Dynamics and Control of Mechanical Systems
Prof. Ashitava Ghosal
Department of Mechanical Engineering
Indian Institute of Science-Bengaluru

Lecture-11
Angular Momentum, Spinning Tops and Gyroscopes

In the next lecture, I will look at angular momentum of rigid body. And in particular, we will look at two examples, we will look at what is a spinning top and we look at gyroscopes.

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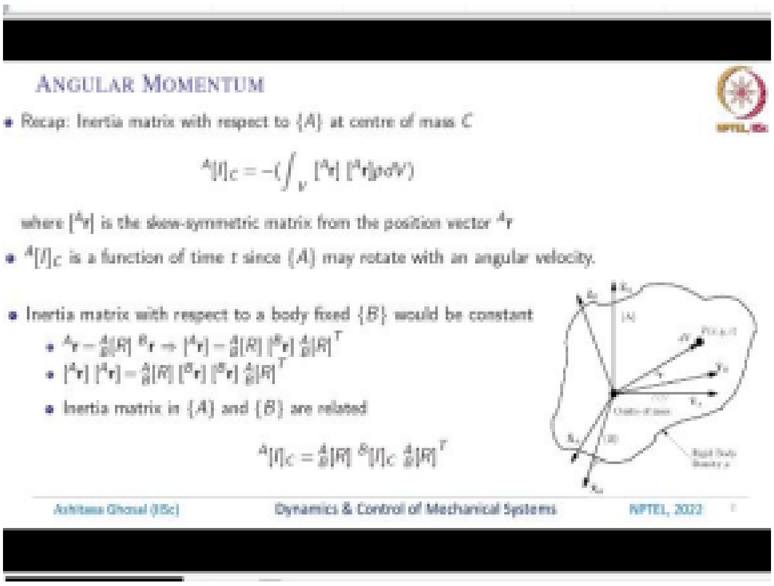

So, let us recapitulate what is angular momentum, to understand what is angular momentum, we first need to look at what is the inertia matrix of a rigid body with respect to the mass center. So, the mass center remember is located in a way such that the mass. So, our C is the integral of r into dV volume integral divided by the total mass and the moment of inertia is given by this ${}^{A}[I]_{C} =- \left(\int_{V} [A\,r][\ A\,r]\rho dV \right)$ and what is $A\,r$?

It is the skew symmetric form of this vector $A\,r\ and\,[A\,r]$ is a skew symmetric matrix for the position vector $A\,r$. So, we had done this earlier. So, this is the inertia matrix for these rigid bodies about the mass center with respect to this A coordinate system. This is a function of time since A may rotate with some angular velocity. So, in order to look at the inertia, it is better to compute the inertia in a body fixed coordinate system.

So, we have another coordinate system $\hat{X}_B\ \hat{Y}_B\ \hat{Z}_B$ which is fixed to the body for the moment it is located at the center of mass instantaneously at the same place. So, now, this

$A\, r$

vector can be written in terms of $BA[R]$, it is a rotation matrix of B with respect to A into $B\, r$. So, the skew symmetric form of this vector $A\, r$ can be written as $BA[R]$ $B\, r$ into $BA[R]^T$.

So, this is just like any other matrix, how a tensor is transformed between two different coordinate systems A and B. So, $[A\, r]$into $[A\, r]$ the product of these two skew symmetric matrix can be written as $BA[R]$ $[B\, r]$ $[B\, r]$ $BA[R]^T$. This is again skew symmetric matrix is converted from one coordinate system to another coordinate system. So, the inertia matrix in A and B are related in this form.

That the inertia matrix with respect to the A coordinate system is related to the inertia matrix in the B coordinate system pre multiply by this rotation matrix $BA[R]$ and forced multiplied by $BA[R]^T$. This we had seen earlier.

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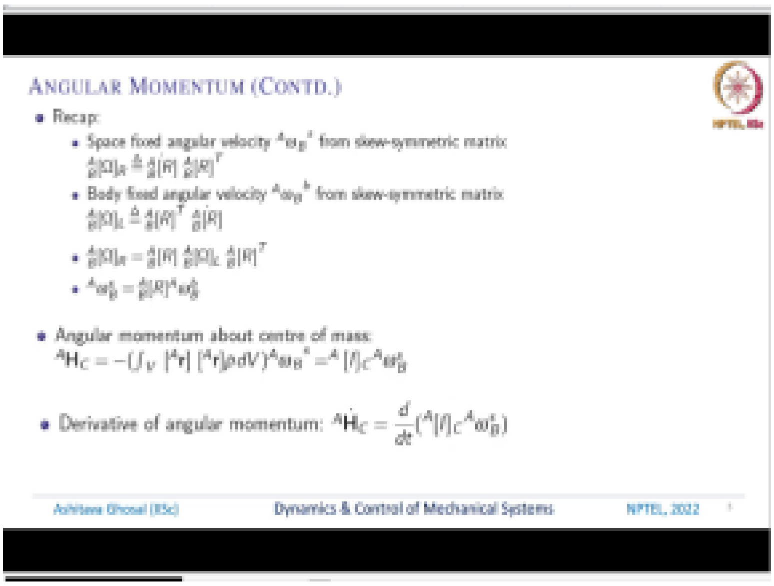

Let us recap also a little bit about the angular velocity vector. So, we had derived one angular velocity vector from the rotation matrix which was $\dot{[R]}$ $[R]^T$. So, $\dot{[R]}$ $[R]^T$. This we called as the $BA[\Omega]_R$, this from here we could obtain a skew symmetric matrix $BA[\Omega]_R$ and from that we could extract the angular velocity vector $A\omega_B^s$ and we said that this was the space fixed angular velocity vector.

