Robotics: Basics and Selected Advanced Concepts
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Lecture - 40
Kinematics and Dynamics of WMR on Uneven Terrain

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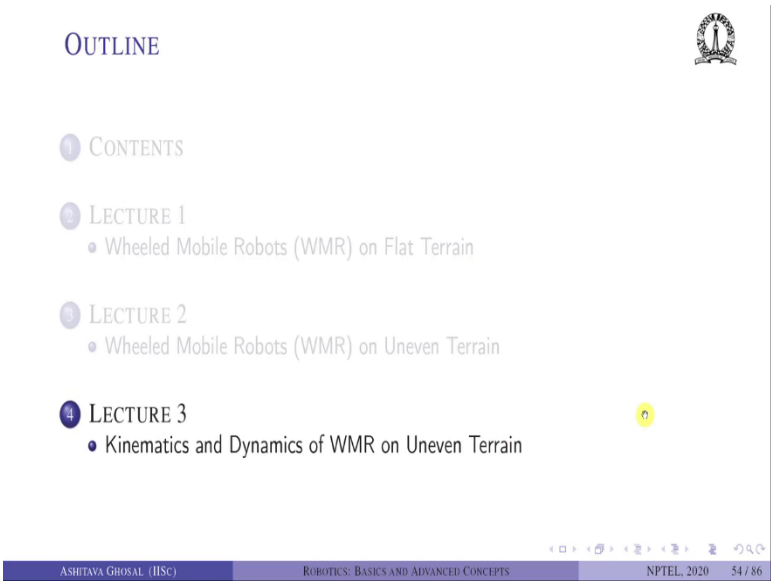

Welcome to this NPTEL lectures on Robotics – Basic and Advance Concepts. In these lectures, we are looking at wheeled mobile robots. In the first lecture, I had looked at a wheeled mobile robot moving on a flat terrain; in the second lecture, we had looked at how to model the wheeled mobile robot moving on an uneven terrain. In this lecture, we look at the Kinematics and Dynamics of a Wheeled Mobile Robot on Uneven Terrain ok.

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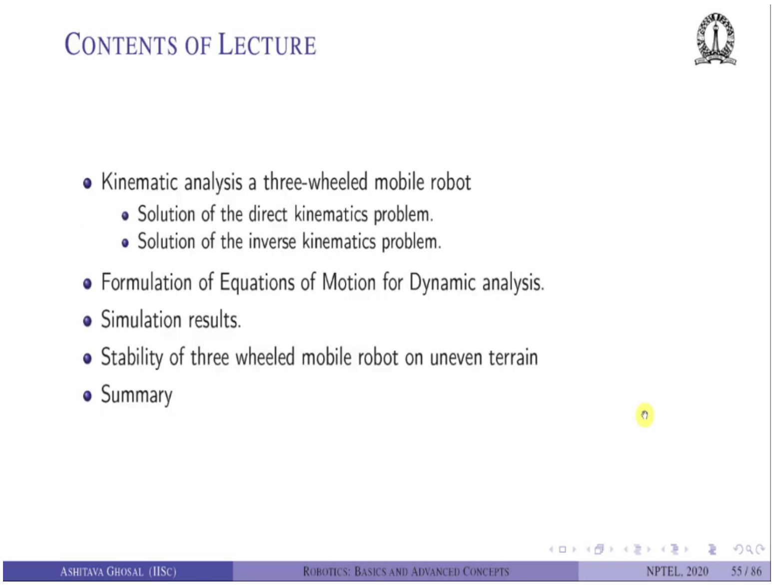

So, the contents of this lecture are the following. We will look at the kinematic analysis of a three-wheeled mobile robot. We will look at the solution of the direct kinematics problem; we will look at the solution of the inverse kinematics problem. Then we will look at the formulation of equations of motion for dynamic analysis. Then I will present some simulation results, and then I will look at the stability of a three-wheeled mobile robot on uneven terrain ok.

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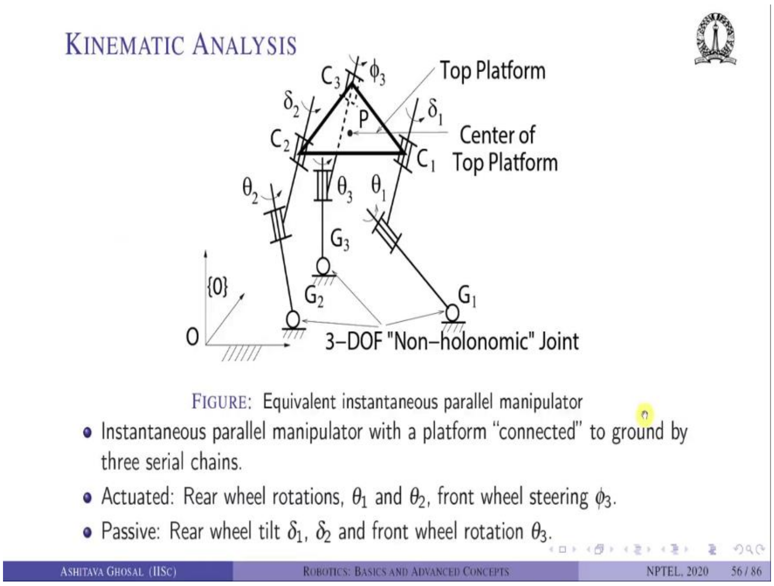

So, as I had mentioned in the last lecture, we will model a wheeled mobile robot as an instantaneous parallel manipulator. So, we have a top platform and although we do not have a fixed platform, but we have these three ground contact points. And we will represent these ground contact points as 3 degree of freedom non-holonomic joints ok.

So, in a sense, G_1, G_2, G_3 determines the 'fixed' base. As the wheeled mobile robot move, this G_1, G_2, G_3 will change with respect to the reference coordinate system which is $\{0\}$. And then there are these two rotary joints in each leg.

So, one rotary joint allows lateral tilting, one rotary joint represents a rotation of the wheel, in the front wheel there is no lateral tilting, but there is steering. So, ϕ_3 is the steering. δ_1 and δ_2 are the lateral tilt; and θ_1, θ_2, θ_3 are three rotations of the wheel. So, as I said we are going to look at this wheeled mobile robot as an instantaneous parallel manipulator with the platform connected to the ground by three serial chains.

There are three actuated joints which are θ_1, θ_2, and ϕ_3. So, θ_1 and θ_2 are the two rear wheeled rotations. So, we can think of some motors, may be a half motor which is rotating the wheels. And wheels steering is in the front wheel which is ϕ_3. And then there are these three passive joints. One of them is the wheel rotation of the front wheel, and then there are these two rear wheel tilts denoted by δ_1 and δ_2.

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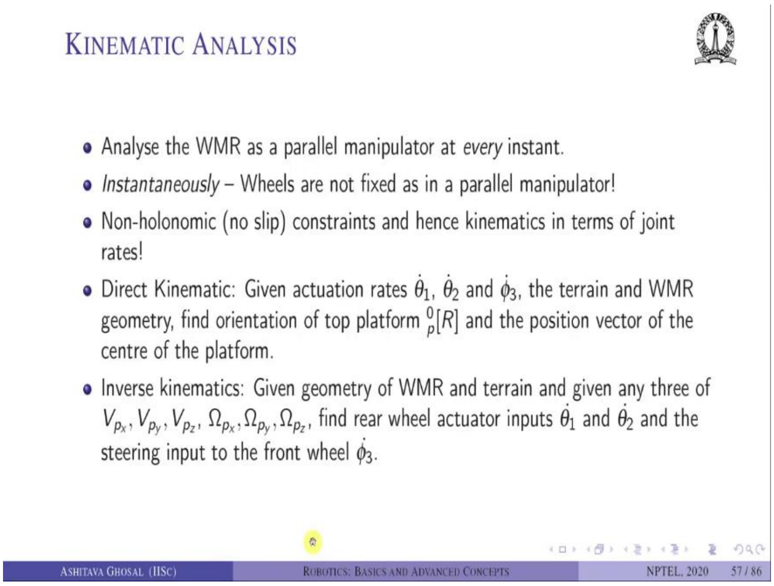

So, we are going to analyze the WMR as a parallel manipulator at every instant. Why? Because the ground contact point G_1, G_2, G_3 are changing. So, instantaneously we are going to think of it as a parallel robot. So, the wheels are not fixed as in a parallel robot in a typical say 3 RPS robot or some others steward co platform. The base points are fixed with respect to the reference coordinate system. In this case, the base points are moving, but at each instant it looks like a parallel robot.

We have non-holonomic no slip constraints, and hence the kinematics is in terms of joint rates. Remember we have said $v_x = v_y = v_z = 0$, and $v_z = 0$ because the wheels cannot lift off from the ground from the contact points. So, the kinematics are basically in terms of the rate of change of these generalized coordinates. In the direct kinematics problem, we are given the actuation rates which are $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\phi}_3$.

$\dot{\theta}_1$ and $\dot{\theta}_2$ are the in the two rear wheels; $\dot{\phi}_3$ is the steering. We are also given the terrain and the WMR geometry. We want to find the orientation of the top platform and the position vector of the center of the platform. So, we want to find the orientation of this top platform and the position of this top platform with respect to a fixed coordinate system. We are given $\dot{\theta}_1$, $\dot{\theta}_2$, and $\dot{\phi}_3$.

In the inverse kinematics, we are given the geometry of the wheeled mobile robot the terrain. And given any three of V_{p_x}, V_{p_y}, V_{p_z}, and ω_x, ω_y, ω_z, we know that it has 3 degrees of freedom, remember this is a 3 degree of freedom model. So, we can only give 3 of these 6 quantities; 3 of this 6 linear velocity and angular velocity combined.

And the goal is to find the rear wheel actuator inputs $\dot{\theta}_1$, $\dot{\theta}_2$, and the steering input to the front wheel. So, this is how the direct and inverse kinematics problems are defined. And as I mentioned since the constraints are in terms of non-holonomic constraints, we need to derive this direct and inverse kinematics in terms of velocities.

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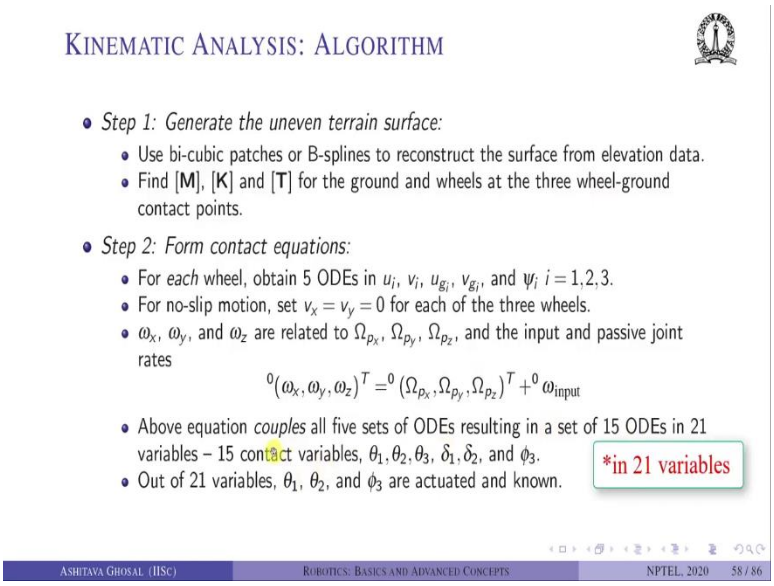

So, what is the algorithm? The first step is of course, to generate the uneven terrain surface. You have to tell me what is the uneven terrain. So, we can use either bi-cubic patches or B-splines to reconstruct the surface from the elevation data. So, somebody has given you some points what is the location of the point in X and Y and the height, we can use this elevation data to obtain the equation of a surface.

