentioned earlier and because we are using the Lagrangian formulation to derive these equations, friction and dissipative terms are not there in this equation. If I want to add friction and dissipative terms to these equations, we have to do it in an adhoc manner.

So, we have to add some friction term after this, so the friction term will be like maybe $(c_1 + c_2$– $\dot{\theta}_1$, if you assume that the friction is constant plus some term which is like viscous friction. Similarly we can have some other $(c_1 + c_2)\,\dot{\theta}_2$ for the second equation but that has to be added manually in an adhoc manner. The Lagrangian formulation does not automatically give you the friction and dissipative terms and we have discussed this earlier. The Lagrangian formulation is for conservative systems not for dissipative systems.

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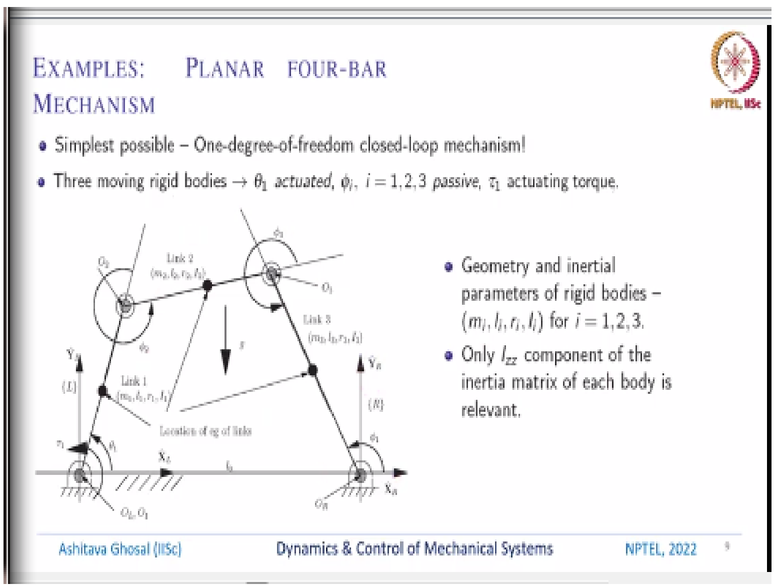

Similarly, for this closed loop simplest possible mechanism which is a 4 bar mechanism, we can derive the equations of motion. Remember, it is a 1 degree of freedom closed loop mechanism, there are 3 moving bodies 1, 2 and 3, we have only 1 actuation, so that is the torque τ_1 and it is rotating θ_1. The other 3 angles ϕ_1, ϕ_2 and ϕ_3, they are passive, there is only one single actuating torque.

So, similar to this previous example for each link we have m_1, l_1, r_1 and I_1 and again i_1 is the z component because this is a planar motion. For link 2 we have m_2, l_2, r_2 and I_2 and for link 3 it is m_3, l_3, r_3 and I_3 these angles are given in this form because we are assuming that we are using relative angles. So, θ_1 is relative to this x_1 and ϕ_2 is relative to the direction, this is the direction of the link and the next link is this.

So, if you assume or if you take the convention that counterclockwise is positive ϕ_2 will be like this, ϕ_3 will be like this not this smaller angle. It does not matter eventually if you stick to a consistent configuration convention then you will get everything will be fine. And again there is a gravity which is acting in this direction and the location of the CG is along the link similar to the previous case.

So, the geometry and inertial parameters of the rigid bodies are m_1,l_1, r_1 and I_1 for i = 1, 2 and 3, so there are 3 links. And again I_{zz} component of the inertia matrix for each body is relevant, I_{xx} and I_{yy} is not relevant because the motion is in the one plane.

