Introduction to Soft Matter
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Lecture No 09
Mechanical Analogues

So, welcome everybody. Welcome to another lecture on Introduction to Soft Matter. Last time we were discussing the concept of ageing and how that affects the material response. So, let us just quickly look back at that 1 more time. And the idea was very simple. And what we are saying is that.

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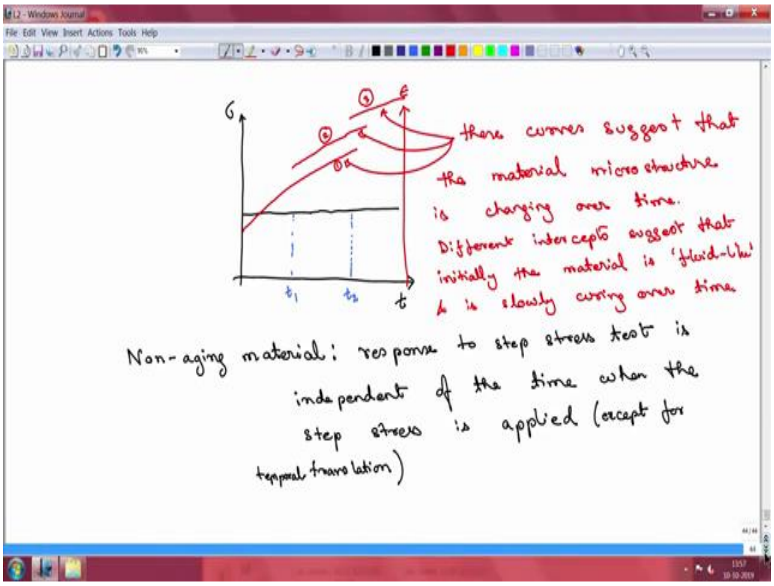

We wanted to understand whether the response of a material is time independent or not. And let us say you are going into the lab and you are going to do conduct an experiment over there. Now, when you do that, let us say you have 500 ml of some solution or 600 ml of some solution. And then you take 200 ml of that, and you start doing an experiment where you apply a constant stress test. And then you are going to measure the system response.

What happens is, after a time of let us say t 1, a friend of yours comes in. And they say that I would like to repeat the exact same experiment as you are. So, but as he came in at the slight delay, he starts his experiment, the same experiment at t 1 time. Another friend of your comes at time t2, you had already anticipated this. And so what you had done is the 600 ml of the material you had already divided it into 200 ml 200 ml, okay.

So, you are starting to do some experiments with your fluid and then the second, the first friend comes in and you hand over him some material at t1 time. And then the second friend comes in and you hand him some material at time t2. And each of you conducts an experiment separately and measures the system response. So, let us say, in your case you get a system response, so we will measure epsilon in red, you get a system response like this.

So, this curve is the first one, so we will just right maybe 1. The friend of yours who is doing an experiment instead gets a slightly different curve. Maybe this is the curve. This is what his data looks to him. The third person is also doing an experiment and he comes up with a totally different curve for his own experiment. Now, had this material been non-aging you would have expected the material response to be independent of the time of the experiment itself, okay.

So, for a non-aging material so, a non-aging material is a material such that the response to a step stress test and we are just saying step stress test as an example, it could be any other test as well is independent of, is same, just a second, is independent of the time when the step stress is applied except for a translation in time. So, except for the translation, temporal translation.

But in an aging material, like this example, what happens is that the material responses do not look as if they are simply translated in time. You can see at least I have not given you the functional form of the graphs, but at least you can see that the intercepts of these are different, intercept of 1, intercept of 2 and intercept of 3 are different. So, this material which is given by this curves.

So, these three curves, these curves suggest that the material micro structure, micro structure is possibly changing over time the different intercepts, intercepts suggest that initially the material is somewhat soft, material is somewhat soft and is sort of fluid like is put it in this fluid like and is slowly curing over time.