Similarly, we could also obtain a body fixed angular velocity vector from the skew symmetric matrix $BA[\Omega]_L$. So, basically instead of $[R]$. $[R]^T$ equals identity we started $[R]^T.[R]$ is equal to identity and we could obtain another skew

symmetric matrix which is $[R]^T$ $[\dot{R}]$ and again just like any other matrix or tensor.

The left and the right skew symmetric matrices are related by means of rotation matrices $BA[\Omega]_R$ $[R].BA[\Omega]_L$ $[R]^T$. And the angular velocity vectors $A\omega_B^s$ in the space fixed formulation. So, $A\omega_B^s$ is nothing but $A\omega_B^b$ in the body fixed reference system pre multiply by a rotation matrix. So, this we had seen earlier. We had also seen that the angular momentum about the center of mass could be written in this form.

So, C is the center of mass and with respect to some A coordinate system, it is the ${}^AH_C =-\left(\int_V [A\,r][\,A\,r]\rho dV\right)$ And then this whole thing was multiplied by this $A\omega_B^s$. So, this quantity here as I have shown in the last slide, it is nothing but the inertia matrix. So, the angular momentum is nothing but the inertia matrix times this angular velocity vector. So, this is a 3 by 3 matrix into 3 by 1.

So, we get a vector with three components here. So, the derivative of the angular velocity matrix can be denoted by $A\dot{H}_C$. So, this is nothing but the derivative of the right side. So, we have d by dt of inertia matrix into angular velocity vector. So, again A an s and B and all these things are consistent, to just to make it consistent that we are multiplying vectors and matrices and we are doing operations on vectors and matrices all in the same coordinate system.

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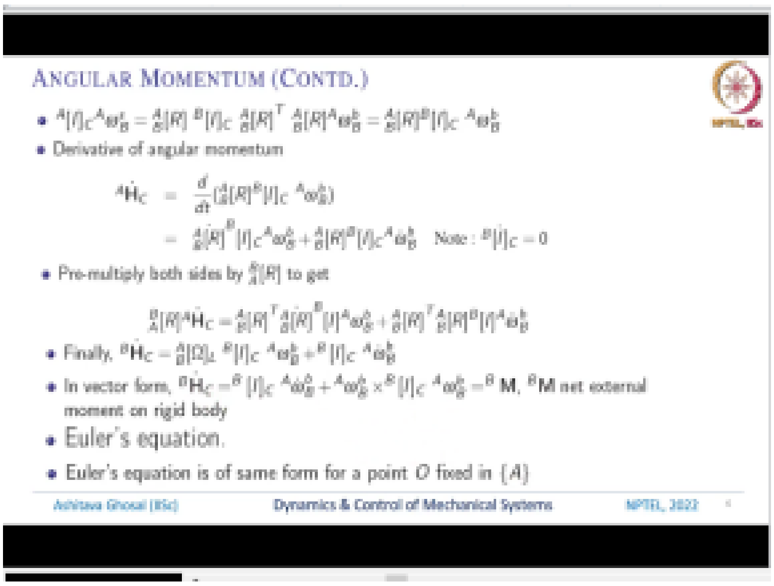

So, now, from that expression, we can show that this ${}^{A}[I]_{C}\, A\omega_{B}^{s}$ can be written in terms of $BA[R]$ into ${}^{B}[I]_{C}$ into $BA[R]^{T}$. So, ${}^{A}[I]_{C}$ is this part here and then $A\omega_{B}^{s}$ can be written as $BA[R]\ \ A\omega_{B}^{b}$ body fixed angular velocity. So, I am just rewriting this in these terms here and this angular velocity space fixed can be written in terms of body fixed.

So, now, what you can see is this $BA[R]^{T}$ and $BA[R]$ they will give an identity matrix. So, hence we are left with $BA[R]\ {}^{A}[I]_{C}$ into $A\omega_{B}^{b}$. So, because these two will become identity. So, the derivative of the angular momentum $A\dot{H}_{C}$ which is the derivative of the left hand side is nothing but the derivative of these three terms $BA[R]\ \ {}^{B}[I]_{C}$ into $A\omega_{B}^{b}$.

So, if you again use chain rule, so, the first time will be $[\dot{R}]$ into ${}^{B}[I]_{C}$ into $A\omega_{B}^{b} + BA[R]$ into the derivative of this is 0. So, the last term is this into derivative of $A\omega_{B}^{b}$ so, which is written here. So, in the body fixed coordinate system the inertia does not change. So, if you are sitting on the body and you see these components of inertia they are not changing with time.

So, hence the ${}^{B}[\dot{I}]_{C}$ derivative of this is 0. So, now, you pre multiply both sides by $AB[R]$ So, which is what $BA[R]^{T}$. So, that this is the inverse of the rotation matrix. So, if you pre multiply by this, so, again you can see this A and A will sort of cancel and you will be left with this derivative of the angular momentum in the B coordinate system and the right hand side will have now $BA[R]$ R transpose into $BA[\dot{R}]$ into this inertia matrix into $A\omega_B^b$.