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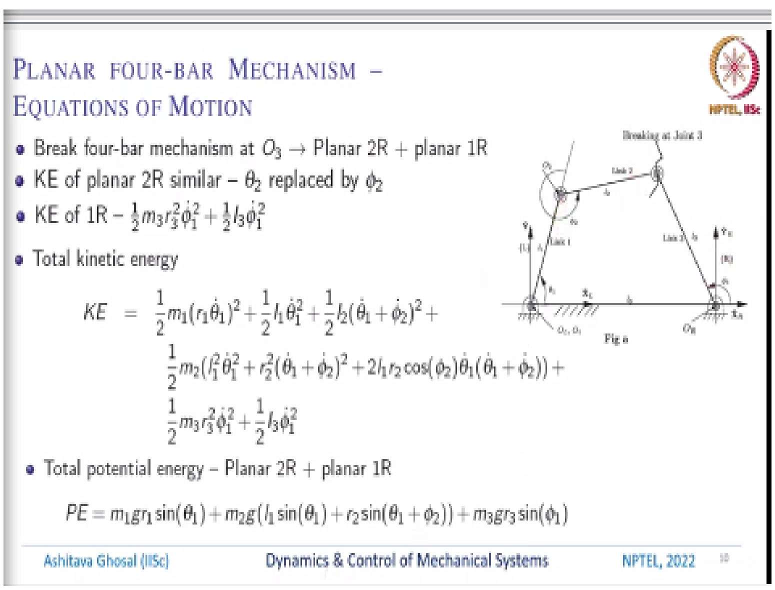

So, we have to derive the equation of motion, we break this 4-bar mechanism at O_3, so we break it here. And as a result once you break it you will have a 2R chain a serial chain and a 1R chain, very simple. We have done this earlier also when we looked at the kinematics of a 4-bar mechanism. So, the equations of motion can be very simply derived for this 2R chain and this 1R chain.

So, this kinetic energy of the planar 2R is very, very similar to what we did earlier few minutes back. So, all we need to do is θ_2 has to be replaced by ϕ_2, so this is a 2R chain and where the variables are θ_1 and instead of θ_2 we have ϕ_2. And this is a single link with 1 degree of freedom which is ϕ_1 to 1 rotation. So, the kinetic energy of the 1R is $\frac{1}{3} m_3 r_3^2 \dot{\phi}_1^2 + \frac{1}{2} I_3 \dot{\phi}_1^2$.

So, this is like $\frac{1}{2} I \omega^2$ and this is like $\frac{1}{2} m V_c^2$. So, the center of mass is somewhere here at a distance r_3, so the velocity will be $r_3 \dot{\phi}_1$, so the kinetic energy will be $\frac{1}{3} m_3 r_3^2 \dot{\phi}_1^2$ and of course $\frac{1}{2} I_3 \dot{\phi}_1^2$, where I is the z component of this link about the CG. So, the total kinetic energy will be the sum of the kinetic energy of this 2R chain and + 1R chain.

So, this first 4 terms very similar to what we have derived earlier is for the 2R chain and this last one is for the 1R chain and as before we see that there are terms which are like $\frac{1}{2} m_1 \left(r_1 \dot{\theta}_1\right)^2 +$ $I_1 \dot{\theta}_1^2 + I_2 \left(\dot{\theta}_1 + \dot{\phi}_2\right)^2$ because now we are replacing θ_2 by ϕ_2. And also we have a term which is $\cos \phi_2$ which depends on $\cos \phi_2$, previously in the 2R example we had $\cos \theta_2$.

So, as I said all we need to do is whatever the equations we have obtained for the 2R we replace θ_2 by ϕ_2. The potential energy is also a scalar and it is nothing but the potential energy of this 2R chain + 1R chain. So, this will be like $m_1 g\, r_1 \sin \theta_1$, we have assuming that this is the 0 potential energy surface, so the CG of this link 1 is at a distance r_1, so the height is $r_1 \sin \theta_1$, so we will get $r_1\, \theta_1\, into\, m_1$ g.

The height of this link 2 is $l_1 \sin \theta_1 + r_2 \sin(\theta_1 + \phi_2)$ and what is the height of this third link? It is $r_3 \sin \phi_1$. So, we can very easily compute what are the potential energies of 2R and 1R then we just add them.

