Sound and Structural Vibration
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Lecture – 11
Derivation of Coupled Roots Using Asymptotic Method

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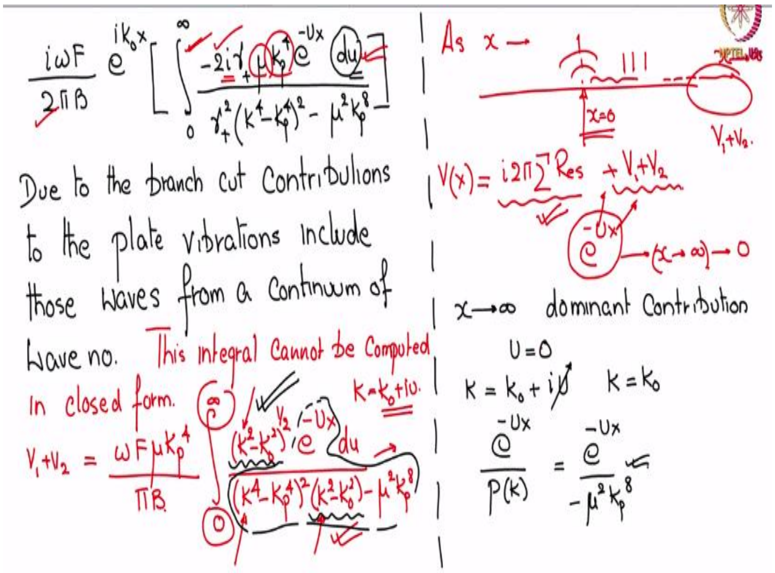

Good morning. Welcome to this next lecture. We are doing the velocity field calculation using our branch cut contour. So, we reached this position where we are seeing that one part of the contribution to the velocity field of the plate is due to a continuous range an integral over that wave number from 0 to ∞. So, now this integral the way it is this integral cannot be computed in closed form.

It has to be done numerically, but here we are going to find some approximate value to it. So, the contribution $v_1 + v_2$ is given by,

$$v_1 + v_2 = \frac{\omega F \mu {k_p}^4}{\pi B} e^{ik_0 x} \int_0^\infty \frac{\left(k^2 - {k_0}^2\right)^{1/2} e^{-Ux} dU}{\left(k^4 - {k_p}^4\right)\left(k^2 - {k_0}^2\right) - \mu^2 {k_p}^8}.$$

We should not forget that $k = k_0 + iU$. So k comes here, k comes here, k comes here, so I am not changing that, it will become horrendous. Now we will find as x extends to ∞, x approaches ∞ that means I have this 1-D plate excited by a line force. So, I have some vibrations happening, then sound field being generated and so forth and the sound field is interacting. So, as I move far away from my source as x tends to ∞, I want to see the velocity field.

So, let us not forget actually, let us not forget that my total velocity field, just want to remind here is first of all the sum of residuals which we are yet to calculate, plus $v_1 + v_2$. So, we are looking at $v_1 + v_2$ only now. This residue part has not come in yet, we will do that. We will do that the residue part a little later. I said that is a separate study. So we are looking at $v_1 + v_2$ and $v_1 + v_2$ is this integral and I say that this is not integrable in closed form.

So we see what happens as you move further away from the forcing. Now we have e^{-Ux} in the numerator, so as x tends to ∞ this goes to 0. And so as x tends to ∞, the dominant contribution comes from $U = 0$. So, I had said my $k = k_0 + iU$ and so if U happens to be equal to 0 my k should be equal to k_0. So, in here k will be equal to k_0 or close to k_0 and k will be equal to k_0 here.

So, in the approximate sense I will have

$$\frac{e^{-Ux}}{p(k)} = \frac{e^{-Ux}}{-\mu^2 {k_p}^8}.$$

So, this part and the denominator we write it as this. So, there is still this portion. So, what do we do?

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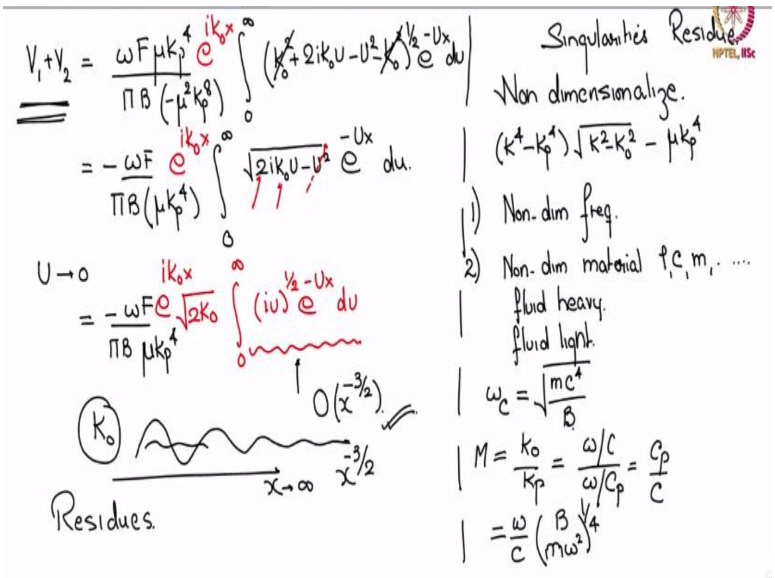