So, in such materials, in materials which are aging. So, in aging materials, the time when the material was created, often called the time of creation of the material becomes very important relative to when you are doing the experiment. Okay?

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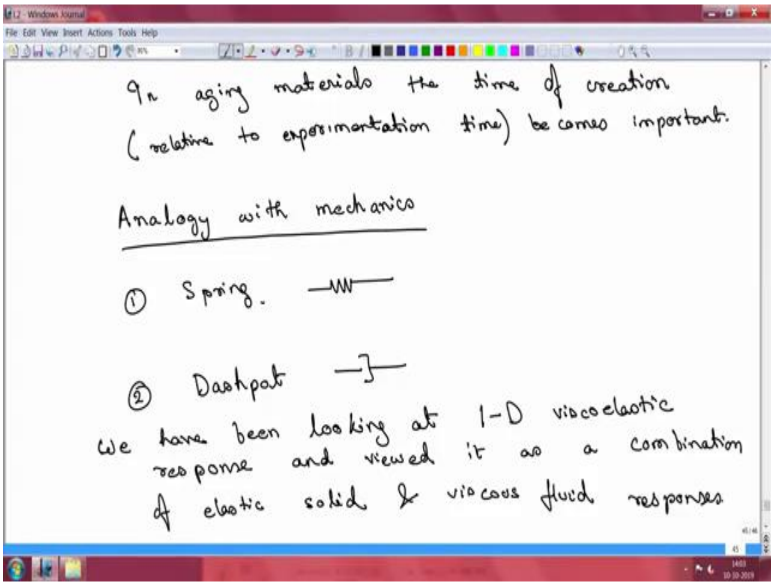

So, in aging materials you have to, in aging materials the time of creation relative to experimentation time, experimentation time becomes important. So, as I told you before that there is no fundamental law that requires a material to be aging or non-aging. So, whenever you have a material it is for you to determine whether, you determined, it has to be determined experimentally, whether the aging process is really taking place or not.

We will mostly concern ourselves with non-aging materials, except perhaps in a few sample problems here and there. But, now, you now know that the important concepts that are involved in some of the linear viscoelastic idea would also have the idea of non-aging material.

So, the material response should be the same over time. So, when we are discussing viscoelastic city, we discussed that viscoelastic response is a response that is intermediate to two classical responses. The first classical response is that of a linear elastic solid and the second is that of a viscous fluid.

Now, when we had done that, we had looked at various graphical representations of how the viscoelastic response and the two classical responses look like. We looked, since we looked at the graphical response, there was another important cue that we did not discuss at that point. And that was an analogy with mechanics that I would like to discuss right now. So, what do I mean when I say analogy with mechanics?

Is that we did discussed graphical representations. But is there anything in mechanics that reminds us of or suggests that the viscoelastic response can be a function, could be some combination of those Mechanical analogs. So, one mechanical analog that we want to discuss is the spring. The spring is a construct that we apply quite often in mechanics where we know that a spring when it is an equilibrium it does not exert any force, but when it is disturbed from equilibrium, it exerts a restoring force on a massive body right.

And if it is a linear spring, then there is the force. The restoring force that it applies is equal to, is proportional to the displacement from equilibrium of the spring. The other is the dashpot. Some people call it dampener it is also represented by this symbol. The dashboard is usually use is another construct which is used to signify dissipation in mechanics. So, here the dissipation is proportional to the velocity.

So, let us see whether, so we have been looking at, okay just quickly going back to this, this page, okay. So, till now we have been okay so we have been looking at 1 dimensional viscoelastic response and viewed it as a combination of elastic solid and viscous fluid, fluid responses. So, let us see how far we can carry this analogy. Okay.