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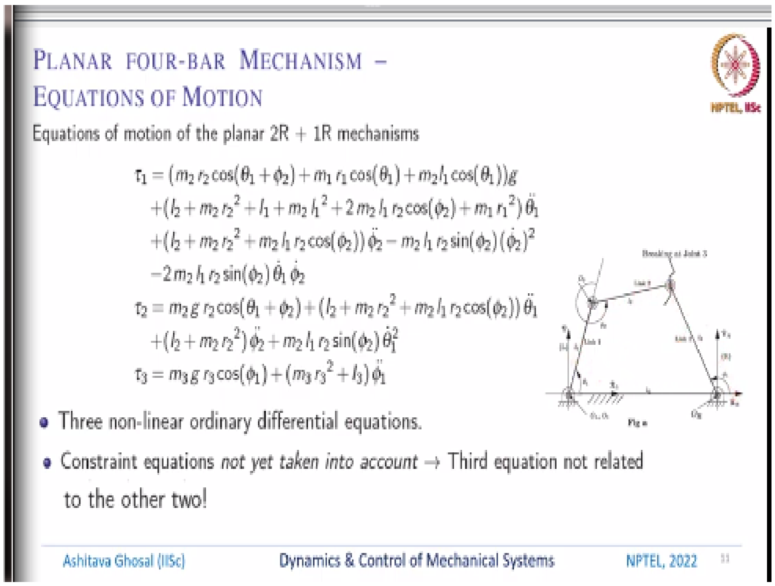

And once you add them and do the Lagrangian formulation we can obtain the equations of motion and this was discussed last week also. So, I showed you how we can obtain all the terms in the equation of motion using Maple and that is exactly what has been done here. So, we have used Maple to derive the kinetic energy, the potential energy and then assemble them together and we obtain 3 equations of motion.

So, there is a τ_1 which is given by functions of sum I_2 and $m_2 r_2^2$ and then all this $\ddot{\theta}_1$ and then I_1 and $m_2 l_1^2$, exactly the same as earlier. And similarly one τ_2 which is given by $(I_2 + m_2 r_2^2)\ddot{\theta}_1$ and then $\dot{\theta}_1^2$ and then some gravity term. Here also you can see that there is a gravity term which is $m_2 r_2 \cos(\theta_1 + \phi_2)$ g, so here it is written slightly differently, the g is taken out.

And the third equation is $m_3 g r_3 \cos \phi_1 + (m_3 r_3^2 + I_3)\ddot{\phi}_1$. So, we can derive the equations of motion of this system which is planar 2R + 1R because remember we have broken it. So, these are 3 non-linear ordinary differential equations. The important thing is the third equation in it is current form here is nowhere connected to the first 2 equations. So, you see here τ_3 is there, ϕ_1 is there and $\ddot{\phi}_1$ is here, we know there is only one independent variable because it is a 1 degree of freedom system.

So, we need to somehow bring in the constraints now and then all these 3 equations will become coupled. So, right now the third equation is not related to the other 2 but once you bring in the constraint equations it will become coupled. So, here it is broken, so as you can see, so the equation for this and the equation for this somehow, we have to go back and couple them together.

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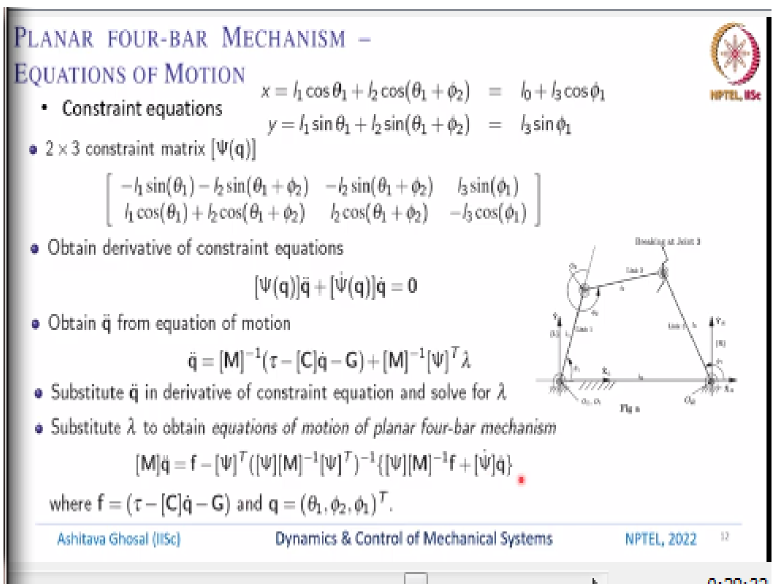