Once we find the equations of the surface, we can find the metric, the curvature form, and the torsion for the ground and the wheels at the three-wheeled ground contact point. So, at every point, wherever there is contact, we find this $[M]$, $[K]$ and $[T]$ because that is required in the contact equations.

Then we form the contact equations. So, for each wheel obtain 5 ordinary differential equations in $\dot{u}_i, \dot{v}_i, \dot{u}_{gi}, \dot{v}_{gi}$ and $\dot{\psi}$. Remember we derived this contact equations in the derivatives of $\dot{u}$, $\dot{v}$ and $\dot{\psi}$. For no slip, we set $v_x = v_y = 0$, and we compute $\omega_x, \omega_y, \omega_z$ as they are related to the angular velocity of the platform. So, which is Ω_{p_x}, Ω_{p_y}, Ω_{p_z} and the input and passive joint rates.

So, the angular velocity $(\omega_x, \omega_y, \omega_z)^T$ which appear in the contact equations can be related to the angular velocity of the platform plus the angular velocity of the inputs. So, again the inputs are $\dot{\theta}_1$, $\dot{\theta}_2$ and the $\dot{\phi}_3$, steering. Above equation couples all five sets of ODEs. So, for each wheel, there are 5 ODEs. This equation couples all the 5 sets of ODEs, and we are resulting in 15 ODEs in 25 variables.

So, we have 15 contact variables remember u_i, v_i, u_{gi}, v_{gi}, and ψ_i times 3, 15 contact variables. And then we have this θ_1, θ_2, θ_3, δ_1, δ_2 and ϕ_3. Out of this 21 variables, θ_1, θ_2, and ϕ_3 are actuated and known ok. So, we have 15 contact variables plus 6 from the WMR itself; but out of this 6, 3 are known. So, we need three constraint equations. We have 21 variables, 3 are known, but we have only 15 ODEs ok.

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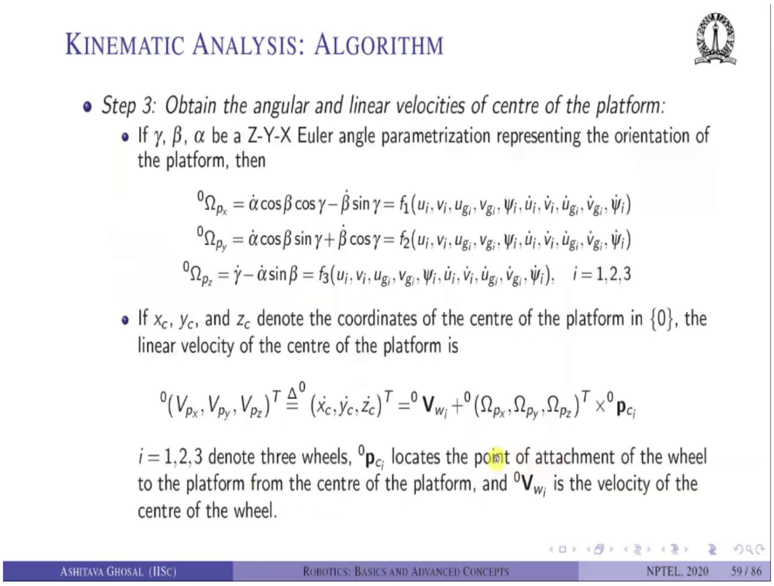

So, we obtained the angular and linear velocity of the center of the platform. So, let us assume some γ, β, α is some Z-Y-X Euler angle representation of the orientation of the top platform. Then the $\left(\Omega_{p_x}, \Omega_{p_y}, \Omega_{p_z}\right)^T$ which is nothing but the platform angular velocity along X, Y and Z will be related to $\dot{\alpha}\cos\beta\cos\gamma - \dot{\beta}\sin\gamma$ and so on.

We have done this earlier. We have seen how we can take simple rotations about Z, Y and X, multiply them, multiply these three rotation matrix get a rotation matrix. And then do $[\dot{R}][R]^T$ to obtain the angular velocity of the top platform ok. We are doing exactly the same thing. And it turns out that we will get $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$ in some particular way.

And clearly these are related to all the 15 joint variables u_i, v_i, u_{gi}, v_{gi}, ψ_i for each of the wheels, and their derivatives ok. If $(x_c, y_c, z_c)^T$, denote the coordinates of the center of the top platform in reference coordinate system $\{0\}$, then the linear velocity of the center of the platform can is given by derivative of x_c, y_c, and z_c.

And this can be obtained as linear velocity of the wheel coordinate system. Just remember there is a wheel coordinate system for each wheel at the center of the wheel and times Ω_p cross the distance between the CG of the wheel ok. So, ${}^0\boldsymbol{p}_{c_i}$ locates the attachment of the wheel to the platform from the center of the platform. And ${}^0\boldsymbol{V}_{w_i}$ is the velocity of the center of the wheeled.

So c_i is not the CG, it is the location of the point of attachment of the wheel to the platform. Later on we will also need the velocity of the CG, and location of the CG. We will come to that.

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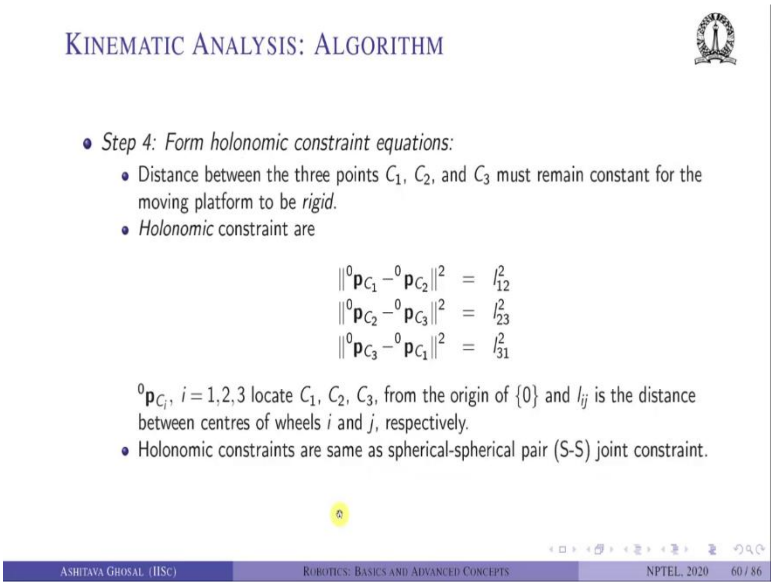

We form this holonomic constraint equations. What do we mean by holonomic? They are not related to the derivatives of the generalized coordinates. So, basically what are these three holonomic constraint equation that the distance between C_1, C_2, C_3 must remain constant because the top platform is rigid. So, $\| {}^0\boldsymbol{p}_{c_1} - {}^0\boldsymbol{p}_{c_2}\|^2 = l_{12}^2$ and so on.

So, ${}^0\boldsymbol{p}_{c_i}$, for $i = 1,2,3$ locates C_1, C_2, C_3 from the origin of the reference coordinate system. And l_{ij} is the distance between the centres of the wheel i and j. So, these holonomic constraints are very similar to S-S joint constraints. So, let us look at these constrain once more. So, I have C_1 to C_2, the distance will be constant; C_1 to C_3, the distance would be constant; C_3 to C_2, the distance will be constant because otherwise the platform is not rigid ok.

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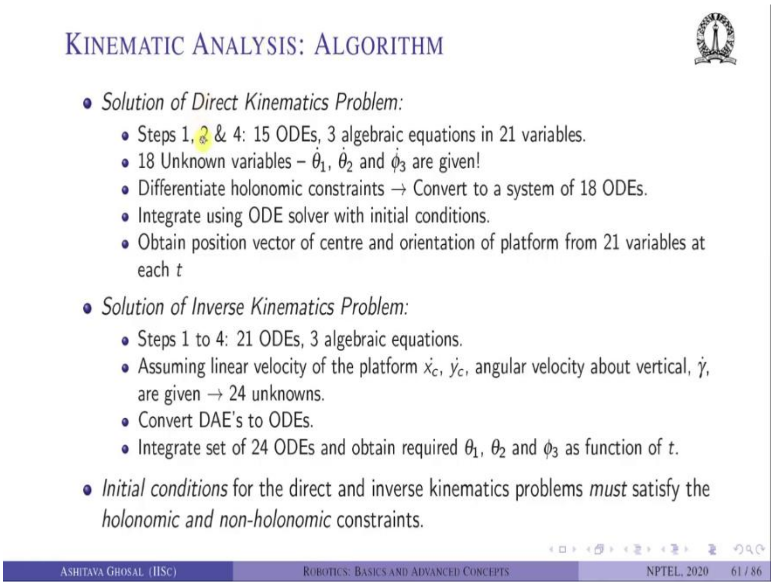

So, now we can write the algorithm how to solve the direct kinematics problem? So, first is from steps 1, 2 and 4, we have 15 ODEs plus 3 algebraic equations, 3 holonomic constraints in 21 variables. There are 18 unknown variables, why? because $\dot{\theta}_1$, $\dot{\theta}_2$ and $\dot{\phi}_3$ they are the rotations of the two rear wheels and the steering of the front wheels are given.

So, we differentiate the holonomic constraints convert to a system of 18 ODEs in 21 variables, out of which 3 variables are known. And we solve these equations using an ODE solver. We integrate with a ODE solver with initial conditions. And then we obtain all these 15 variables u_i, v_i, w_i and so on.

All these locations of the each wheel, and every two parameter is known. And obtain the position vector the center and orientation of the platform from the 21 variables at each instant of time. Remember at each instant of time, the ground contact point is changing. How about the inverse kinematics problem?

We do steps 1, 2, 4. Again we have 21 ODEs plus 3 algebraic constraints. We assume that the linear velocity of the platform $\dot{x}_c$, $\dot{y}_c$, and the angular velocity of about the vertical $\dot{\gamma}$ are given. So, we have 24 unknowns. Again convert the DAEs into ODEs. Now, we have a set of 24 ODEs required with $\dot{\theta}_1$, $\dot{\theta}_2$, $\dot{\phi}_3$ as a function of time ok. So, we solve this 24 ODEs, and obtain θ_1, θ_2, and ϕ_3 as a function of time.