And then here we have $AB[R]$ into this $BA[R]$ which is $BA[R]^T$ into $BA[R]$. So, this again becomes an identity matrix and we will have B I and $A\dot{\omega}_B^b$ of that. So, finally, we can now see that this is the angular momentum in the B coordinate system and what is this? This is $[\dot{R}]$ $[R]^T$. So, this is the left skew symmetric matrix into ${}^B[I]$.

And then we have this $A\omega_B^b$ which is here and then these two becomes identity.

So, we have ${}^B[I]_C$ into $A\dot{\omega}_B^b$, it is a derivative of the body fixed angular velocity vector. So, in vector form, this is the same as this, but this quantity can be written as ${}^B[I]_C$into $A\dot{\omega}_B^b$. So, this is the term coming here and this quantity here, remember,
this is a skew symmetric matrix. So, this is what like omega cross I omega. So, this part is I omega instead of taking a skew symmetric matrix, I can say this is like a cross product.

So, what do we have here? The derivative of the angular momentum in the B coordinate system is something like I into the derivative of the angular velocity plus omega cross I omega and this whole thing will be equal to the net external moment which is acting on the rigid body. So, this is well known. So, like linear momentum is the change of linear momentum is equal to the external force.

The change of angular momentum is equal to the external moment, which is acting on the rigid body; this is the well known Euler's equation. An interesting fact about Euler's equation is that it need not be about the center of mass. The Euler's equation also is valid or has a similar form, if it is about a point O which is fixed in A. So, I could have written $B\dot{H}_C$ O which is but the point O has to be fixed in A, need not be about the center of mass.

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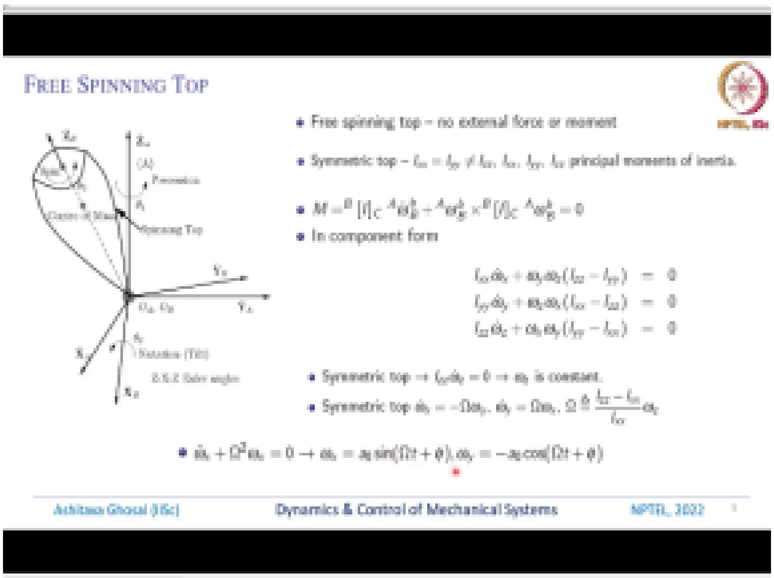

So, let us look at an example. This is an example of a free spinning top. So, what we have here is a top which is spinning and this point is fixed in the O_A and O_B are at the same place. So, we have an $\hat{X}_A$, $\hat{Y}_A$ $\hat{Z}_A$ axis and this top has a body fixed coordinate system which is $\hat{Z}_B$ which is along this direction as shown here, then there is an $\hat{X}_B$ and there is a $\hat{Y}_B$. So, there are these names for these different axes and the angles.

So, the rotation about Z axis, this θ_1 is called precession. So, if you have this top which is spinning, then it is also rotating about the $\hat{Z}_A$, there is also a tilt of the stop about the $\hat{X}_B$ axis. So this is either angles, this is about the moved x axis. So, this angle is θ_2 and this is sometimes called nutation, it is also sometimes called tilt and that is the last angle which is spin of the stop and this is θ_3 about the Z axis.

So, this is a classic example of Euler angles. So, Z-X-Z Euler angles. So, in this free spinning top, there are no external forces or moments which are acting. The symmetric top is a special case of general top in which what happens is that I_{xx} of this inertia matrix of this top about XX, XX is remember integral of y square plus z square is same as I_{yy} and both of these are not equal to I_{zz}.

So, I_{xx}, I_{yy}, I_{zz} at the principal moments of inertia of this top about this $\hat{X}_B$ $\hat{Y}_B$ $\hat{Z}_B$. So, now, we can write the Euler equation, which is that ${}^{B}[I]_{C}$ that inertia matrix with respect to the body fixed coordinate system into $A\dot{\omega}_{B}^{b}$. So, this is like alpha rate of derivative of the angular velocity vector I α + ω× I ω should be equal to the external moment, but we are looking at a free spinning top with no external forces or moment hence, this is equal to 0.

So, if you now expand this, so what is this is I_{xx}, I_{yy}, I_{zz} what is $A\omega_{B}^{b}$? It has components of ω_x, ω_y, ω_z? So, if you expand all this, you will get $I_{xx}\ \dot{\omega}_x$ because this is the derivative of the angular velocity + $\omega_y\ \omega_z\ (I_{zz} - I_{yy}) = 0$. So, that is one equation. Likewise, we will get another equation which is $I_{yy}\ \dot{\omega}_y + \omega_z\ \omega_x\ \ (I_{xx} - I_{zz}) = 0$.