So, how do I obtain this coupling? We go back and look at the constraint equation. So, what are the constraint equations? It is x = $l_1\cos\theta_1 + l_2\cos(\theta_1 + \phi_2) = l_0 + l_3\cos\phi_1$, this is seen earlier. y component is of that point where we are broken is $l_1\sin\theta_1 + l_2\sin(\theta_1 + \phi_2) = l_3\sin\phi_1$. And we can differentiate these equations and then we get this 2 by 3 constraint matrix.

So, when you differentiate the first one you will get $-l_1\sin\theta_1$, so partial differential with respect to θ_1 then partial derivative with respect to θ_2 of ϕ_2 and the partial derivative with respect to ϕ_1.

So, the first partial derivative, so once you take it on this side will become sin ϕ_1 and then this one will be -sin ϕ_1 partial derivative with respect to ϕ_1 we will get -l_3 cos ϕ_1.

So, this is a 2 by 3 matrix, we can obtain the derivative of this matrix and write the constraint equations. So, this is psi into $\dot{q} = 0$ when you take the derivative, we have $\psi\ddot{q} + \dot{\psi}\,\dot{q} = 0$. And this again we have discussed earlier, so we introduced Lagrange multipliers for this loop closure constraint equations then we get an equation of motion which looks like $\ddot{q} = M^{-1}\left(\tau - [C]\dot{q} - G\right) + M^{-1}[\psi]^{T}\lambda$ we substitute $\ddot{q}$ back in the constraint equations.

And solve for λ and then obtain the equations of motion, so this has been done earlier. So, this is just a review, I have shown you what we can do using Maple and we can obtain all these terms and then we can obtain this equations of motion. So, in this figure just to go back and see the steps, I have broken this joint at joint 3, so then the X coordinate of this point here this $l_1$cos θ_1+ l_2 cos(θ_1+ϕ_2), so this is the x coordinate here.

This will be equal to l_0 + l_3 cos ϕ_1 component along the x axis, so both of these 2 will be equal. So, for Y we have l_1 sin θ_1 which is the vertical height and then this is l_2 sin(θ_1 + ϕ_2) that should be equal to l_3 sin ϕ_1. So, these are the 2 loop closure constraint equations when we break it at joint 3. And then again we take the partial derivatives and find the constraint matrix then we take the derivative of $\psi\,\dot{q} = 0$ we get $\psi\ddot{q} + \dot{\psi}\,\dot{q} = 0$.

And then mechanically very, very straightforward it will lot of effort but nevertheless it is a very, very mechanical way of deriving the equations of motion. So, hence we obtain the equations of motion and for this 4-bar mechanism in this form which is $[M]\,\ddot{q} = f - [\psi]^{T}\left([\psi]M^{-1}[\psi]^{T}\right)^{-1}$ and then $[\psi]M^{-1}f + [\dot{\psi}]\dot{q}$.

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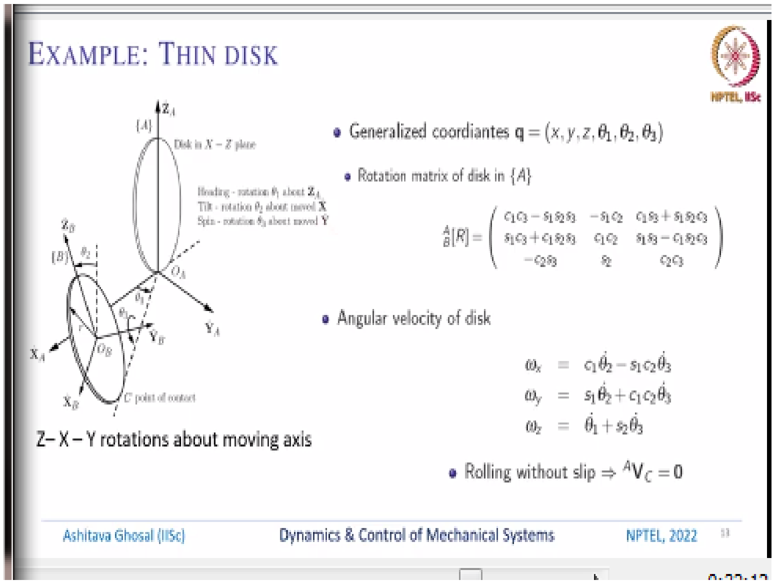