In the direct kinematics, we have 18 ODEs; in the inverse kinematics, we have 24 ODEs. Why, because there is a differential equation for the 6 degrees of freedom of the top platform which is appearing. Again, the initial conditions for the direct and inverse kinematics problem must satisfies the holonomic and non-holonomic constraints. We cannot arbitrarily choose all these generalized coordinates initial conditions. They must satisfy all the constraints.

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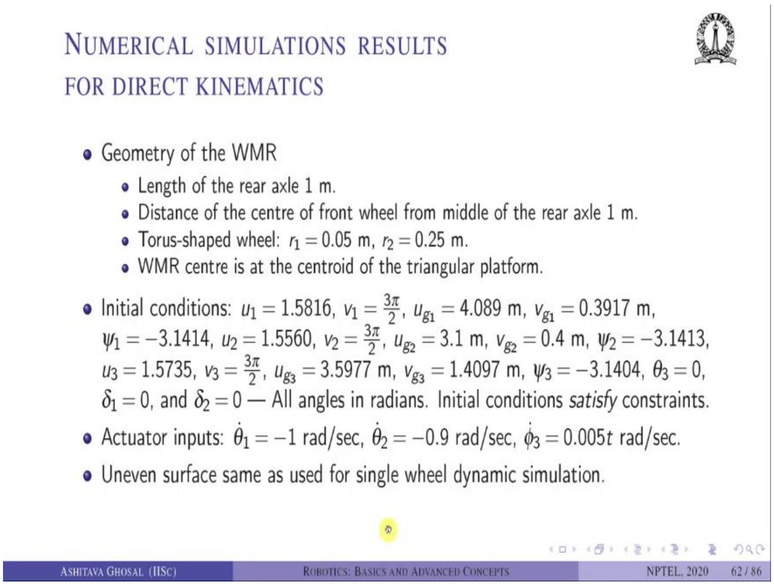

So, let us look at some numerical simulation results for direct kinematics. We have geometry of the WMR. Length of the rear axle is 1 meter; distance of the center of the front wheel from the middle of the rear axle is 1 meter, so two rear axle 1 meter. Centre point of the axle to the front wheel is also 1 meter. Torus-shaped wheeled are similar to what we have done earlier, r_1 the small radius is 0.05 meters, $r_2 = 0.25$ meters.

And WMR center is at the centroid of the triangular top platform. We choose a set of initial conditions. So, $u_1 = 1.5816$, $v_1 = 3\pi/2$ and so on. So, this basically these tell you where the point of contact is initially between the three wheels and the ground, and what is the location of the X-axis between each wheel ground contact point, the angle ψ, and also all the tilt angles ok.

These sets of values of initial condition, so u_1, v_1, u_{g1}, v_{g1}, ψ_1, u_2, v_2 and so on, they are checked to satisfy the constraints ok. These initial values must satisfy the initial constraints which are holonomic and non-holonomic constraints. We have actuated inputs which was $\dot{\theta}_1 = -1$ radian per second; $\dot{\theta}_2 = -0.9$ radian per second; and $\dot{\phi}_3 = 0.005t$ radians per second.

These are chosen arbitrarily. We could have chosen some other rotation rates of the rear wheels and the steering rotation rate ok. So, this uneven surface is same as the one which is used for single wheel dynamic simulation.

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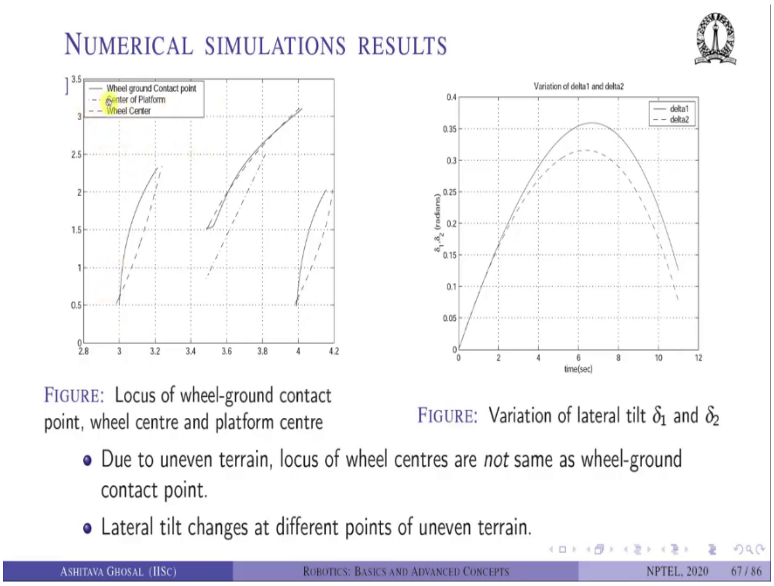

So, we have these simulation results. So, wheel 1 the dark line shows the wheel ground contact point of the wheel 1 and 2 and 3. The you know bigger dash lines shows the wheel center, and the dot dash lines shows the center of the platform ok. So, since these wheels are moving on uneven terrain, all these curves will look different because the wheels are tilting.

So, as you can see the center of the platform is this line, the wheel 3 looks like this; and the wheel ground contact point looks like this ok. Wheel 2 and wheel 1 the wheel ground contact point and the motion of the wheel centers are different. We can also find δ_1 and δ_2 which are the lateral tilt of the two rear wheels.

So, as you can see that δ_1 and δ_2 are adjusting in a way or they are behaving in a way such that $v_x = v_y = 0$, there is no slip at the wheel ground contact points. So, the locus of wheel centers are not the same as wheel ground contact point. This is uneven terrain and there is lateral tilt. The lateral tilt changes at different points of the uneven terrain, which is expected.

Due to the uneven terrain the distance between the two contact points in the rear wheel will be different at different instance of time. And hence the lateral tilt will be different.

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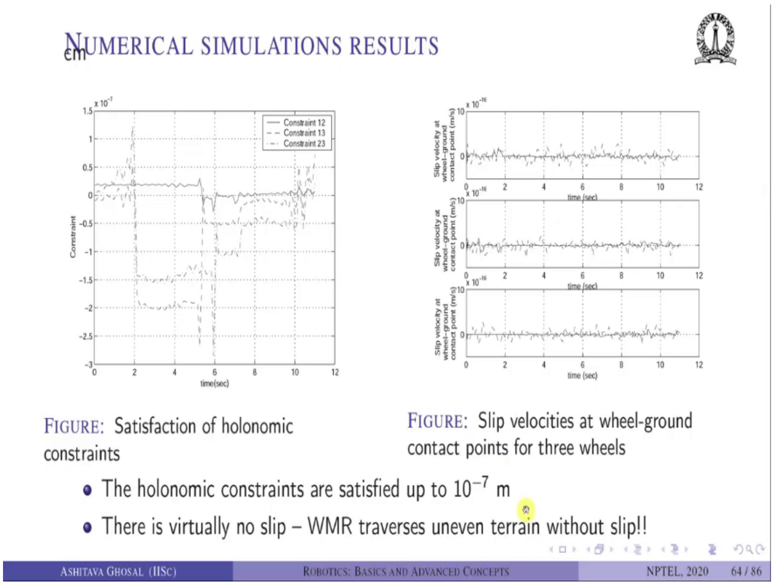

We can also plot and see whether the constraints are satisfied during the simulation. These are the three holonomic constraints. So, basically the distance between $C_1 - C_2$, $C_2 - C_3$, and $C_3 - C_1$ are constant. We can also plot the slip velocities which is v_x and v_y. So, as you can see all of them are more or less satisfied. So, we have very small constraints violation.

We also have very small slip velocities ok. The holonomic constrains are satisfied up to 10^{-7} meters, there is virtually no slip. The WMR traverses uneven terrain without slip.

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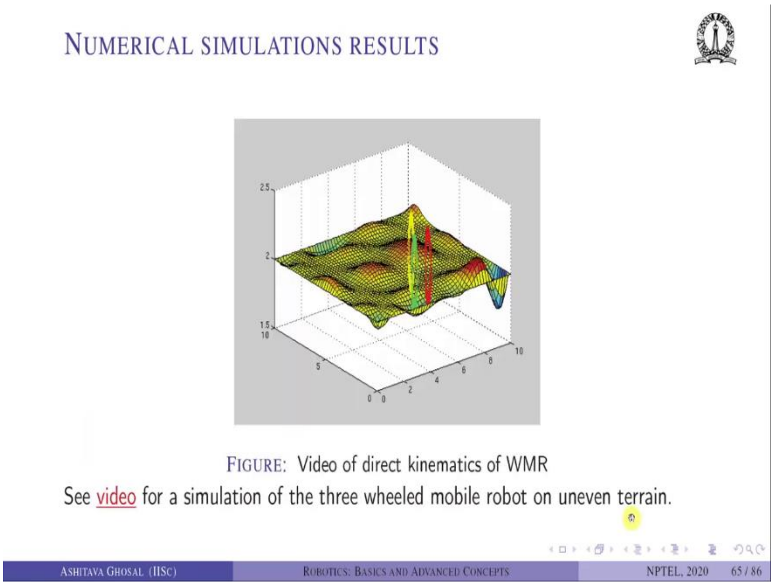

This is a video showing the direct kinematics of the three-wheeled robot. And I will show that at the end, I will show it with some other videos ok. Basically, this is a surface and this yellow, green, and red are the three wheels. And you will see in the video that it can go over this uneven terrain.

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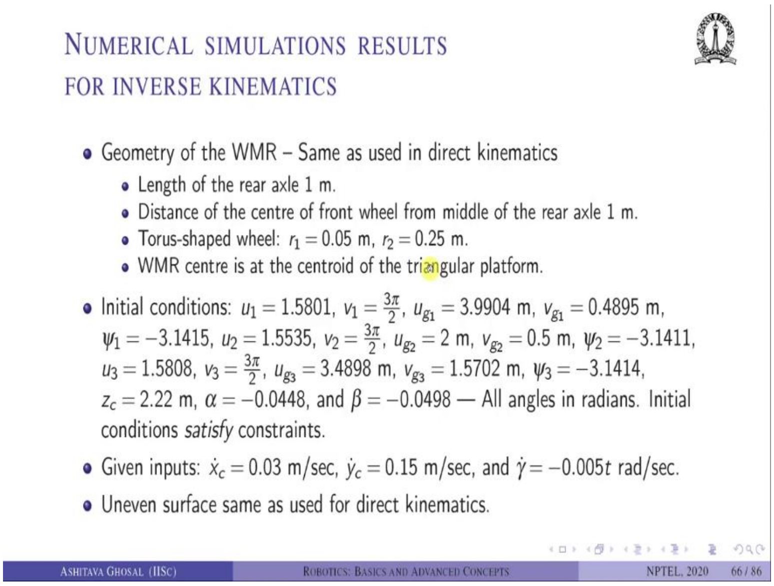

We can also do the inverse kinematic simulations. Again, the geometry of the WMR is same as used in the direct kinematics. Basically, the length of the rear axle is 1 meter; the distance of this on the center of the front wheel to the middle of the rear axle is 1 meter;

torus-shaped wheels $r_1 = 0.05$, $r_2 = 0.25$. WMR center is at the centroid of the triangular platform. Again, we choose these initial conditions to satisfy the constraints.