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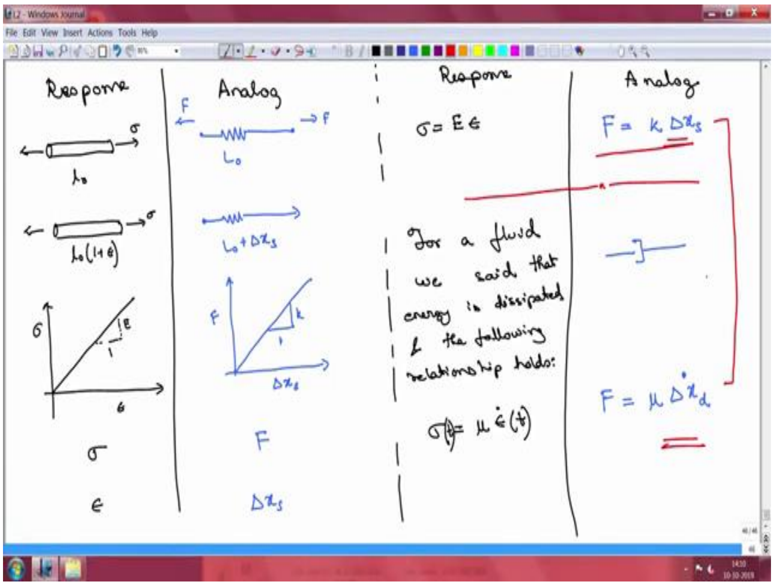

So, just maybe, okay, what I am going to do is just I am going to break up this page into two halves as we have a lot of space here. So, let us look at our response and let us see if our analogues will work here. So, the first response is the response of the elastic solid, right? What we had is let us say we are looking at a one dimensional response. So, let us say you have a rod of length l naught and you apply some normal stress to it and this results in a deformation and this deformed state, the length has now increased and it is given by 1 plus epsilon where epsilon is your strain.

Similarly, in our analogous system, we have a spring that initially at some L naught size or L naught is the equilibrium length, then you apply a force to the spring when you apply the force the spring deforms. Now, becomes let us say delta xs, delta xs standing for the spring. In case of the solid, we know that we have the stress of the strain, which are linearly related to each other.

And they are given. And this linear response is characterized by a simple proportional relationship between sigma and epsilon. And the same is the case for our spring where you have this restoring force which is a function of the displacement and this slope gives you information about the spring constant, right?

So, which implies that so, if you try to draw this analogy, what we are implying is that this stress in our system is analogues to the force in mechanics. So, it is analogous to force and your displacement then becomes analogous to the strain. So, the relationship so, we have to, let us go over to this side.

So, in this case, in the case of the elastic body, we had this relationship and now we can see say that this relationship is analogues. I am just trying to keep it consistent, so I will use the same color. So, your force is equal to k times delta xs. So, this equation now, this entire equation, this becomes our mechanical, mechanical analog to the elastic solid.

So, similarly, if we go for the viscous dissipations case or the case of the fluid body, what we see that there is that the relationship we had so for a fluid we said that energy is dissipated and the following relationship holds, the following relationship is sigma equal to mu times. So, if we want to use the dashboard as our analog for the viscous dissipation in the fluid, then here, your equation will now be some mu times of delta x, let me call it delta xd which is the displacement of the dashpot where the dot represents the time derivative.

So, these two equations are important because what the makers realize is that if the elastic body can be considered analogues to a spring and if the viscous fluid can be considered analog us to a dashpot then the viscoelastic response can be considered as some linear combination of the spring and the dashpot. And that is indeed the case so there are many different many models, which use these two analogs and they use these two analogs to build the viscoelastic or predict the viscoelastic response. Okay.