And the third component of this vector equation is $I_{zz}\ \dot{\omega}_z + \omega_x\ \omega_y\ \ (I_{yy} - I_{xx}) = 0$. So, this is a symmetric top. So, hence $I_{zz}\dot{\omega}_z$ is 0. So, you can see that I_{yy} will be equal to I_{xx}. So, hence this term will go to 0. So, $I_{zz}\ \omega_z = 0$. So, which means what ω_z is constant and then if ω_z is constant we can see that $\dot{\omega}_x$ will be equal to minus some another $-\ \Omega\ \omega_y$.

And $\dot{\omega}_y$ will be this $\Omega\ \omega_x$ where this $\Omega = I_{zz}$ - I_{xx} divided by I_{xx}. So, just by looking at these two equations and putting ω_z as constant and then simplifying and noting that I_{xx} and I_{yy} are same. So, what we get here is two differential equation $\dot{\omega}_x$ is something constant into ω_y and $\dot{\omega}_y$ is some constant into ω_x.

So, both of these two equations if you take another derivative of this and then substitute back what is $\dot{\omega}_y$, you will get $\ddot{\omega}_x + \Omega^2\omega_x = 0$. You can see that, this derivative is $\ddot{\omega}_x$ is - $\Omega\ \omega_y$. So, $\dot{\omega}_y$ will be this $\Omega\ \omega_x$. So, you can substitute here. So, we will get $\Omega^2\omega_x = 0$. So, this Ω^2 is constant remember ω_z is constant and this I_{xx} and I_{zz} these are known constants.

So, hence this ω_x component is some $a_0\, sin\ (\Omega\, t\ +\ \phi)$. So, this is the simple second order equation with Ω^2 is positive. So, hence the solution is some sine $sin\ (\Omega\, t\ +\ \phi)$ and ω_y can be written as mean ω_y is $\dot{\omega}_x$ divided by $-\Omega$. So, you will get another equation which looks like this which is $-a_0 \cos(\Omega t + \phi)$ whereas, $\omega_x + a_0 \sin(\Omega t + \phi)$.

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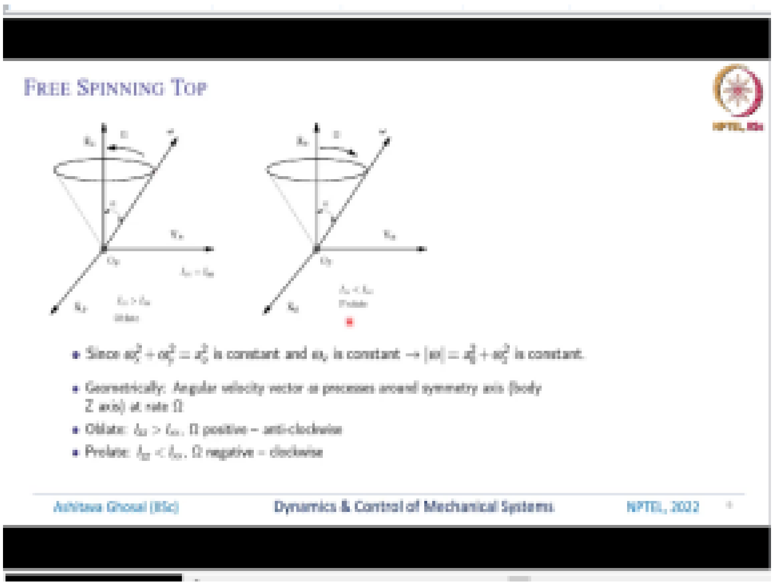

So, we can show what is happening to this ω and what is actually Ω? So, basically if you have I_{zz} greater than I_{xx}. So, this omega vector with components ω_x, ω_y, ω_z is rotating about this cone at some angle α and if I_{zz} is greater than I_{xx} the direction is this way, if I_{zz} is less than I_{xx} the direction is this way. So, since $\omega_x^2 + \omega_y^2 = a_0^2$ is a constant and ω_z is a constant.

So, this magnitude of omega is also a constant, magnitude of omega is $a_0^2 + \omega_z^2$ it is also a constant. So, basically what is happening geometrically? That this angular velocity vector omega precesses around the symmetric axis the body z axis at a rate Ω and it goes anti clockwise if $I_{zz} > I_{xx}$ and it goes clockwise. If $I_{zz} < I_{xx}$, this is called oblate and this is called prolate.

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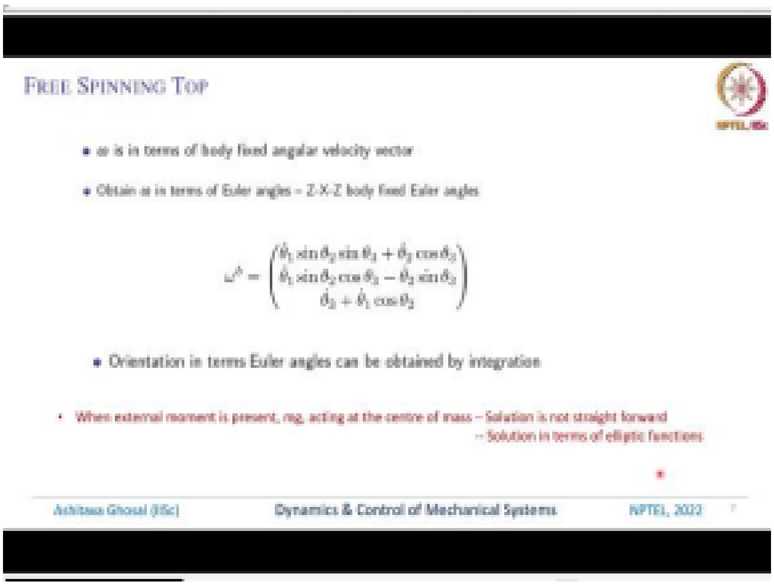

So, the ω is in terms of the body fixed angular velocity vector, we can obtain ω using Z-X-Z Euler angles. Remember Z was first was precession, then it was tilt and then it was spin for this top. So, the angular velocity vector in terms of θ_1, θ_2 and θ_3 and their derivatives can be obtained in this form, this is the standard Z-X-Z body fixed Euler angles.