So, I am not going to go into each and every term, but we have obtained u_1, v_1, u_2, v_2, u_{g1}, v_{g1}, and so on everything such that they satisfy the constraints. The inputs are $\dot{x}_c = 0.03$ meters per second, $\dot{y}_c = 0.15$ meters per second, and $\dot{\gamma} = -0.005t$ radians per second.

So, I want a velocity along the X-direction, X-component velocity as 0.03 meters per second, the Y-component of the center of the top platform should have a 0.15 meters per second, and it should rotate by some $0.005t$. The uneven surface is same as used in direct kinematics.

Again, we can plot the wheel ground contact point which is the dark line, the center of the platform which is the dot dash line which is seen here, and the wheel center. So, for the three wheels, so this is one wheel, this is another wheel, this is the front wheel. And this is the platform center. You can plot the trajectory of all these points ok.

Likewise, you can find the plot of δ_1 and δ_2 which are the lateral tilts. So, again due to uneven terrain, locus of wheel centers are not same as wheel ground contact point. If the wheel was rolling on a flat terrain straight or taking a turn, this wheel center and the contact point should be on top of each other. And if you are looking from the top, they should be along the same trajectory.

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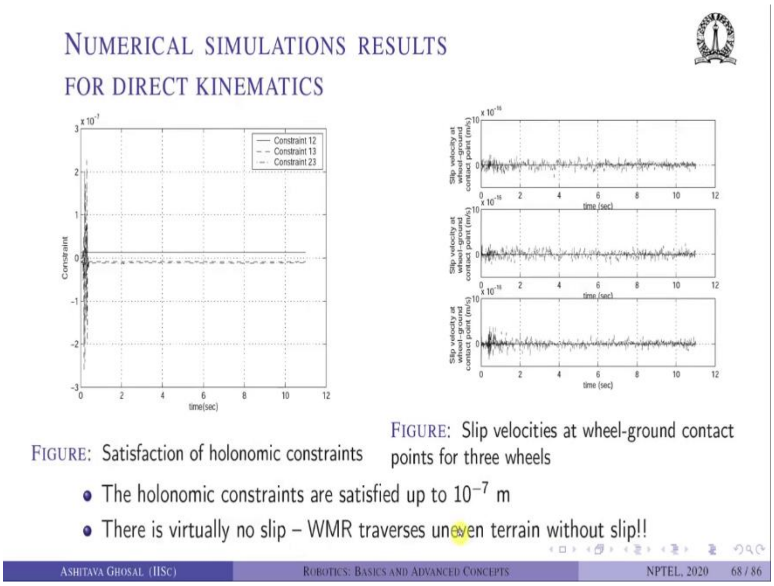

Then again, the lateral tilt changes at different points of the uneven terrain as expected. We can also plot the constraints as a function of time. We can plot this constraints between C_1 and C_2, C_2 and C_3, and C_1 and C_3 the distance. And we can see that it is very small it is of the order of 10^{-7}.

We can also plot the slip velocities. The slip velocity at the three wheel ground contact point. So, this is for one wheel, this is for another wheel, this is for the third wheel. And we are plotting v_x and v_y, all of them are very, very close to 0 which is like 10^{-16}.

So, the holonomic constraints are satisfied up to 10^{-7} meters, there is virtually no slip. The WMR is traversing this uneven terrain without slip which is what we wanted. Remember as I said that on an uneven terrain if the axle length is constant, then the wheels will slip. However, we have proposed a way of letting this torus shape wheel tilt laterally, and the distance between the wheel ground contact points of the two rear wheels will adjust such that there is no slip. And this is a proof by simulation.

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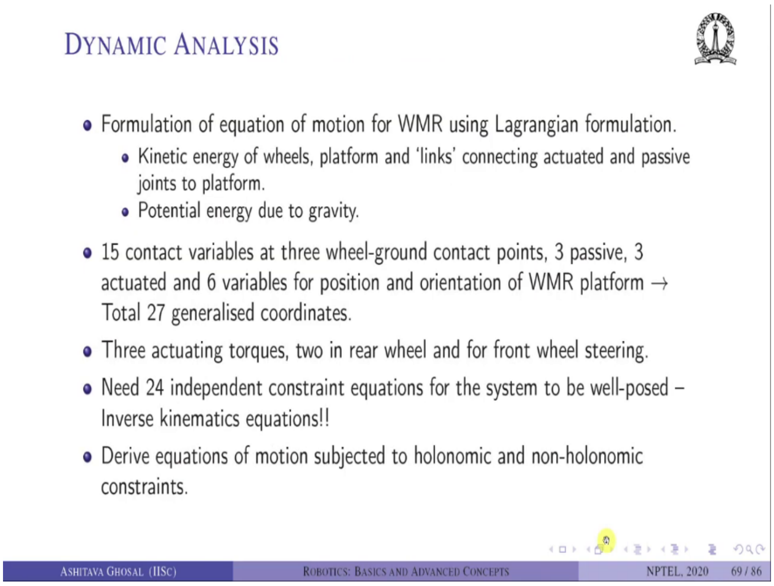

Let us continue. We can also perform the dynamic analysis of this WMR. We can derive the equation of motion for the WMR using the Lagrangian formulation ok. The kinetic energy of the wheels platform and links connecting the actuated and passive joints to the platform need to be computed.

The potential energy is due to gravity. Again in the dynamic analysis, we have 15 contact variables of the three wheel ground contact points, 3 passive variables, and 3 actuated variables, and 6 variables for the position and orientation of the WMR platform. So, totally there are 27 generalized coordinates.

There are three actuating torques, two in the rear wheel and one for the front wheel steering. So, we need 24 independent constraint equations for the system to be well-posed. We have 27 generalized coordinates, 3 are given. So, we need 24 constraint equations. So, as you can see this is a very, very complex system.

We have large number of differential equations; we also have large number of constraints. So, we can derive these equations of motion subjected to the holonomic and non-holonomic constraints using the Lagrangian formulation. This was done ok.

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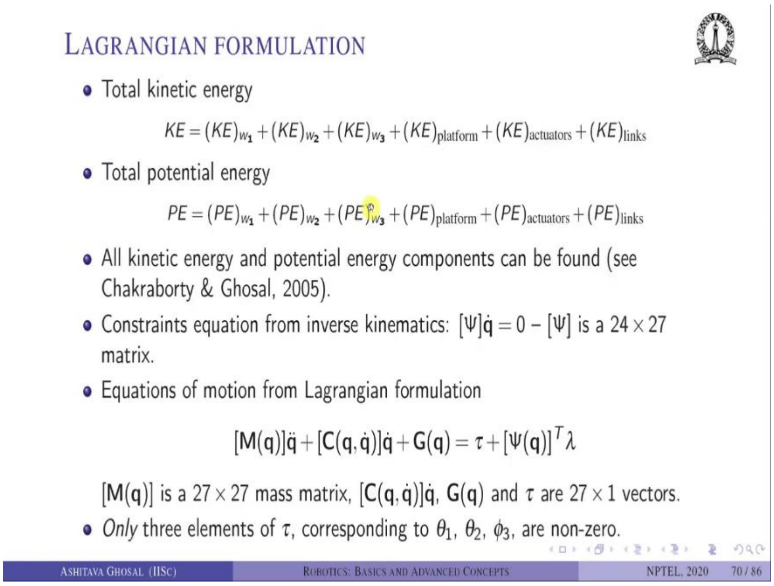

Just to give you an idea how complicated it is. The kinetic energy is kinetic energy of wheel 1, wheel 2, wheel 3, kinetic energy of the platform, kinetic energy of the links of the actuator, kinetic energy of the links also. We also have potential energy of wheel 1, wheel 2, wheel 3, platform actuator then links.

So, all the kinetic energy and potential energy can be found. It is a humongous task. It is very laborious and long, but nevertheless it can be found the ideas are very simple. Kinetic energy is $\frac{1}{2}mV_c^2 + \frac{1}{2}I\omega^2$, and potential energy is like mgh.

We can also find the constraint equations from inverse kinematics which is $[\Psi]\dot{q} = 0$, where $[\Psi]$ is a 24×27 matrix. And hence eventually we have a set of differential equations which are 27 of them. So, the mass matrix is 27×27, Coriolis term is 27×1 gravity term.

Then there are these torques, and then there is this $[\Psi]^T\lambda$. So, these are the constraint equations and λ's are the Lagrange multipliers ok. So, only 3 out of these 27 elements in this τ are nonzero. So, the ones corresponding to θ_1, θ_2, and ϕ_3 are nonzero, because they are given all others are 0. There are no torques corresponding to the other generalized coordinates.

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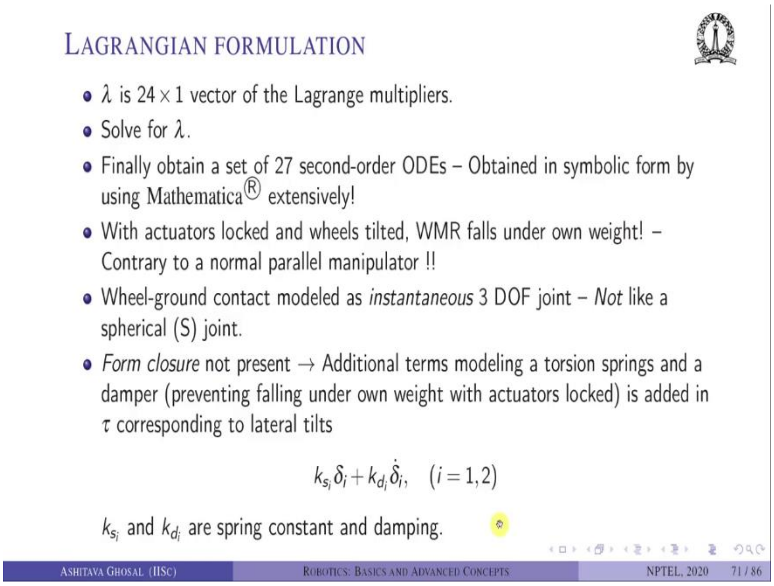

So, λ is a 24×1 vector of Lagrange multipliers. We can solve for λ. Finally, we obtain a set of 27 second order ODEs, and these were obtained in a symbolic form by using Mathematica. It took a lot of time. It is huge and humongous set of simplifications and equations which were used to obtain this 27 second order ODEs.

With actuators locked and wheel tilted, WMR falls under its own weight because this is to do with non-holonomic constraints. Contrary to a normal parallel manipulator, if you lock the actuated joints, it becomes a structure. In the case of a three degree of freedom WMR because the constraints at the wheel ground contact points are non-holonomic, basically they restrict the velocities.

But they do not restrict the configuration of the whole parallel manipulator. And as a result the whole platform manipulator will fall, and this was seen during simulations. So, as I said this is only because the wheel ground contact points are instantaneous three degrees of freedom joints, they are not like spherical joints.