The last example or recapitulation of equation of motion is that of a thin disk. So, we have a thin disk which is originally in the XZ plane and then there is a rotation θ_1 which is called the heading. So, rotation about Z A axis then the next rotation is about θ_2 which is about the moved x axis and then the final spin which is θ_3 which is about the moved y axis. So, basically we have generalized coordinates for this problem as x, y, z, θ_1, θ_2, θ_3.

x, y, z is the center of this disk, so this is the location of this point O_B with respect to O_A, so that is the vector x, y, z. And because there are these 3 rotations which are happening about Z axis, then X axis, then Y axis, so we can use this Euler angle formula, Euler angle rotations about Z,,X and Y and obtain a rotation matrix of this disk with respect to the A coordinate system, so this is given by $BA[R]$ and we will have this 3 by 3 rotation matrix.

And as expected it will contain sin θ_1, sin θ_2, cos θ_1, cos θ_2 and cosθ_3 and sin θ_3 in some particular order. And again in the last week in the Maple example I have shown you how to derive this rotation matrix, we have done it much, much earlier in this course also but using Maple we can find the rotation matrix about Z, then the subsequent rotation about X and then the third rotation about Y.

We multiply these 3 rotations in the order of they are occurring and we will get this rotation matrix. So, once we have this rotation matrix we can do $[\dot{R}]\ [R]^T$ as shown again using this example of Maple last week I can find out the skew symmetric matrix $[\dot{R}]\ [R]^T$. Then from this skew symmetric matrix extract the x, y and z component of the angular velocity vector.

So, this is ω_B angular velocity of this B coordinate system; with respect to the A coordinate system and this is the space fixed angular velocity vector. So, $\omega_x = c_1\dot{\theta}_2 - s_1c_2\dot{\theta}_3$ and so on, $\omega_z = \dot{\theta}_1 + s_2\dot{\theta}_3$. So, we have all these angular velocity vectors which we can compute once we know what is the rotation matrix.

Then finally we impose the condition that the velocity of this point c which is the point of contact with respect to the A coordinate system with respect to the reference coordinate system is 0. So, this disk cannot slide in this direction, it can only roll about this dotted line, it cannot slide perpendicular to the dotted line, that is not allowed. We are solving the problem of a thin disk which is rolling without slipping.

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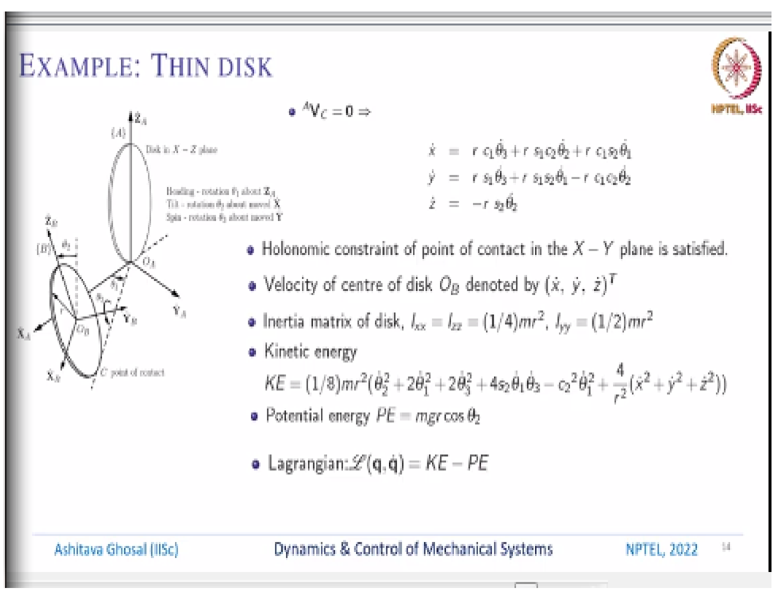