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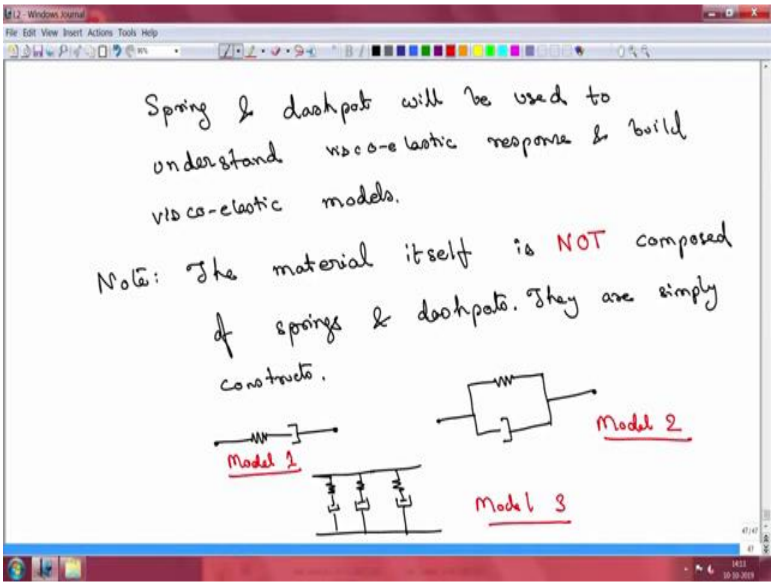

So, spring and dashpot will be used to understand viscoelastic response and build viscoelastic models. Now, we must clarify here that the material itself is not made of the spring and dashpot, they are just imaginary constructs that we can use to model viscoelastic response. So, please note the material itself is, I will just use red to highlight, is not composed of springs and dashpots they are simply constructs.

So, what we want to do is essentially at some point, we will represent our viscoelastic response with some linear combination, and I am just drawing some possibilities. You can have even more complicated scenarios sort of cramped for space here. So, based on the continuum mechanics of the continuum responses of viscoelastic body, we can build different models by putting the spring and the dashboard in some linear combination.

And I have just given you three different examples here in which they have been combined in different ways. And these can all represent different viscoelastic responses. Okay. The mathematical details of the these responses we will see at a slightly later stage, but what we are going to do is that these analogs are later on going to help us build continuum level models for viscoelasticity.

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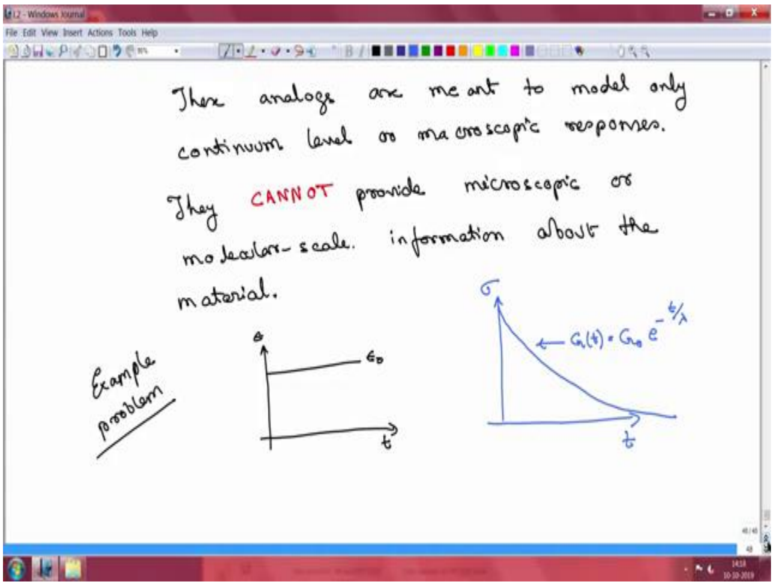

So, before we sort of end our discussion on analogs we will just note that these analogs are meant to model only continuum level or macroscopic big responses. They cannot model or they do not have any information about the microscopic details of the material, so they cannot provide and maybe here and just they cannot, they cannot provide microscopic or molecular scale information about the material.

So, what we have done till now we figured out that there are two analogs of the elastic, for the elastic response and the fluid response we can use two different analogs to model such responses and most likely we will be able to use a linear combination to give us some good models for viscoelasticity. The details of that, in mathematical sense, we will see at a slightly later stage.

