Sound and Structural Vibration
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Module No # 09
Lecture No #42
Panel Radiation Model using Monopoles

Good morning and welcome to this next lecture on sound in structural vibration we were summarizing radiation from a rectangular panel in a baffle.

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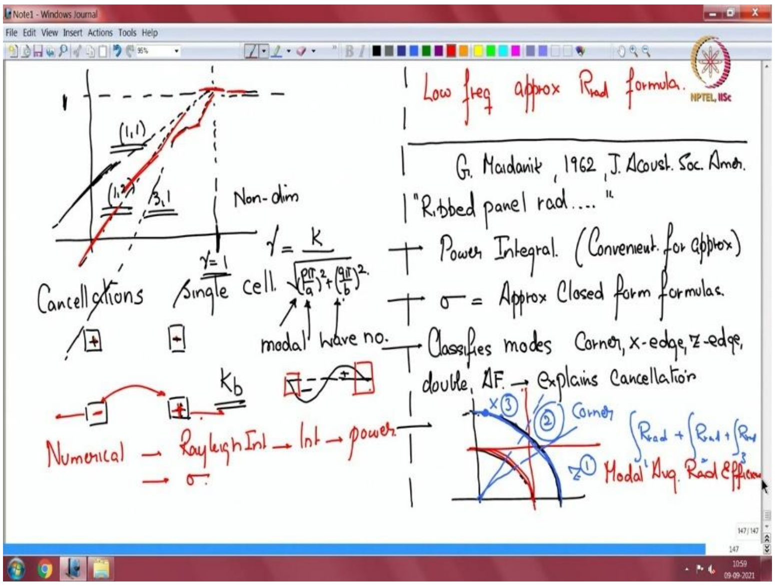

And we were talking of the work by C.E. Wallace in floating the radiation resistance of modes. So, let me add to that work is largely numerical that means compute Rayleigh integral then from there you compute intensity and then you compute power over a hemisphere and then you compute radiation efficiency or radiation resistance numerical. However, at low frequencies as I have mentioned before he has given approximate radiation resistance expressions formula which we have seen for an, add on mode earlier.

The next or rather the main key work in this area which actually should be mentioned earlier I suppose is by G Maidanik. So, there is a 1962 key paper published in Journal of Acoustical Society of America title is something like ribbed panels ribbed panel radiation efficiency or sound radiation I will give you the exact title maybe next time you type this in Google Scholar you will immediately get it.

So, this is a very key paper, so they are also a power integral is formulated acoustic power integral is formulated in a very different manner. So that it is convenient for approximations closed form approximations some special relations are used and then for radiation efficiency, approximate formulas are approximate closed form formulas are presented for various frequency ranges.

For example, whereas Wallace does a numeric over this. So, Maidanik does an asymptotic expression for this range, some more other expression for this range. Some expression for that range, something for this range, this range here and finally, close to coincidence and beyond. So, in local frequency regimes he finds approximate formulas.

And more importantly, he classifies mods as corner radiators, as you know edge radiators either X edge radiator or Z edge radiator or double edge radiator or acoustically fast, he classifies the mods in this manner and explains the cancellation. For example, many cases arise beyond what I have spoken here for odd mods you get all of them radiating. In the same sign, but if you have even mods.

You could have different sign at the corners this in this? The corners you could have a different because even mode is like this then even mode, $p = 2$ looks like this p equal to do is even it looks like this, so it is negative on one side positive on the other side. So, if you look at cancellations at the edge, you will have one sign for the right side, one sign for the left side.

So, you could have opposite signs, now, further things arise if these opposite signs sit close together, if they are close together, they nearly cancel each other. Whereas if they are a little bit apart, then cancellation is less so. So, these additional final cases arise. And he discusses all these cases and the related radiation efficiency formulas for all the cases. Now, finally, finally, what he does is that? Realistic radiation involves all the modes at a certain frequency.

It is not modal it is not you that you fix the amplitude and talk of cancellation, or You know do not worry about cancellation, but worry about resonance. It is not that so realistic radiation is all the modes are radiating together. So, if we look at the k_b quarter circle, where the modes are sitting then you will have in your let us say your acoustic circle is somewhere here then this is the bounds of acoustic circle.

And then so, on this k_b he assumes that this line is started with modes not as discrete points as we saw earlier not as discrete points but a distribution of continuous distribution of modes with equal energies, then he computes radiation efficiencies for each of these regions, because these modes will be added these modes will be X edge and these mods will be corner and so forth. So, he separately computes radiation resistance or efficiency for each of the regions. Let us say 1, 2 and 3 and comes up with what is called a modal average radiation efficiency.