So, this $AV_C = 0$ will give rise to these 3 equations which is nothing but $\dot{x}$, $\dot{y}$, $\dot{z}$ which is the velocity of this point will be related to $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ and of course θ_1, θ_2 and so on, by these 3 expressions. So, how did I get this? Very easy, so we obtain the velocity of this point plus $\omega \times r$ will be the velocity of this point c and then we set that equal to 0.

So, velocity of this point contains $\dot{x}$, $\dot{y}$, $\dot{z}$, ω is what we had derived in the last slide it has ω_x, ω_y, ω_z, it contains $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\theta}_3$ and the angles also and then we simplify and we will get these 3 constraint equations. Strictly speaking there is a holonomic constraint also involved which is that this point of contact is always in the X-Y plane so basically this disk cannot leave this plane.

So, Z coordinate of this point of contact must be 0, so if you do this $AV_C = 0$ that is automatically satisfied, so we do not really have to worry about that there is 1 holonomic and 2 non-holonomic constraints, we are taking all of them into account by these 3 constraint equations. So, as I said the velocity of the center of the disk O_B is $\dot{x}$, $\dot{y}$, $\dot{z}$, the inertia matrix of the disk is I_{xx}, so the disk is in the X-Z plane.

So, $I_{xx} = I_{zz}$ is $\frac{1}{4} m r^2$, the inertia about the Y-axis $is \frac{1}{2} m r^2$, these are available in any standard textbook for a thin disk. The kinetic energy can be written as $\frac{1}{2} m V^2$, $V$of the center of mass of this disk + $\frac{1}{4} I \omega^2$. And if you do the simplifications you will get this expression for kinetic energy which is $\frac{1}{8} m r^2$ and then you will have some terms which is $\dot{x}^2$, $\dot{y}^2$, $\dot{z}^2$, this is very clear.

So, this 4 and 8 will cancel and r^2 and r^2will cancel, so you will have $\frac{1}{2} m \dot{x}^2$, $\frac{1}{2} m \dot{y}^2$, $\frac{1}{2} m \dot{z}^2$ that is very straightforward, $\frac{1}{2} m V_c^2$. The $I \omega^2$ part will be a little bit more complicated and you will get this. So, this is written in a compact form that is why this $\frac{1}{8}mr^2$ is taken outside and then you have($\dot{\theta}_2^2 + 2\dot{\theta}_1^2 + 2\dot{\theta}_3^2 + 4 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_3 - c_2^2 \dot{\theta}_1^2$), so this can be derived.

Once you know these elements of the inertia matrix and also what is the ω. The potential energy is $mgr \cos cos\, \theta_2$, this is also sort of clear. If you can see that this is the disk r, so $r \cos cos\, \theta_2$ is how much is the height above this X-Y plane, so we will get $mgr \cos cos\, \theta_2$. We can obtain the Lagrangian which is kinetic minus potential energy.