So, we have $\dot{\theta}_1 \sin\theta_2 \sin\theta_3 + \dot{\theta}_2 \cos\theta_3$ that is the X component and the Y component is $\dot{\theta}_1$ sin θ_2 + θ_3 and - $\dot{\theta}_2$sine θ_3 and the Z component is $\dot{\theta}_3 + \dot{\theta}_1 \cos\theta_2$. So, the orientation in terms of the Euler angles θ_1, θ_2 and θ_3. So, θ_1 is about z, θ_2 is that tilt, Z is again the spin, we can obtain by integrating.

So, once we know what is the left hand side, we can integrate these three differential equations and find out what is θ_1, θ_2 and θ_3. So, what have we done that for this free spinning torque we can solve the differential equations and I can show you how the top is spinning and how it is precessing about the z axis and in that sense it is tilting and everything can be solved in closed form.

So, this is one of those rare examples of a 3D case of something which is spinning and which we can obtain the orientation in close form because we know what is angular velocity from the previous slide and then we can find out what is theta dots from this and theta by integration. When the external moment is present, so, for example, there is a weight of mass times gravity which is acting at the center of mass then we cannot solve these equations.

Nonlinear equations is not straightforward. However, a lot of people have worked on these tasks and there is something called as an elliptic function which I do not want to get into, I can find the solution when there is a mass mg, which is acting at the center of this top and then the solution is in terms of elliptic functions.

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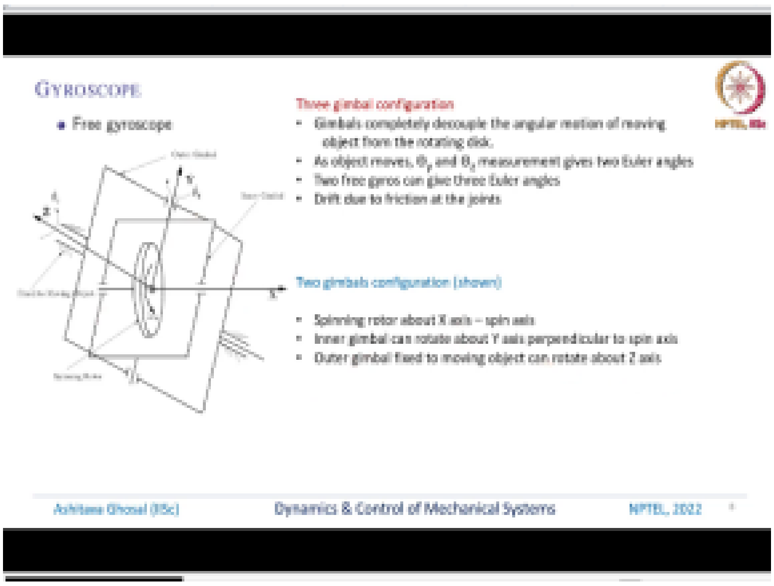

Let us take another example; this is the example of a gyroscope. And we will look at what is called as a free gyroscope. So, there are many kinds of guided scope, there is something which is called as a three gimbals configuration. So, what is a gimbal? So, gimbal is this ring like thing. So, there is a spinning disk which is mounted on this ring like thing. And this ring in this example could rotate about the x axis.

So, this gimbal itself can mount it on another outer gimbal and then that outer gimbal can also rotate. So, we could have some rotation of the disk about x axis and this gimbal can rotate about the y axis by $\dot{\theta}_y$ and then this outer gimbal can again rotate about $\dot{\theta}_z$. So, this is an example of a two gimbal configuration where you have an inner gimbal and an outer gimbal and a spinning rotor.

So, we will come to that little later what it is used for. So, basically, if you had a three gimbals configuration, which is not shown here, these gimbals completely decouple the angular motion of the moving object from the rotating disk. So, this whole thing is mounted, this hash lines are the moving object. So, this is fixed on let us say a car or an airplane or a missile or something which is moving.

And then inside this moving object, we have all these rings or gimbals and then finally, there is a rotating disk. So as the object moves θ_y and θ_z, so we could measure what is θ_y and θ_z and it gets to Euler angles. So remember, you can see here it is about the rotating or move Y and Z axis. The X axis is the spin axis; this X axis itself is rotating by this gimbal.

So this rotating disk can tilt and that is the rotation about y and then this whole thing can rotate about the Z. So if I could measure what is happening at these two rotations about Y and Z as the body moves, I can somehow figure out what are these two rotations. So, these are the two Euler angles corresponding to the moving body. So, if you have two free gyros and they are mounted in a way such that one of them gives Y and Z, the other one will give some X and Y.