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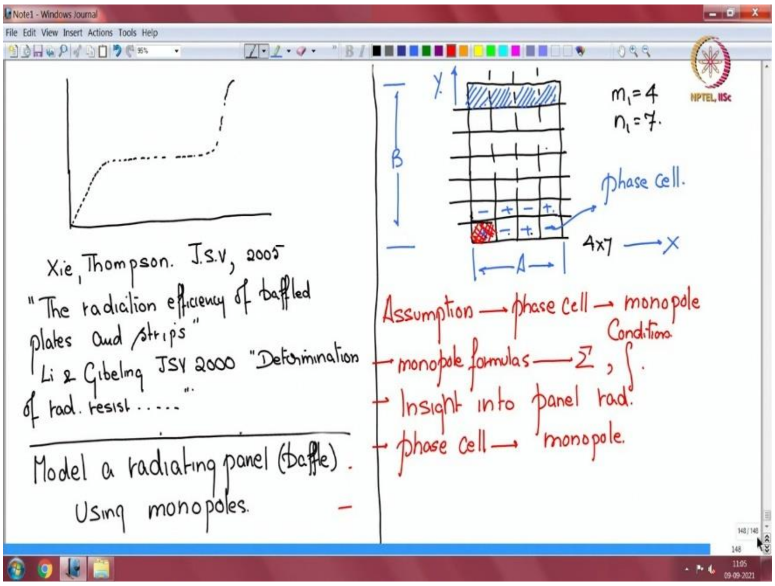

So, this gives a picture of a panel radiating with all its modes in some average sense. So, the idealized modal average radiation efficiency in some non dimensionalized coordinate looks like this. Now along these lines there have been papers written so one is by Xie Thompson in journal of sound and vibrations in the year 2005 and it is somewhat title like this the radiation efficiency of baffled plates and strips.

There is another also relevant by Li and Gibeling in JSV year 2000 which is somewhat title determination of radiation resistance of a rectangular plates something like that. These are relevant papers about which if I have time, I will make some comments later. Now I want to do one thing here which is not correct which is 3, 1 is the odd even mode and 1, 2 is a add even mode.

So, 3 1 looks a lot more like this and 1 2 looks more like this 1 2 is more inefficient than 3 1 is more cancellation. So, that is we modal now, we will modal a radiating panel his sound radiating panel of course placed in a baffle using monopoles what do we mean by that? What we mean is that there is this panel let me say it is currently vibrating with draw it first 1, 2, 3, 4, 5, 6 and 7, so 1, 2, 3, 4. So there is 4 and 1, 2, 3, 4, 5, 6, 4 by 7 mode. So, we will say m_1 is 4 and n_1 = 7. So, in this case I am take changing the coordinates a little you take this as x this is y.

And this whole panel now has dimension B in this direction and this whole panel has capital A in this direction. So now individual cells are radiating these are cells modal phase cells so we will call this a phase cell. Why because 1 is $+$ 1 is $-$ 1 is $+$ 1 is minus the whole cell is in one phase for that this will be minus plus and so forth. So, the each of these is called a phase cell.

Now, if we make the assumption, if we make the assumption that a phase cell satisfies a monopole condition. Then we can use monopole formulas then what we can use monopole formulas. And once you use monopole formula it is more of a summation than an integral you some of some all the monopoles then you might get some additional insight that is of course. The main point here additional insight into panel radiation.

That means we have to satisfy a criterion that this phase cell is a monopole, so mainly phase. So, as long as that is correct, we will have accurate results phase cell must be a monopole. So, now, how to set a criteria?

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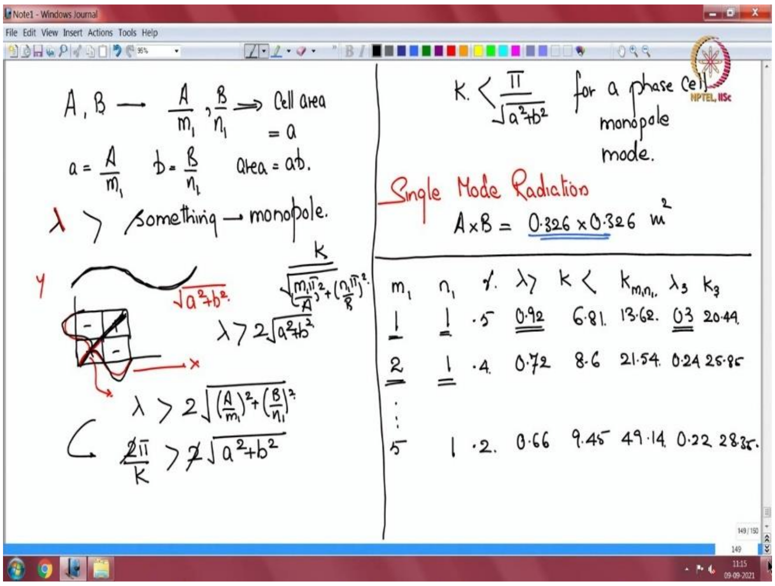