And there is something called form closure which is not present ok. We need additional terms modeling a torsion spring and a damper to prevent the top platform falling under own weight with actuators locked. And hence this is added. So, we add corresponding to the lateral tilts some talk which is $k_{s_i}\delta_i + k_{d_i}\dot{\delta}_i$, $i = 1,2$.

So, there is a damping and there is a spring stiffness added to the lateral tilt. So, these are some constants. So, this is added in a way so, that it does not fall in its on its own weight.

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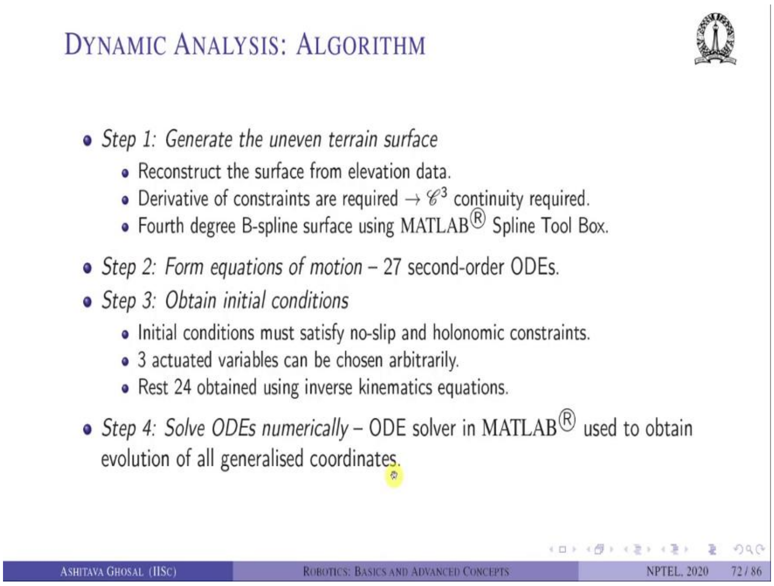

So, here is the algorithm to do the dynamic analysis of a three-wheeled mobile robot going over uneven terrain with wheels model as torus shape wheels, and also the wheel ground contact does not have slip. So, first just like kinematics, we generate the surface. We reconstruct the surface from elevation data. We find the derivatives of the surface. We need C^3 continuity; because we need to go to dynamics we have the derivatives of many of these terms.

We used a fourth degree B-Spline surface using MATLAB Spline Toolbox, and the surfaces were generated. We form the equations of motion these are 27 second order ODEs. We obtain initial conditions. The initial conditions must satisfy no slip and holonomic constraints.

The three actuated variables can be chosen arbitrarily. The rest 24 are obtained using inverse kinematics equation ok. So, to obtain the initial conditions, we need to solve the inverse kinematic equations. And then we solve all these ODEs numerically in MATLAB, to find how the generalized coordinates are changing in time.

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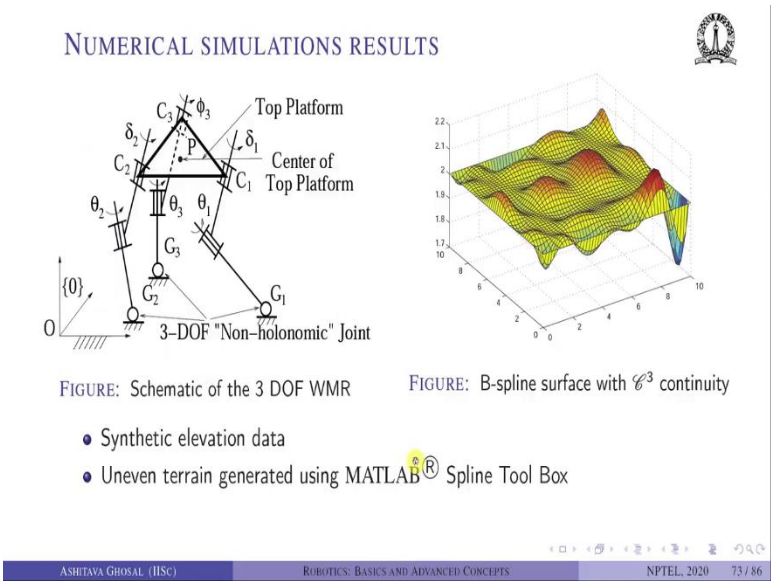

So, a few numerical simulation results, so this is the three-wheeled mobile platform, again this is the model G_1, G_2, G_3 are the three non-holonomic contact points. There are 15 variables at each in these 3, 5 in each, then there is θ_2, δ_2, θ_3, ϕ_3, θ_1, δ_1, and then there are the 6 variables of the top platform ok.

This is the surface which was use earlier also. And we are going to use the same synthetically generated surface with C^3 continuity for simulation. This is not a real surface; this is generated in MATLAB.

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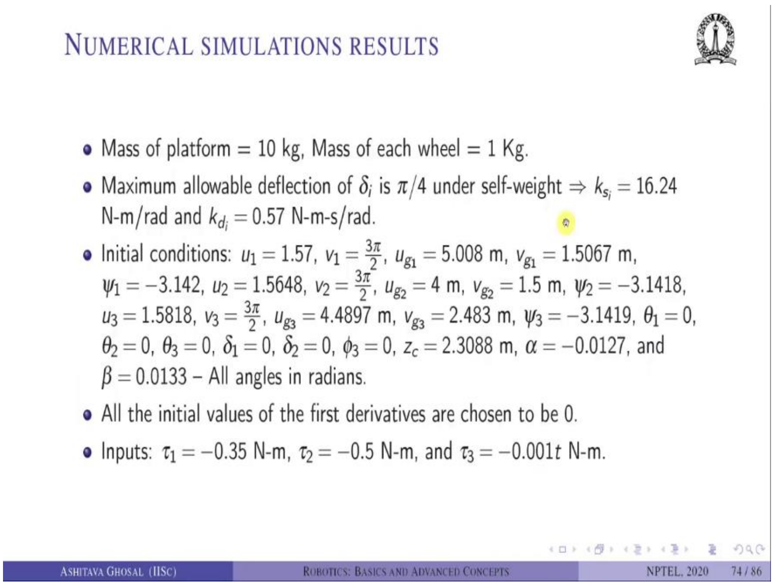

The mass of the top platform is assumed to be above 10 kgs, mass of wheel 1 kg because in dynamics we need to know masses. The allowable deflections due to lateral tilt is $\pi/4$, because we need to assume some spring constant to prevent lateral tilt beyond $\pi/4$ radians. And it turns out $k_{s_i} = 16.24$ Newton meter radian, and $k_{d_i} = 0.57$ Newton meter second per radian ok. So, this is k_{s_i} is obtained for self-weight, and k_{d_i} is chosen such that there is some damping.

The initial conditions are again obtained such that it satisfies the constraint equations both the holonomic and non-holonomic constraint equations ok. All the initial values of the first derivatives are chosen to be 0. So, this robot or this wheeled mobile robot is starting from rest.

And we have arbitrarily chosen $\tau_1 = -0.35$ Newton meters, $\tau_2 = -0.5$ Newton meter, and $\tau_3 = -0.001t$. So, it is tracking some trajectory with the steering there is as a function of time ok. We can choose other values. This was chosen by the student.

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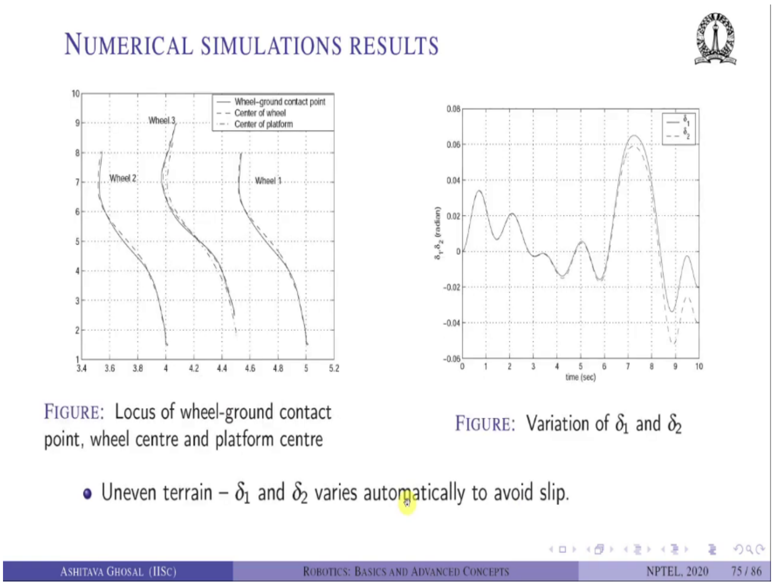

So, we can solve these 27 ODEs, and then we can plot what the wheel 1 is doing, what the wheel 2 is doing, and what the wheel 3 is doing, so wheel 1, wheel 2 and wheel 3, wheel 3 is the steering wheel. We can also see what the center of the platform is doing again using this dot dash line. At the center of the wheel and the wheel ground contact points are again different basically because it is moving on the uneven terrain.

There is also δ_1 and δ_2 which is tilting the two rear wheels are tilting, so that there is no slip. And we can plot δ_1 and δ_2 as a function of time ok. So, coming back to it, δ_1 and δ_2 are passive ok. So, they vary or adjust automatically to avoid slip. We can see that the error in the holonomic constraints are very small of the order of 10^{-6}.

So, this is the constraint between C_1 and C_2, this is the constraint between C_1 and C_3 and this dot dash is a constraint between 2 and 3. As you can see it is very small, it is 10^{-6}. We can also check v_x and v_y for the slip velocity at all the three wheels, at the wheel ground contact point. And as you can see they are also very small this dash line is for v_y, v_x is the solid line, they are all like 10^{-7}, of the order of 10^{-7}.

So, all the constraints are satisfied at least up to 10^{-7}, distance meters and this is something else velocities. The three-wheeled mobile robot can hence to be shown to travel an uneven terrain without slip because the slips are so small, the constraints are satisfied. And as you can see it is tracing some path on the uneven terrain ok.