So I can find all the three Euler angles of this moving object. So, basically what is happening is with two free spinning gyros, I can obtain what is the Euler angles or the rotation of this rigid body or this body on which these two gyros are mounted. There is of course, this issue of some friction at these joints at these rotary joints. So, then due to friction, there will be some drift, but nevertheless, conceptually, if I can measure what is the rotation about the y axis and z axis in this figure.

And for another gyro rotation about X and let us say Y, so together, I have X, Y and Z three rotations. So, in this two gimbals configuration, the spinning rotor is about the X axis, this is called as the spin axis, the inner gimbal can rotate about the Y axis and the outer gimbal fixed to the moving object can rotate about the Z axis.

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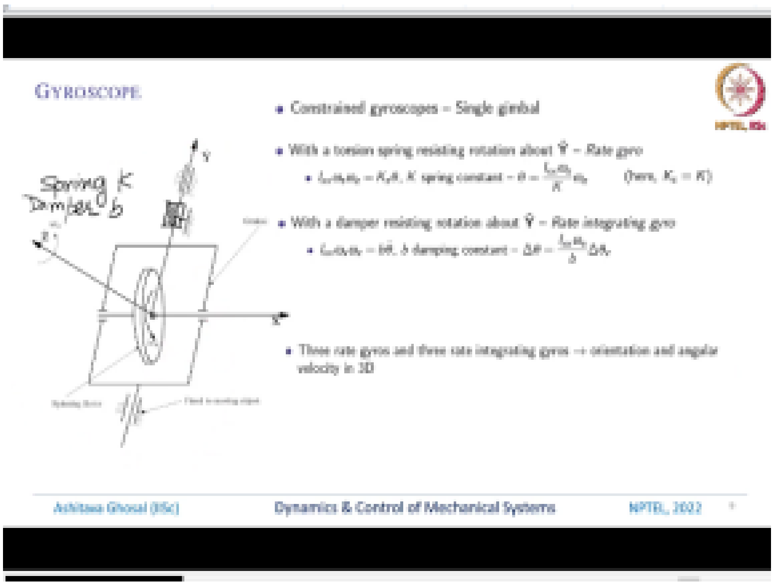

We can also have a gyroscope where in the Y axis; this motion is constrained by a spring or a damper. So, this is a single gimbal configuration, these are called constraint gyroscopes. So, basically, this disk is spinning about the X axis, but the motion about the Y axis, there is a spring K and a damper. So, with the torsion spring resisting the motion about this Y axis, these are called rate gyros.

So, basically, what we have is $I_{xx}\,\omega_s\,\omega_z = K_s\theta$. So, there is some theta rotation happening or the Y axis there is a torsional spring. So, $K_s\theta$ will be equated to $I_{xx}\,\omega_{spin}\,\omega_z$. So, the spring constant is K and from here we can see that theta is given by $(I_{xx}\omega_s)$/ K into ω_z. So, if I can measure what is this rotation I can find out ω_z because I know how much, at what speed this disk is spinning that is driven by some motor.

And if I know what is I_{xx} and if I know what is the spring constant measuring theta will give me ω_z. So, measurement of an angle gives the angular velocity along the Z axis. So, it is interesting to see that the angle is rotating about the Y axis, but it is measuring this ω_z which is the component about the Z axis. If you also have a damper which is resisting this motion about the Y axis, these are called rate integrating gyros.

Because then what will happen is this $I_{xx}\,\omega_s\,\omega_z = b\,\dot{\theta}$, here it was $K_s\theta$. So, this is a $b\,\dot{\theta}$. So, basically what it means is $\Delta\theta = \Delta\theta_z$. So, we are assuming ω_z is some $\Delta\theta_z$ divided by delta t. Similarly, here $\Delta\theta$ divided by delta t. So, these two t's will go. So, if you could measure this $\Delta\theta$, I can find what is the integral of this ω_z.

This is $\Delta\theta_z$ that is why these are called rate integrating gyros. So, here theta was proportional to the angular speed or component about the Z axis. Here the rotation of the changes rotation is proportional to the change in rotation about the Z axis. So, again if you have three rate gyros and three rate integrating gyros, we can obtain both the orientation and angular velocity in 3D. So, with three of these rate gyros I can find ω_x, ω_y, ω_z if I mounted correctly about the X,Y and Z axis.

Similarly, if you have three of these, then I can find the orientation. So, this is an interesting fact that with three rate gyros and three rate integrating gyros you can find the orientation and angular velocity of the object on which these gyros are mounted in 3D.

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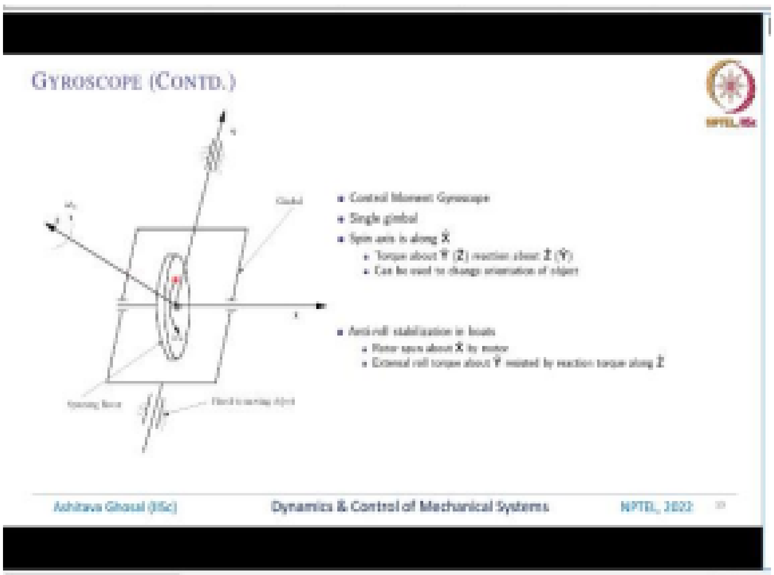

Finally we have this two other kinds of gyros; these are called a control moment gyroscope. So, this is a single gimbal, the spin axis is again about the Z axis. So, this is the disk which is rotating at some ω_s constant speed. So, if there is a torque about the Y-axis the reaction can be seen about the Z-axis and the other way around. So, if there is a torque which is acting on this rigid body on which this gyroscope is mounted.