So, we have the dimension of the panel is a A and B. So, the phase cell dimensions are A over m_1 and B over n_1 which means the cell area phase cell area is given by a well, let us see $a = \frac{A}{m_1}$, $b = \frac{B}{n_1}$ and phase cell area is equal to ab. Now, of course the acoustic wavelength lambda must be greater than something some cell dimension.

Something related to the cell for it to be a monopole at least a monopole definition. So, if we look at some corner region of the panel. Let us say this is the corner and this is one cell, and this is the next cell in the column next cell this is plus this is minus plus. So, then a wavelength in this direction is this wavelength in the x direction is this and a wavelength in the y direction is this.

And so, one diagonal dimension is what? one diagonal dimension is twice I mean $\sqrt{a^2 + b^2}$ there is one diagonal dimension. So, if we want to make lambda bigger than some wavelength or some wavelength like dimension. So, it has to be greater than this in the x direction that in the y direction. So, we make lambda greater than twice the diagonal.

So, this is the diagonal we will be making bigger than twice the diagonal it is a good enough measure, you are doing an approximation. So, it is a good measure to do that means what happens now? What happens is that lambda is greater than $2\sqrt{\left(\frac{A}{m_1}\right)^2 + \left(\frac{B}{n_1}\right)^2}$ and lambda is what how is it related to wavelength lambda.

So, we have $\frac{2\pi}{k}$ greater than $2\sqrt{a^2 + b^2}$. So, that means, the acoustic wave number k has to be less than this $\frac{\pi}{\sqrt{a^2+b^2}}$ a phase cell what a phase cell to behave like a Monopole in that modal in that mode, that is important. So, again we are looking at modal radiation we are looking at a single mode radiation.

So, now, we chose a dimension A cross B a panel of this dimension two square panel meter squared and let us just see some values of this lambda and k and how they satisfy. So, the mode m_1 and n_1 mode and we have γ which is of course $\frac{k}{\sqrt{\left(\frac{m_1\pi}{A}\right)^2+\left(\frac{n_1\pi}{B}\right)^2}}$. Then we have lambda which should be greater than some value then we have acoustic wavenumber which should be less than some value.

So, we look at the 1,1 mode for this particular panel in air we will find that lambda has to be greater than 0.9 meters. If lambda is bigger than that many meters then a single cell will be a monopole, because 1,1 means the whole panel is a cell actually, right? There are no nodal lines. So, the lambda is also quite big, then k has omega over c so, it has to be low frequency. So, it has to be less than 6.81.

We are doing this for air, and the k_p value the modal they have number value is 13.62. This modal let us call it $k_{m_1 n_1}$. Let us say $m_1 n_1$ not confused with the infinite panel. Similarly, if we do m_1 is 2, n_1 is 1. So, this γ is 0.5 about then themselves the cells have become shorter. And therefore, what happens is? Lambda has to be only greater than 0.72. So, now, the wave number can be higher frequency can be a little bit higher.

So, it comes to 8.6 and this of course is 21.54 here. And similarly, if we go down, we look at m_1 is 5 and n_1 is 1. Here the lambda has to be bigger than 0.66, k has to be less than 9.45 this is 49.14, γ starts to go down. Now, we can put a further refinement on this which is this 1 cell. This is not a whole plate is 1 cell

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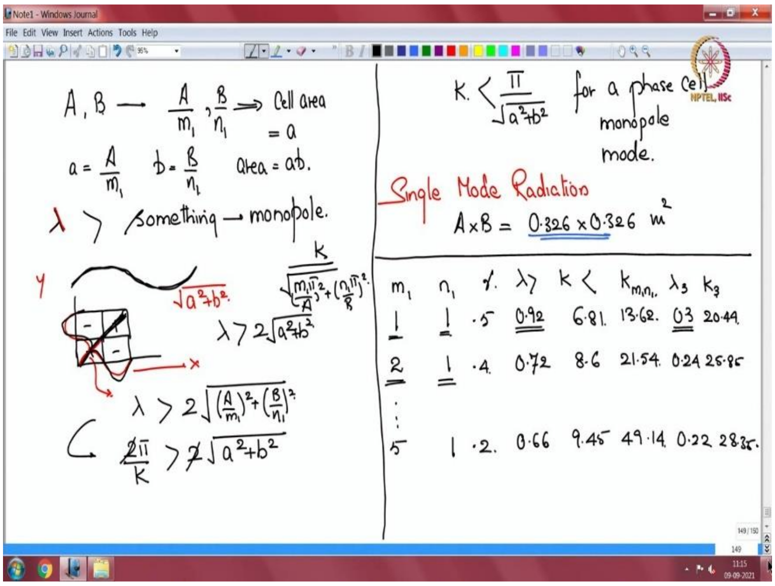