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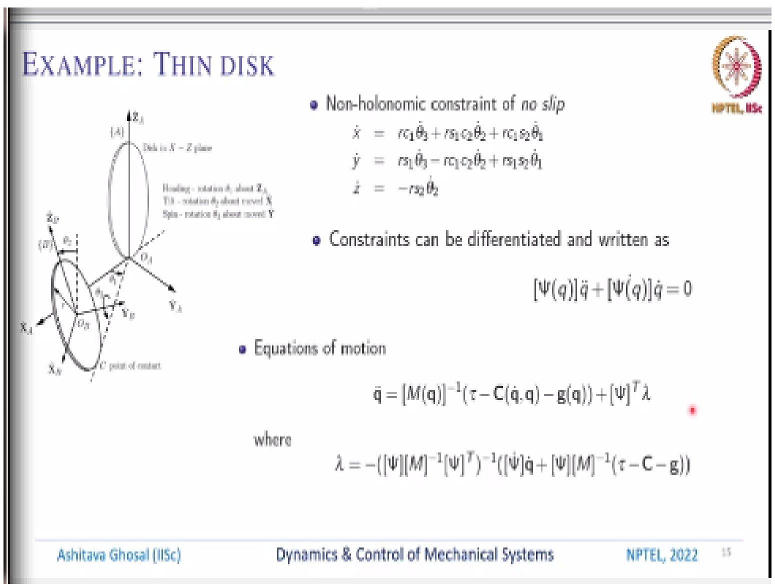

And then we can continue and derive the equations of motion and then we can take into account these non-holonomic constraints of no slip. We can differentiate these constraints standard approach, we get $[\psi]\ddot{q} + [\dot{\psi}]\, \dot{q} = 0$. So, this is like $\psi\, \dot{q} = 0$, so we can reorganize this. So, here q is a 6 by 1 vector, so this is ψ will be 3 by 6 and $\psi\, \dot{q}$will be appropriately you can find the dimension.

So, then the equation of motion again using the approach which we have discussed now several times with constraints is $\ddot{q} = M^{-1}\left(\tau - [C]\dot{q} - G\right) + [\psi]^T\lambda$, where λ at the Lagrange multiplies, there are 3 of them because of these 3 constraints. And then we can find the expression for the Lagrange multiplier which is $\left([\psi]M^{-1}[\psi]^{T}\right)^{-1} \left([\dot{\psi}]\,\dot{q} + [\psi]M^{-1}\left(\tau - [C]\dot{q} - G\right)\right)$.

Same story, whatever we have done earlier. We can compute the Lagrange multiplies, substitute back in this second order ODE and obtain the equations of motion.

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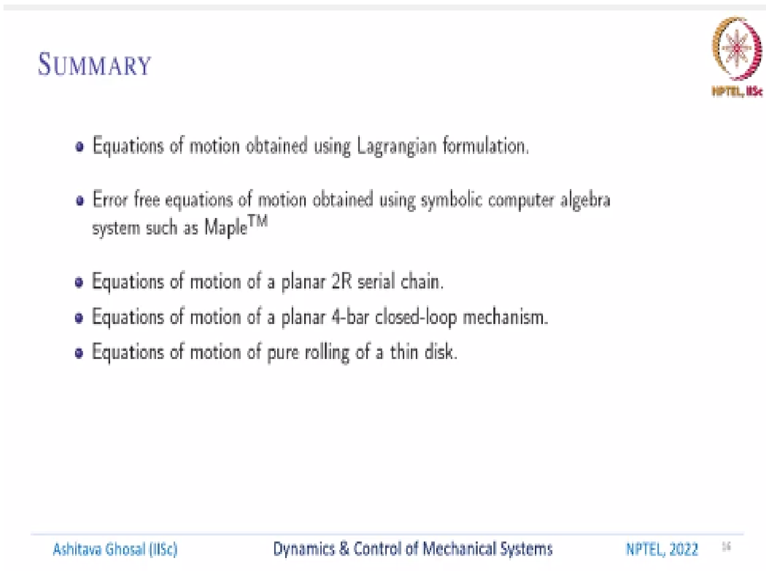

So, in summary we can derive the equations of motion of a multi-body system using the Lagrangian formulation. We can obtain error free equations of motion using symbolic computers algebra systems such as Maple. And in this lecture I have shown you the equations of motion of a planar 2R serial chain. The equations of motion of a planar 4-bar closed-loop mechanism and also the equation of motion of a pure rolling of a thin disk.

So, this one there are no constraints, this one there are loop closure constraints which are holonomic constraints. And in the case of this pure rolling of a thin disk we also have non-holonomic constraints involving derivatives of the generalized coordinates.