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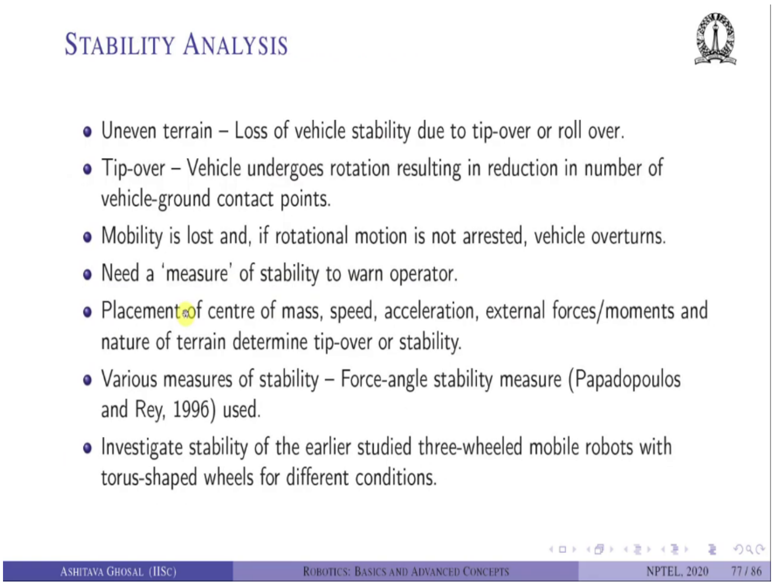

So, next let us look at stability of this three-wheeled mobile robot and uneven terrain ok. So, on an uneven terrain, the loss of vehicle stability could be due to tip-over or roll over ok. Tip-over means the vehicle undergoes rotation resulting in reduction in the number of vehicle ground contact points ok, say instead of three you can have less. Mobility is lost and if the rotational motion during tip-over is not arrested, the vehicle will overturn.

So, first thing is we need to find or define and measure of stability to warn the operator, or if you are on the vehicle to see that it is tipping over or it is about to tip-over ok. Like in any vehicle the placement of the center of mass, speed, acceleration, external forces and moments, and the nature of terrain will determine the tip-over or stability ok. It is not only one parameter; various parameters are involved to obtain the stability of a mobile robot on uneven terrain.

There are various measures of stability which has been developed ok. We are going to use something called the force angle stability measure. This was first developed in 96 ok. And we will investigate the stability of the previously developed three-wheeled mobile robot with torus-shaped wheels for different conditions.

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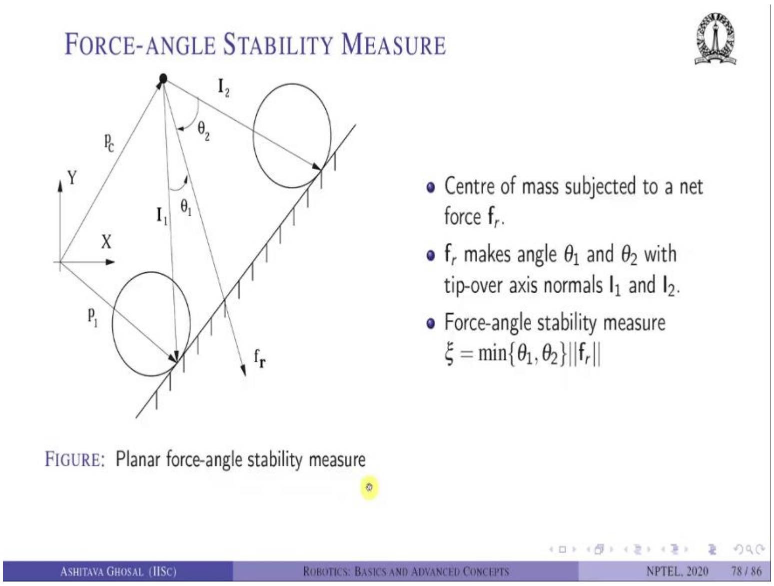

So, what is the force angle stability measure? Ok. So, let us look at the center of mass which is subjected to a force $\boldsymbol{f}_r$. So, this is the center of mass of the platform that is the force which is acting. This is a terrain ok. The $\boldsymbol{f}_r$ makes an angle θ_1 and θ_2 with the tip-over axis normals $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$. So, these are the two wheels, there are these axis normals $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$. And it makes an angle θ_1 and θ_2, so θ_1 and θ_2.

The force angle stability measure tells you that this variable ξ, which is minimum of θ_1, θ_2 into $\boldsymbol{f}_r$. So, this ξ should have some range of values. So, basically what it eventually will boil down to the fact is that this angles θ_1 and θ_2 should not be such that this $\boldsymbol{f}_r$ is outside this $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$, roughly speaking ok.

And we have this reference coordinate system X-Y, and we can locate the point of contact which is $\boldsymbol{p}_1$ in this case, and we can also locate the center of that platform which is $\boldsymbol{p}_c$.

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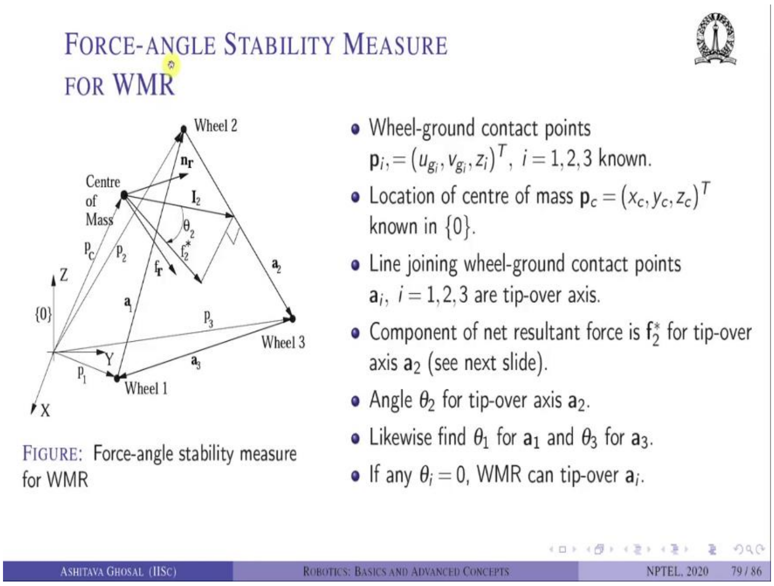

So, in the case of a three-wheeled mobile robot, so we have wheel 1, wheel 2, wheel 3, we can locate the wheel ground contact points by u_g, v_g and z_i for each of the three wheels. The location of the center of mass $\boldsymbol{p}_c = (x_c, y_c, z_c)^T$. The line joining the wheel ground contact points is $\boldsymbol{a}_i$. So, $\boldsymbol{a}_3$ is the line which joins wheel 1 to wheel 3; $\boldsymbol{a}_2$ is 2, 1, 3; and $\boldsymbol{a}_1$ is between wheel 1 and wheel 3. And then we have this force which is acting at the center of mass which is $\boldsymbol{f}_r$.

So, the component of the net resultant force is $\boldsymbol{f}_2^*$, for tip-over axis $\boldsymbol{a}_2$. So, the angle θ_2 for the tip-over axis a_2; likewise we find θ_1 for $\boldsymbol{a}_1$, and θ_3 for $\boldsymbol{a}_3$. So, there are these three angles which tells you the angle between the normal $\boldsymbol{I}_1$, $\boldsymbol{I}_2$, $\boldsymbol{I}_3$, and the resultant force $\boldsymbol{f}_2^*$. This is for $\boldsymbol{a}_2$; similarly for $\boldsymbol{f}_1^*$ and $\boldsymbol{f}_2^*$.

So, if any $\theta_i = 0$, the ω_r can tip-over $\boldsymbol{a}_i$. So, basically what is happening? So, if this $\theta_2 = 0$ so which means what, $\boldsymbol{f}_2^*$ is along this normal $\boldsymbol{I}_2$. So, now the WMR can tip-over $\boldsymbol{a}_2$ about this wheel ok. So, that is the basic idea. We will calculate these angles and make sure they are not 0. If they are coming close to 0, we note that tip-over about a certain line is going to occur ok.

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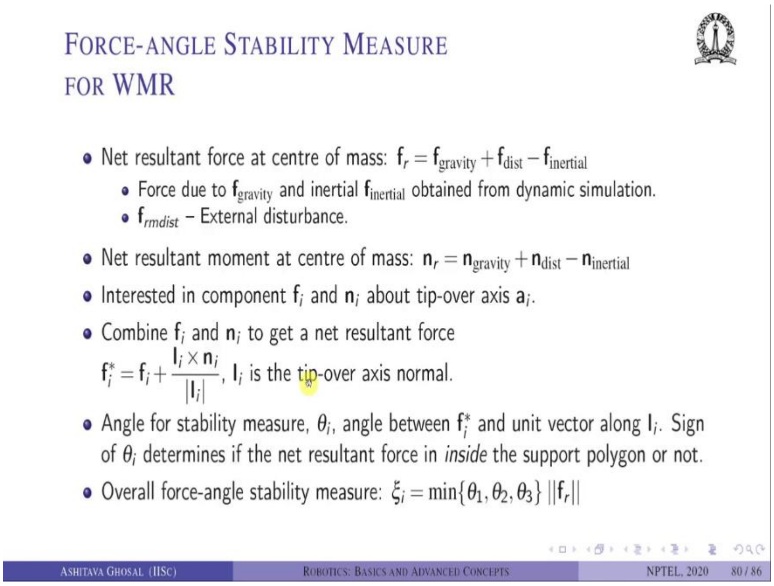

The net resultant force can be computed also. So, this $\boldsymbol{f}_r$ is given by the force due to gravity, the force due to disturbance minus the force due to the inertia ok. So, force due to gravity and inertia are obtained from dynamic simulation. The $\boldsymbol{f}_{dist}$ is some external disturbance which we need to assume. The net resultant movement at the center of mass is due to gravity, the disturbance moment and the inertial moment.

We are interested in component of $\boldsymbol{f}_i$ and $\boldsymbol{n}_i$ about the tip-over axis $\boldsymbol{a}_i$. So, will there are three of them between. So, 1-2, 2-3, 3-1, it can tip-over any one of these axis. So, we want to find the force for that tip-over axis and also the angle. So, we, so we know this $\boldsymbol{f}_i$ and n_i about the tip-over axis, we have to combine this $\boldsymbol{f}_i$ and $\boldsymbol{n}_i$ to get a resultant force which is this f_i^*, and this is combined in this way.

So, $\boldsymbol{f}_i^* = \boldsymbol{f}_i + \frac{\boldsymbol{I}_i \times \boldsymbol{n}_i}{|\boldsymbol{I}_i|}$. So, $\boldsymbol{I}_i$ is the tip-over axis normal ok. So, from the dynamic simulation we will get some force $\boldsymbol{f}_i$ and $\boldsymbol{n}_i$, we need to combine them to find the resultant force which is given in this form. And then we compute the angle stability measure θ_i which is the angle between $\boldsymbol{f}_i^*$ and the unit vector along the normal $\boldsymbol{I}_i$.

The sign of θ_i determines if the net resultant force is inside the support polygon or not. And the overall force angle stability measures states is that this is ξ_i which is the minimum of all three. So, it can tip-over any one of these three axis.