Now, we have the

$$v_1 + v_2 = \frac{\omega F \mu {k_p}^4}{\pi B\left(-\mu^2 {k_p}^8\right)} e^{ik_0 x} \int_0^{\infty} \left({k_0}^2 + 2ik_0 U - U^2 - {k_0}^2\right)^{1/2} e^{-Ux} dU,$$

$$= \frac{-\omega F}{\pi B\left(\mu {k_p}^4\right)} e^{ik_0 x} \int_0^\infty \sqrt{2ik_0 U - U^2}\, e^{-Ux} dU,$$

$U \to 0$

$$\frac{-\omega F}{\pi B\left(\mu {k_p}^4\right)} e^{ik_0 x} \sqrt{2k_0} \int_0^\infty (iU)^{1/2}\, e^{-Ux} dU.$$

We look at this part, this part will be an integral which decays as x to the power $\frac{-3}{2}$ after you integrate. So, there is a wave which has a dominant wave number as k_0 okay on the plate, but it decays away and decays away as you move towards ∞ at the rate $x^{\frac{-3}{2}}$. So, we had to weigh this contribution along with the other contributions from residue, so which one is big, which one is small so forth.

But the integrals from the vertical cut on the left and right side at k_0 gives me this part, this behaviour. So, that is one major part of it done, we still have to deal with residues that will deal. Now, let us see we have to move towards the thought about singularities so that we can get the residue contributions which is very important. So, now to move towards that first of all we need to non dimensionalize.

So, denominator at some point we had $\left(k^4 - {k_p}^4\right)\sqrt{k^2 - {k_0}^2} - \mu {k_p}^4$, this was a denominator. So, cutting short the entire discussion we need a one parameter which is non-dimensional frequency we need. The second is a non-dimensional material parameter which can take material properties like fluid density, speed of sound, plate density, and any other parameters.

So that we can weigh that when fluid is heavy how it behaves, if fluid is light, how the situation looks like and from here at high frequencies or low frequencies or around some typical frequencies. So, these two parameters we need. So, now to refresh you the coincidence frequency in this plate problems is given by $\omega_c = \sqrt{\frac{mc^4}{B}}$, c is the speed of sound in the material, B is the flexural rigidity.

Now we will take a value or a parameter called M which is $\frac{k_0}{k_p}$, the acoustic wave number by the plate wave number and if you look at this, this is $\frac{\omega/c}{\omega/c_p}$, so that becomes equal to $\frac{c_p}{c}$. So, if we now elaborate here k_0 is $\frac{\omega}{c}$ and k_p is $\left(\frac{m\,\omega^2}{B}\right)^{1/4}$. So, let me take out onto the next page.

$$M = \frac{k_0}{k_p} = \frac{\omega/c}{\omega/c_p} = \frac{c_p}{c},$$

$$= \frac{\omega}{c}\left(\frac{B}{m\,\omega^2}\right)^{1/4}.$$

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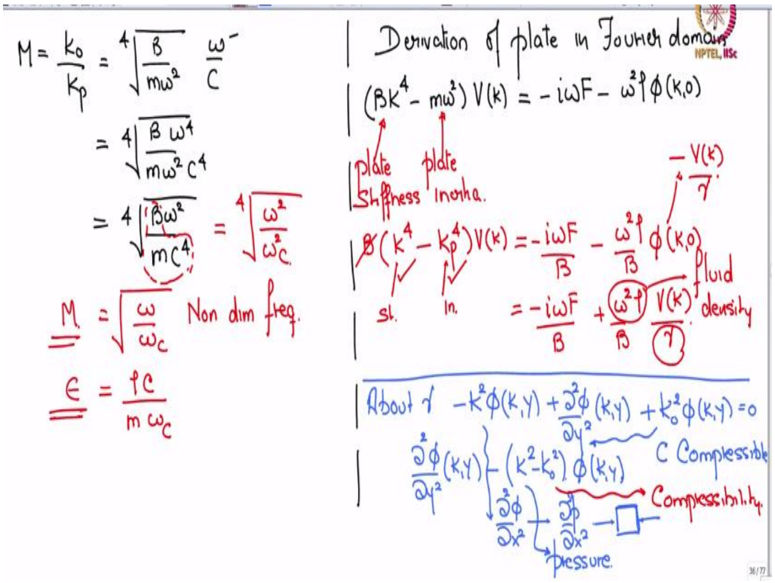

So,

$$M = \frac{k_0}{k_p} = \sqrt[4]{\frac{B}{m\,\omega^2}}\frac{\omega}{c},$$

$$= \sqrt[4]{\frac{B\,\omega^4}{m\,\omega^2\,c^4}},$$

$$= \sqrt[4]{\frac{B\,\omega^2}{m\,c^4}} = \sqrt[4]{\frac{\omega^2}{{\omega_c}^2}}.$$

$$M = \sqrt{\frac{\omega}{\omega_c}}\ .$$

This is the non-dimensional frequency parameter. The other material non-dimensional parameter ϵ let me write it as $\frac{\rho c}{m\omega_c}$, this is the other parameter. So, these are the two we need.