We can further divide this into pieces equally in the x and y direction. So, in this case we have broken into 3 by 3 which is 9 more cells we have broken it. So, if we do that, then the criterion is that this is sub cell is a monopole. So, we have refined, the definition is now refined so we should get better results. So, what that does is that lambda so we will call it lambda 3 has to be greater than 0.3 right because the sub cell is smaller lambda the restriction is smaller.

So, $k3$ the related acoustic wave number has to be less than 20.44 for 2, 1 mode lambda has to be greater than only 0.24 and here only 0.22 and $k3$ has to be less than 25.85 28.35 etc. So, now let us see we will do some at the mathematics. So, let me redraw here we are we will specifically look at for 7 modes just for being specific it is more demonstrated so, 1, 2, 3, 4, 5, 6 I always fall short here 7 the 4 by 7 mode 4 by 7 mode.

So, let us look at the volume velocity. So, what is it we $\tilde{V}_{pq}$ or V_w let us we have chosen somehow the symbol $V_w \sin\left(\frac{m_1 \pi x}{A}\right) \sin\left(\frac{n_1 \pi z}{B}\right)$ as that is the mode m_1 is 4 and n_1 is 7 that is the mode. So, the volume velocity now, what is the volume velocity Q is equal to $\int_0^a \int_0^b V_w \sin\left(\frac{m_1 \pi x}{A}\right) \sin\left(\frac{n_1 \pi z}{B}\right) \, dx \, dy$.

So, let me continue here this is equal to

$$Q = V_w \left(\frac{A}{m_1 \pi}\right) \left(\frac{B}{n_1 \pi}\right) \left[\cos \frac{m_1 \pi x}{A}\right]\Big|_0^{A/m_1} \left[\cos \frac{n_1 \pi y}{B}\right]\Big|_0^{B/n_1},$$

$$= V_w \frac{4AB}{m_1 n_1 \pi^2},$$

this is the volume velocity from a single-phase cell. So, now in trying to approximate using monopoles which will lead to summations and not so much as integral. And still trying to see if a cell is a monopole cell is not always a monopole some frequency it will fail.

So, the smaller and smaller cell you take better you are. So, first idea is we want a volume velocity which is uniform across the cell. This is a cell we want uniform velocity on the cell that should equal the volume velocity here. So, what does this mean? The velocity which is uniform into $\frac{A}{m_1}$ into $\frac{B}{n_1}$ should be equal to this $V_w \frac{4AB}{m_1 n_1 \pi^2}$ So, if you cancel this off there is a correction V_m value is $V_w \frac{4}{\pi^2}$.

$$V_m \frac{A}{m_1} \frac{B}{n_1} = V_w \frac{4AB}{m_1 n_1 \pi^2},$$

$$V_m = V_w \frac{4}{\pi^2}.$$

Now there is a further refinement possible which I will only mention but not go through details. Further refinement possible which is we break this phase cell originally into sub cells you break it into further sub cells 3 by 3, 4 by 4, 5 by 5 and so forth. Further sub cells that now that sub cell will be a monopole which is easier. Because as it its dimension is small so making sub cells equal sub cells so D sub cells in any direction so D cross D so if D is 3 then we have 9 sub cells this gives better results.

There is even a further refinement if you actually look at a mode so it will be like this in one direction where is it like this in the other direction you know etc it has this shape across the plate. So, if a cell happens to be sit somewhere sitting somewhere here a cell sitting here. Then and again, it is divided into sub cells. If I explode that small region that is a phase cell to begin with but within that there are sub cells now.

Then each sub cell is seeing a different amplitude is sub cell is seeing a different amplitude. So, if I accommodate the amplitude change not make it uniform. Give this cell amplitude appropriate for its volume velocity give this sub cell an amplitude appropriate for its velocity and so forth. Then I will much better refinement so one is breaking into sub cells the other is amplitude attributing appropriate amplitudes I will get a much better refinement.

And I will get a so we are we trying to do we are trying to find radiation efficiency right. So, we will get curve as good as Wallace till the coincidence frequency. So now time is up for this lecture so I will close here and continue from the next class.