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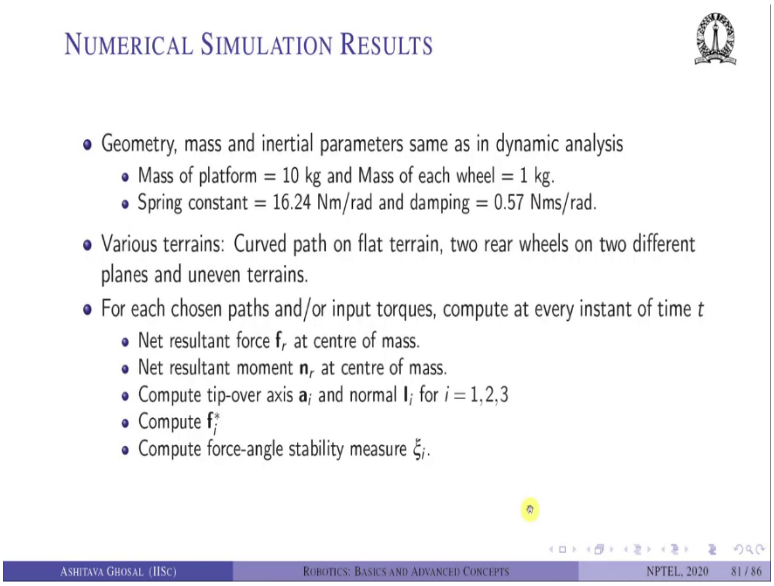

So, we need to find the minimum of θ_1, θ_2, θ_3 and then take a decision about which axis it is going to tip-over. So, let us look at some numerical simulations. So, again the mass of the top platform is 10 kg. We are going to use the same numbers as used in dynamic analysis.

Mass of each wheel is 1 kg. The spring constant we have assumed some springs to prevent lateral tilting and the damping is this. We look at various terrains. We, can look at a curved path on a flat terrain, two rear wheels on two different planes and uneven terrains.

For each of the chosen path and or input torques, compute at every instant of time the net resultant force which is acting at the center of mass. The net resultant moment $\boldsymbol{n}_r$ which is acting at the center of mass. Compute the tip-over axis $\boldsymbol{a}_i$ and the normal $\boldsymbol{l}_i$ at each one of these contact points $i = 1,2,3$. Compute $\boldsymbol{f}_i^*$ by combining $\boldsymbol{f}_r$ and $\boldsymbol{n}_r$. And then you compute the force angle stability measure ok. This should be away from 0. It should be for minimum of θ_1, θ_2, θ_3 could be away from 0.

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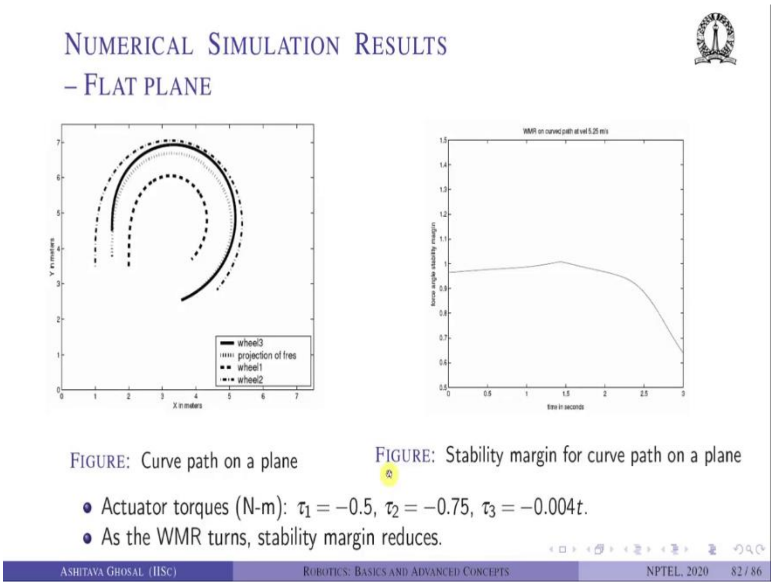

So, here is a simulation of a wheeled mobile robot ok, which is traveling on a trajectory on a flat surface ok. So, the wheel 3 is this dark line. The projection of this resultant force $\boldsymbol{f}^*$ is this dash line which is this one ok. Wheel 1 is this dot dash lined, and the wheel 2 is this one ok. So, what you can see is that force angle stability measure basically the angles behaves like this, and then it comes down to about 0.6 ok.

So, as the WMR turns, the stability margin reduces but it is all still stable it never goes towards 0. And in this example, you have assumed torque as $\tau_1 = -0.5$, $\tau_2 = -0.75$, and $\tau_3 = -0.004t$ to trace this circle. So, basically what have we done, let us summarize.

I have the equations of motion. I can solve the equations of motion to find out what is the net resultant force, and the net resultant moment acting at the CG of the top platform. Then I can find these tip-over axis between $\boldsymbol{a}_1$, $\boldsymbol{a}_2$ and $\boldsymbol{a}_3$, three of those between wheels 1 and 2, 2 and 3, and 3 and 1.

Then I can find the normals. And then I can find the angle between this resultant force and this I_1, I_2, I_3, and then plot all of them – evaluate all of them and show how the wheel is moving on this flat terrain. I can show this is the third wheel. These are the first one and two wheels. And this is that projection of the force vector onto the plane. And then I can compute this – θ_1, θ_2, θ_3, and the minimum of those and plotted.

So, what does it say? That it is reasonably stable it is this wheeled mobile robot can move on a flat terrain in a stable manner. When it is turning, the stability margin reduces a little bit, but nevertheless it is fine.

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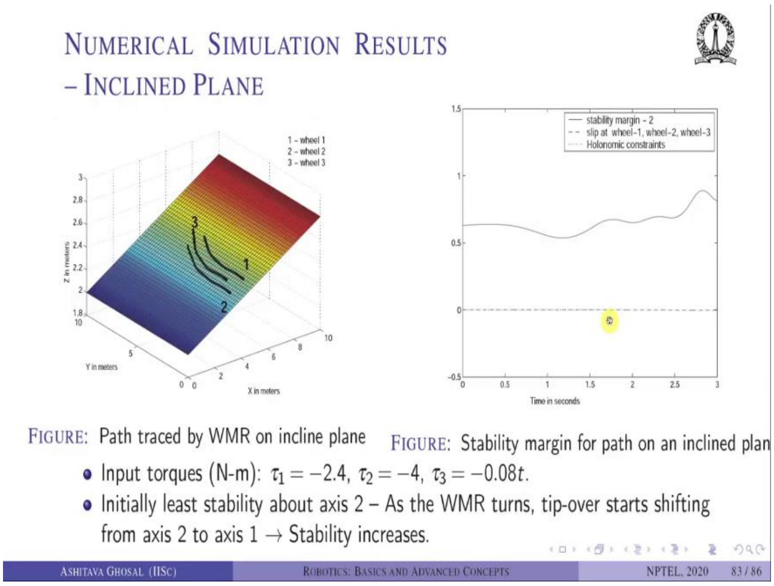

Now, let us look at this wheeled mobile robot on an inclined plane. So, again basically I can plot what the first wheel is doing, the second wheel is doing, and the third wheel is doing ok. So, what we can see here is the stability margin to slip at wheel 1 which is this dash lines 1, 2 and 3, and this the holonomic constraints.

So, all of them are very close to 0. Remember they were like 10^{-7}. So, the stability margin here is also away from 0 ok. So, this wheeled mobile robot on this inclined terrain is still stable for these kinds of inputs again $\tau_1 = -2.4$, $\tau_2 = -4$, and $\tau_3 = -0.08t$.

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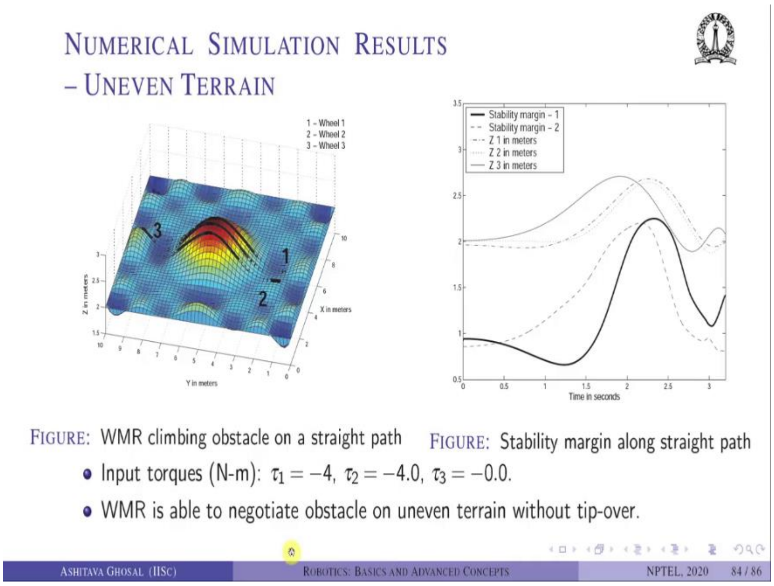

Initially it is least stable about axis 2 – as the WMR turns, the tip-over starts shifting from axis 2 to 1, and stability increases ok. Now, let us look at a wheeled mobile robot on an uneven terrain which looks like this. So, there is a hump here. And we want this wheeled mobile robot to go over this hump. So, what you can see is the first wheel looks like this; this is the path it traverses. The second wheel looks like this, and the third wheel is at the middle ok.

Again the input torques at $\tau_1 = -4$, $\tau_2 = -4$, and $\tau_3 = 0$, as there is no steering it is going straight over this uneven terrain. We are just chosen arbitrarily some numbers to try or different cases. We have tried many, many cases ok. And again the stability margins 1, 2, and the Z-component of this motion is plotted here. So, what you can see is the stability margin 1 and 2, they are all away from 0 occur. And the WMR is able to negotiate this hump which is an obstacle on uneven terrain without tip-over.

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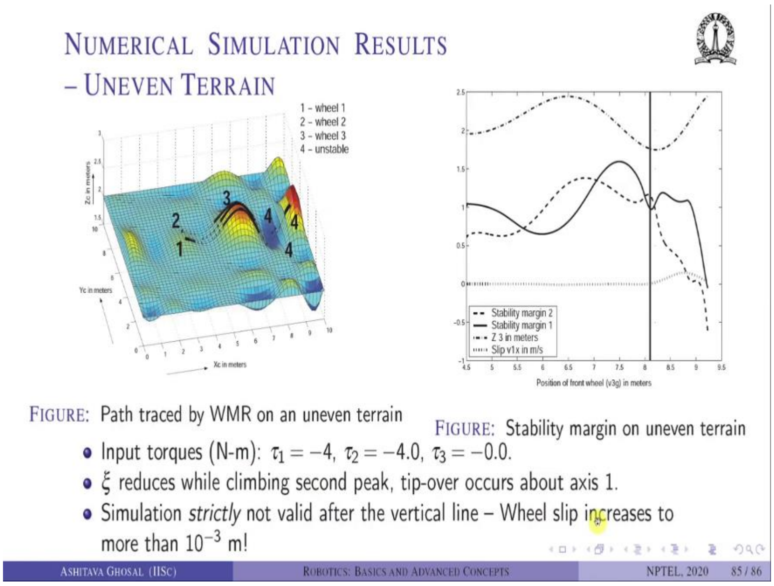

Let us try another trajectory where the surface is slightly different. So, again we have 1 which is wheel 1, 2 which is wheel 2, 3 which is wheel 3, and at 4 there is unstable behavior that is happening which is shown here. So, let us go back a little bit. Again we are applying some torque $\tau_1 = -4$, $\tau_2 = -4$, and $\tau_3 = 0$ – there is no steering ok. The stability margin reduces while climbing the second peak, not this first one, the second peak and tip-over occurs about axis one.