There is a discussion behind it and so forth but these are the two we need. Now, if you recall somewhere in the derivation of the plate in the Fourier domain in Fourier or wave domain we came up across this equation which $(Bk^4 - m\omega^2)V(k) = -i\omega F - \omega^2\rho\phi(k,0)$. Now you should know that this term (Bk^4) comes from plate stiffness like B is flexural stiffness, plate stiffness.

And this term $(m\omega^2)$ comes from plate inertia and if we take B out, suppose we take B out and divide, so I take B out I get

$$\left(k^4 - {k_p}^4\right)V(k) = \frac{-i\omega F}{B} - \frac{\omega^2\rho\phi(k,0)}{B},$$

$$= \frac{-i\omega F}{B} + \frac{\omega^2\rho}{B}\frac{V(k)}{\gamma}.$$

Now let us see about γ . Now, how did we formulate the 2-D verification for the potential, we get

$$-k^2\phi(k,y) + \frac{\partial^2\phi(k,y)}{\partial y^2} + k_0^2\phi(k,y) = 0.$$

Then we have

$$\frac{\partial^2\phi(k,y)}{\partial y^2} - \left(k^2 - {k_0}^2\right)\phi(k,y) = 0.$$

So, this carries $\left(k_0^2\phi(k,y)\right)$ information about the speed of sound and hence compressibility, compressibility of the fluid.

And this term $\left(-k^2\phi(k,y)\right)$ comes from $\frac{\partial^2\phi}{\partial x^2}$ it is similar to $\frac{\partial^2 p}{\partial x^2}$ so that the net pressure across an element. So, this term (k^2) we will consider to be representing pressure and this term $\left({k_0}^2\right)$ to be representing compressibility. So, we have structural stiffness (k^4), structural inertia $\left({k_p}^4\right)$ and a γ over here and ρ over here so which is fluid density. So, what do we get as in the net?

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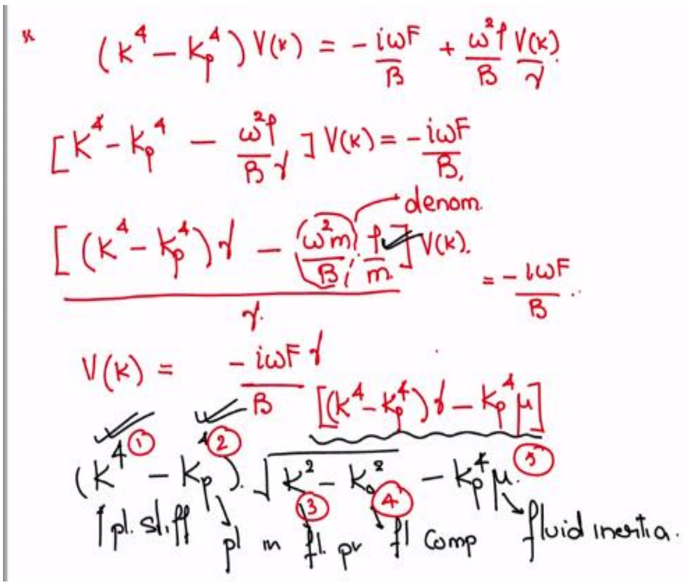

So we will get let us see.

$$\left(k^4 - {k_p}^4\right)V(k) = \frac{-i\omega F}{B} + \frac{\omega^2\rho}{B}\frac{V(k)}{\gamma},$$

$$\left[k^4 - {k_p}^4 - \frac{\omega^2\rho}{B\,\gamma}\right]V(k) = \frac{-i\omega F}{B},$$

$$\frac{\left[\left(k^4 - {k_p}^4\right)\gamma - \frac{\omega^2 m}{B}\frac{\rho}{m}\right]V(k)}{\gamma} = \frac{-i\omega F}{B},$$

$$V(k) = \frac{-i\omega F\gamma}{B\left[\left(k^4 - {k_p}^4\right)\gamma - {k_p}^4\mu\right]}.$$

So, $\left[\left(k^4 - {k_p}^4\right)\gamma - {k_p}^4\mu\right]$ is my denominator now. So the denominator carries the roots of this system. We are going to do control integration, so the roots of the system or poles of the system are important. So, now we have already seen k^4 comes from plate stiffness, ${k_p}^4$ comes from plate inertia.

Then here $\sqrt{k^2 - {k_0}^2}$, k^2 this comes from fluid pressure minus ${k_0}^2$ square this comes from fluid compressibility and $-{k_p}^4\mu$, μ carrying ρ this is fluid inertia. When the plate was in vacuum, the behavior was play between plates stiffness and plate inertia, but now that this is a coupled problem it is a play between these five factors 1(k^4), 2 $\left({k_p}^4\right)$, 3 (k^2), 4 $\left({k_0}^2\right)$ and 5 $\left({k_p}^4\mu\right)$.

These five factors coming in various frequency regimes and various parameter regimes they become more or less important. So, I will close it here, We will continue this from the next class. Thanks.