So, this is fixed to the moving object. If there is a torque about the Z-axis then the reaction can be seen about the Y-axis. So, what is the purpose of this? So, if you spin this and if there is a torque which is acting here then you can use this mechanism to change the orientation of the object. So, you supply one torque here and it will rotate about the Y axis. Likewise you apply one talk about the Y axis and this whole object will rotate about the Z.

So, I can change the orientation of this object due to the torque which is acting in a different direction. You can also use these two what are called as anti-roll stabilization in boats. So, assume that this is like a boat in which there is a spinning rotor and this rotor is spinning about the X-axis. So, now if there is the torque which is acting about the Y axis, this is sometimes called as a roll torque.

This will be resisted by a reaction talk about the z-axis similar to this idea here. So, if you have a gyro which is mounted in this way that this is rotating about this and the boat is going along the y direction. So, if there is a roll which means that the boat is rotating about the y axis depending on how fast you are spinning it will resist that motion, it resists that roll motion.

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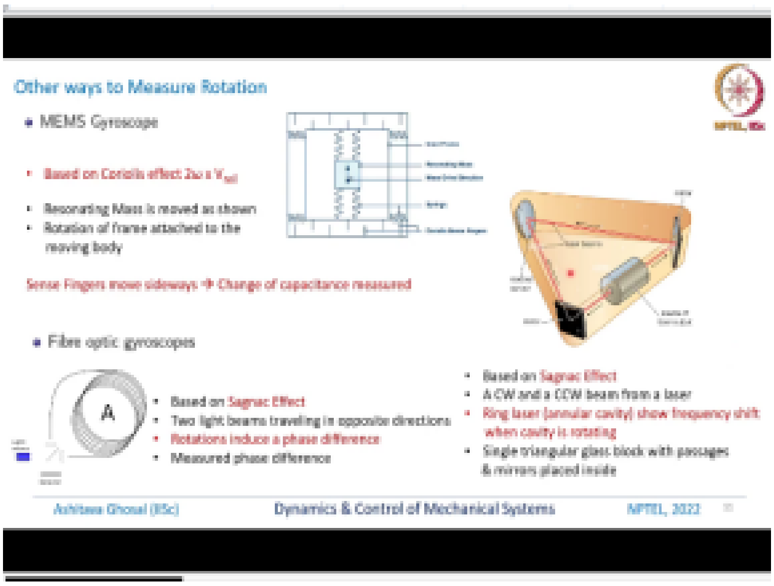

With that we will stop about this top and gyroscope one last word; we want to sometimes measure what is the rotation of an object. So, as I showed you with three rate gyros and three rate integrating gyros we can measure the rotation or orientation and the angular velocities. There are other ways to do it and these slide shows there is something called as a MEMS gyroscope. So, what is a MEMS gyroscope?

We have a fixed body and inside this there is the inner frame and inside this inner frame there is a mass which is going up and down. So, this is the motion of this mass and this mass is supported on springs in this way and this inner frame is also supported on springs in this direction. So, as you can see let us call it x direction and this is y direction. So, if this whole body is rotating about the z direction which is coming out of the page.

And then this mass is translating along this x direction. So, z cross x will give me a force or a motion in the y direction. So, this is 2 $\omega \times V_{relative}$, this is from the coriolis acceleration. So, omega is coming out, $V_{relative}$ is this. So, $\omega \times V_{relative}$ this body will move left or right. So, as it moves left and right there are these fingers, so there is a one set of fingers which are fixed to the outside and then there is another set of fingers which is on this inner frame.

So, what you can see is because of this coriolis component this moving fingers will go towards this fixed fingers and these are charged in the sense that I can measure the capacitance between this finger and this finger. So, what is happening? If I have a angular speed of this whole body which is omega let us say it is along the z axis, the $V_{relative}$ is along the x axis.

So, there will be a motion about the y axis and then this distance between the fixed and the moving fingers will change and we can measure the change in capacitance. So, this change in capacitance after some calibration and so on can be shown to be proportional to this ω. So, we are controlling this $V_{relative}$; this we know by some drive mechanism we can move this up and down and then we measure the change in capacitance then we know what is ω.

This is the way to measure the angular velocity about one direction which is the z direction and if I mount three such devices in which case I can measure the three components. So, if I mount one device such that this is vertically in this plane then the other one is in the other plane xz plane and other one is in the yz plane I can obtain the three components. This is what is called as a MEMS gyroscope.

This is very different from what I showed you about this rotating spinning disk, these are very small and now it is very accurate and they can even fit into your mobile phone. So, in your mobile phone you have gyroscopes and I will show you later we have accelerometers
which can measure the motion of this mobile phone, we can get the acceleration and angular

velocities of this body on which this is mounted.