So, this simulation shows that after some time this at this 8 point something the stability margin is going below 0. So, there is not much meaning what is happening here ok. Also the wheel slip is also increasing a lot. So, after some point, there is wheel slip. So, again there is no point in simulation. Nevertheless, what it is showing here is that the stability margin is going below 0, and hence there is a tip-over.

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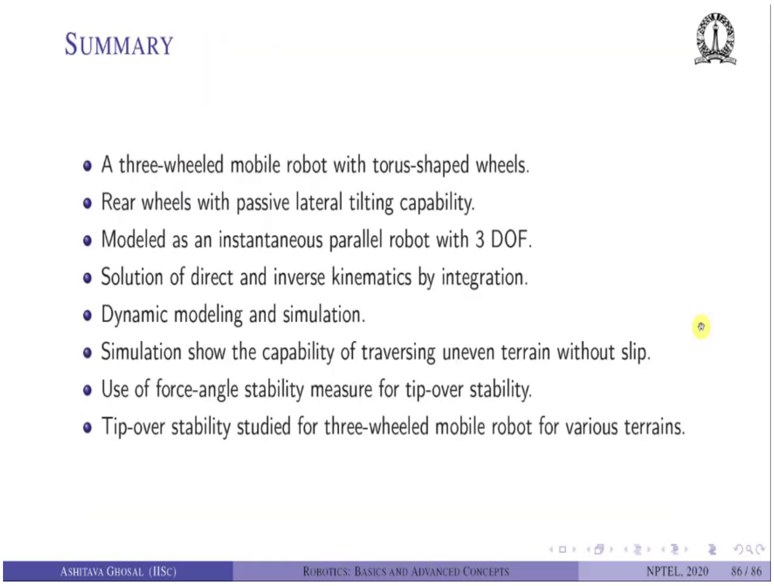

So, in summary a three-wheeled mobile robot with torus-shaped wheels were studied for stability. The rear wheels with passive lateral tilt capability at present. We modeled this three-wheeled mobile robot as a parallel robot with 3 degrees of freedom. The solution of the direct and inverse kinematics was obtained by integration because the direct and inverse kinematics are related to the velocities unlike in a serial robot.

I showed you some dynamic modeling and simulation. I will show you next also some more dynamic modeling and simulation using a software package called ADAMS after this ok. So, in ADAMS, I do not have to do all these things in MATLAB. So, and do all this 27 equations and 24 equations.

So, we can make a CAD model of the three-wheeled mobile robot, we can make a CAD model of the surface, and then we can simulate on the surface. I also showed you how we can traverse this uneven terrain without slip by allowing this lateral tilting. Finally, with force-angle stability measure, I could show you that it could measure or indicate tip-over stability ok.

It is a measure of tip-over stability. And I looked at several trajectories on uneven terrains, on flat terrains, on incline planes, and uneven terrains. And I showed you that we could compute this force angle stability measure. And in one of the examples, I showed you that it could do tip-over ok.

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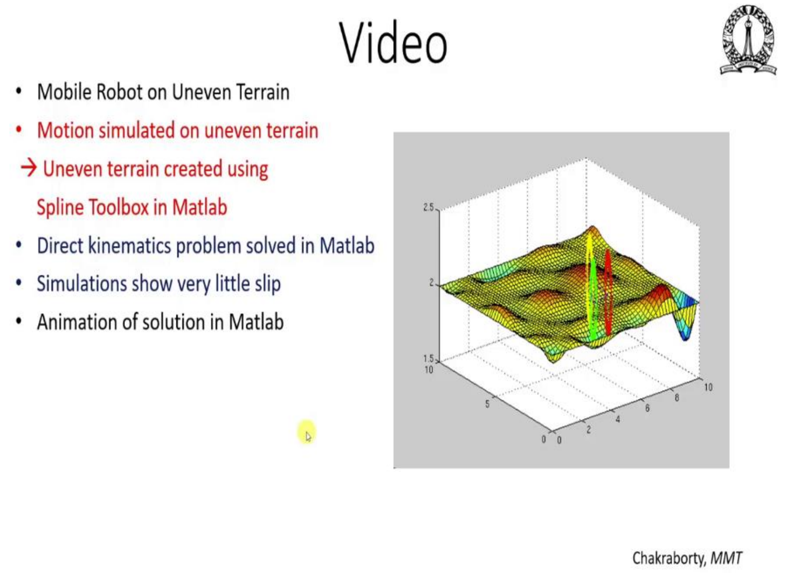

So, with this, we will come in the next after this I will show you some videos. In these set of lectures, we were looking at wheeled mobile robots. In the last, I have discussed and I mentioned that I am going to for some videos of wheeled mobile robots going on uneven terrain ok. So, the first video is on a motion of a three-wheeled mobile robots which was simulated using MATLAB. The uneven terrains were generated using Spline toolbox in MATLAB.

We solve the direct kinematics equations, which was discussed previously. We integrated the first order ODEs and found all the 15 variables, wheel ground contact points, and also how the θ's and the top platform moved. And I showed you in simulation that there was very little slip and the holonomic constraints were satisfied.

So, this solution was generated in MATLAB. And then this in MATLAB, there is a feature to animate the results and this video shows the animation. So, it is looking little crude, but nevertheless you can see that the three wheels are moving on uneven terrain. And I had actually showed you earlier plots of this direct kinematic simulation, and showed that there is no slip.

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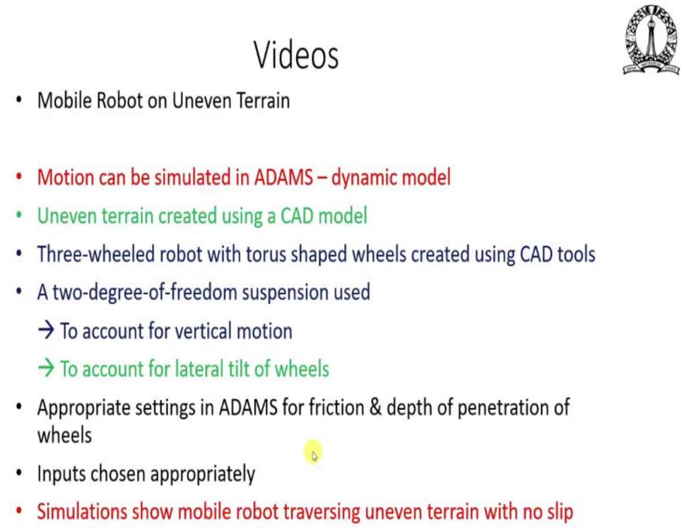

In the next slide, I will show you some videos obtain using something called ADAMS. So, ADAMS is a commercial simulation software package. And again we can create a model of the three-wheeled mobile robot in ADAMS. It is a dynamic model meaning it solves the dynamic equations of motion in its own way. We can create the uneven terrain using a CAD model. The three-wheeled mobile robot with torus-shaped wheels is also created using CAD tools.

Now, in ADAMS to connect, the wheels to the body we need to provide some kind of a suspension. Remember the wheels can tilt laterally and it can go up and down. So, we need a two-degree-of-freedom suspension. And one of my student worked on these two-degrees-of-freedom suspension. And we incorporated a two-degree-of-freedom suspension on this wheeled mobile robot.

So, basically one degree of suspension is to allow the tilt, and one degree of freedom is to account for the vertical motion of the platform and the wheel. So, you choose appropriate settings in ADAMS. So, you need to play round with ADAMS, because you need to set the friction in ADAMS, and something called the depth of penetration of the wheels, so that is the way ADAMS works ok.

You need to make sure that the wheels are always in contact with the uneven terrain by giving this depth of penetration. And also you have to specify a friction. In the MATLAB simulations, I did not specify any friction at the wheel ground contact point because that was the way it was developed ok. We do not need to incorporate the friction at the wheel ground contract point in MATLAB, basically there is no slip. So, there is no friction.

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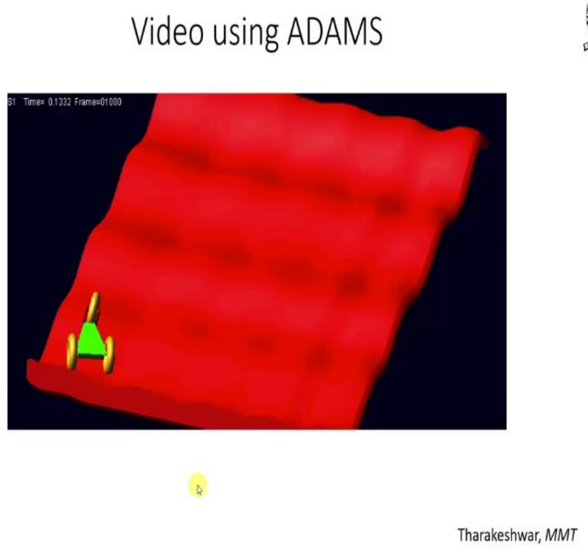

We also need to choose the inputs appropriately in ADAMS, we need to play around with it. So, what you will see now is the motion of a three-wheeled mobile robot going on an uneven terrain which was both the uneven terrain and the robot was created in CAD models.

And then appropriate parameters were set. And then you can see it is moving. So, the important thing to notice here is that this uneven terrain it looks like some wavy surface because we cannot generate very complicated uneven terrains in ADAMS. The important thing to notice is that the wheels are tilting laterally.

So, the front wheel is rotating, and it is and also it is being steered, whereas, the two rear wheels are rotating and tilting laterally. So, as you can see that the motion is quite smooth, there is almost no slip I mean you cannot see that there is no slip in the video. But if you look at the ADAMS simulation and you can plot all the variables again, and you can see that there is no slip.

But definitely this video shows that this three-wheeled mobile robot with torus-shaped wheels, and wheels capable of lateral tilting can negotiate an uneven terrain. This is another uneven terrain and the same three-wheeled mobile robot which is negotiating this an uneven terrain ok. ADAMS is a very, very powerful commercial package.

There are plugins, and there are extensions of ADAMS which can simulate the motion of a car or a four wheeled vehicle, it is called ADAMS car. But this is not using ADAMS car we have just imported the surface from a CAD model, we have also imported a model of this three-wheeled vehicle. Then we have put suitable joints between this green platform and the wheels, and then we are giving some input ok.

So, this is another terrain which was created. And this is that same three-wheeled vehicle negotiating this uneven terrain. Again, I can see that there are these hills and valleys and wavy terrain. And then we can go negotiate the terrain with the wheels always in contact with the terrain ok.

So, the wheels are always in contact with the terrain. It is not floating in air and that is the way to do that is to specify some friction and some depths of penetration. So, the depth of penetration is very small ok. So, with that, we come to an end of this set of lectures on wheeled mobile robots.

Thank you for listening.