There are another very well known way of measuring angular velocity and orientation these are called fiber optic gyroscope. So, basically what you have is a small fibre optic fibre in which you send through a light and this goes round and round and it comes out into a detector. So, now if this object is moving, it is subjected to some kind of an angular velocity. So, the path traveled one light beam which is going like this and another light beam which is going in the opposite direction there will be a phase difference which is introduced.

If there is no angular velocity the phase difference will be something and if there is rotating this whole device then there will be a phase difference. This is called as the Sagnac effect. So, if you can measure the phase difference we can find out what is the angular velocity of this object on which this fiber optic device is mounted. The same story is happening here. This is called as a ring laser gyroscope.

So, again we have a laser which is going in one direction like this and there is another beam which is going in the opposite direction and then this body is somehow rotating then this two laser beams will travel slightly different lengths and if there are these mirrors placed. So, we can see that there is a frequency shift due to this arrangement because due to this rotation this is again based on the Sagnac effect.

So, we have some laser, one counterclockwise beam and one clockwise beam and then this whole object is spinning you can see some bit in this readout sensors you will see some frequency shift happening, in one direction the frequency will be something, in the other direction the frequency will be something else and again the frequency shift is related to the angular velocity of this object.

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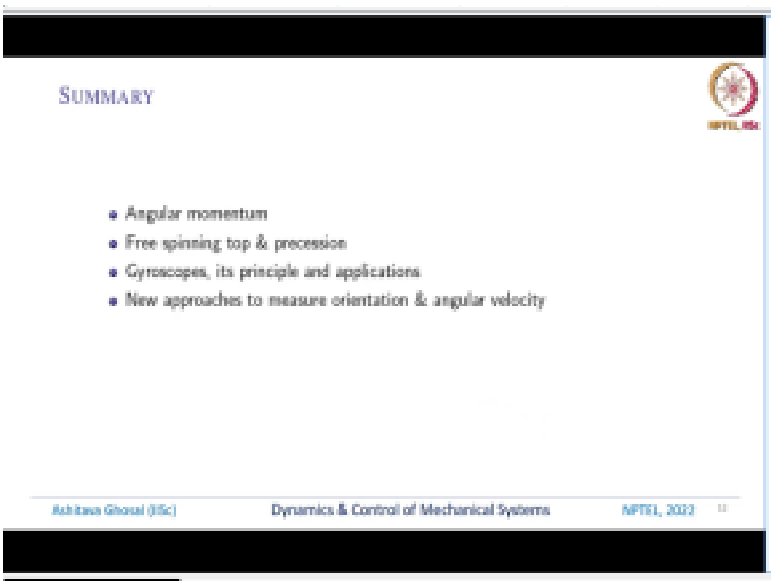

So, in summary, in this set of lectures or in this lecture we have looked at angular momentum and then I showed you how this angular momentum and the rate of change of angular momentum can be related to the external moment and we have this Euler's equation and then I showed you what is a free spinning top and this whole notion of a spinning top which is also processing about an axis and it could be tilting also.

And then the similar ideas this angular momentum and so on can be used to explain the principles of a gyroscope and its application. So, a gyroscope can be used to measure the angular speed or the orientation and it can also be used to change the orientation of a rigid body which is often used in spacecraft or it can be used to stop a rigid body like a boat to roll. And I showed you a little bit about how we can measure orientation and angular velocity using more modern techniques like MEMS or even a fiber optic device or a laser.

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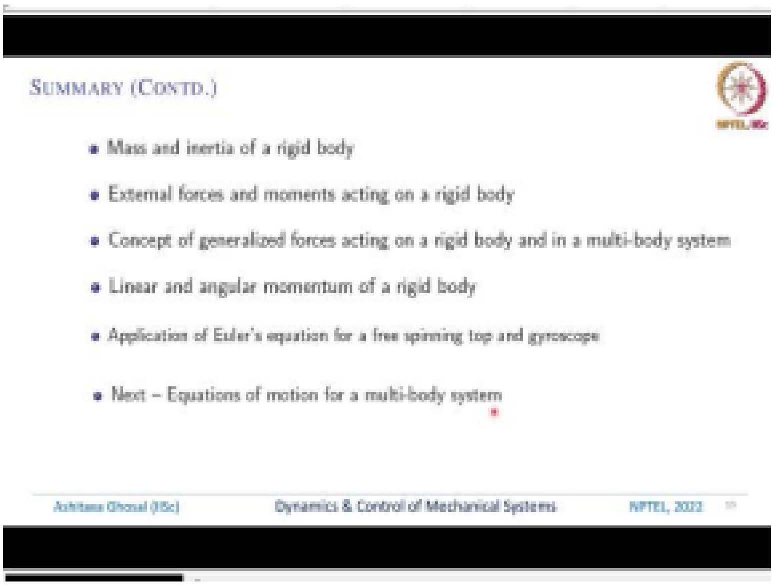

So, in summary for this whole week I have looked at what is mass and inertia of a rigid body, we have looked at external forces and moments acting on a rigid body and then we have looked at the concept of generalized forces acting on a rigid body and in a multi-body system, we have looked at linear and angular momentum of a rigid body and then we had looked at application of Euler equations which is rate of change of angular momentum is equal to the external moment.

And we had applied this for two kinds of examples one is a free spinning top and a gyroscope. In the next week we will look at equations of motion for a multi-body system, thank you.