So, we are also, we also familiarize ourselves with different continuum level responses like the creep response and the, the stress relaxation response. So, let us take some, let us do some example problems, where we get to analyze this a little bit further. So, one example problem that I have prepared for you is a situation where you apply the strep stress test to a material and you experimentally determine to the stress relaxation function looks something like this, where this is given by, this curve is given by sorry minus t by lambda. Okay.

Now, your question is, so this is a given system. So, there is a material and you have done the test and you have experimentally determined that this is, now this information is given to you that the Gt is simply a exponentially decaying function.

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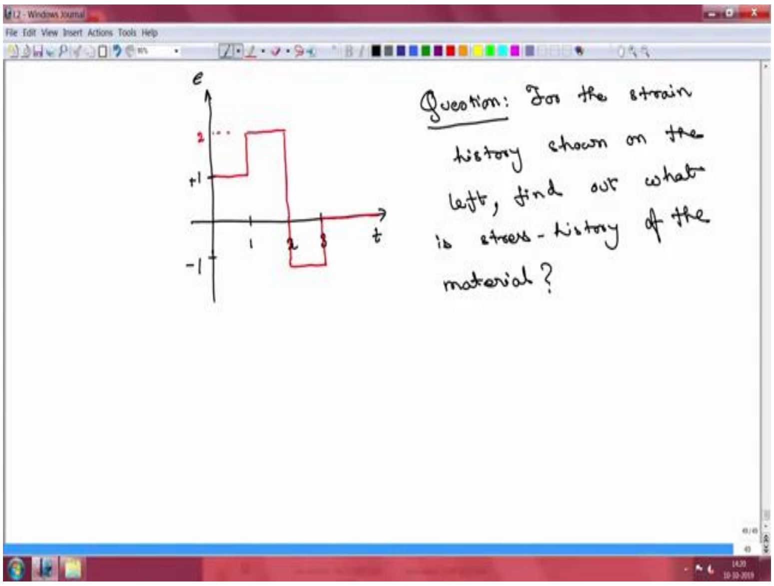

Now, the question is that was being posed for you is that this same material is now going to be subjected to strain where the strain is for a duration of 1 time time delta t equal to 1, the strain is 1, thereafter it increases to a value of 2. So, this value here is 2 when it remains at 2 till t equal to 2 after which decreases to a value of minus 1 and remains at a level of minus 1 till delta t of 1 and at time t equal to 3 it becomes 0.

So, your question is for the strain history shown on the left find out what is the stress history of the material. So, I hope you understood the question that there is a material which is given to you.

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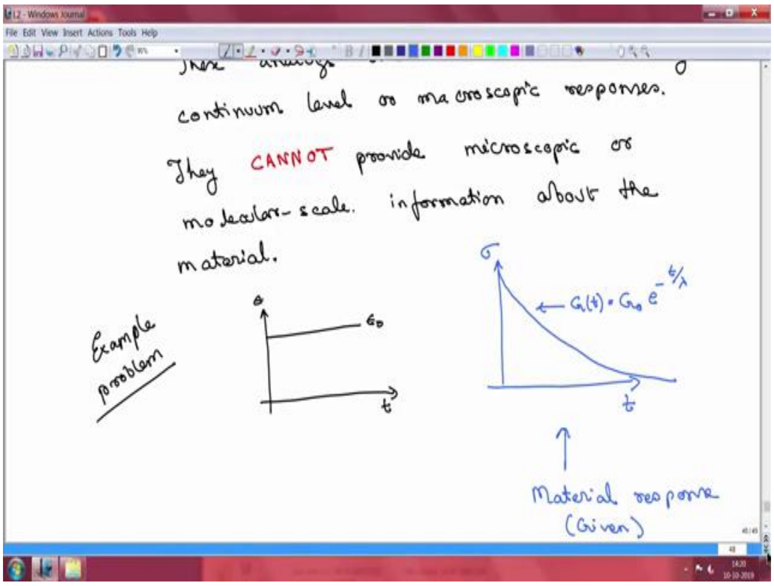

The material response is already shown to you, and this has given to a strep strain test.

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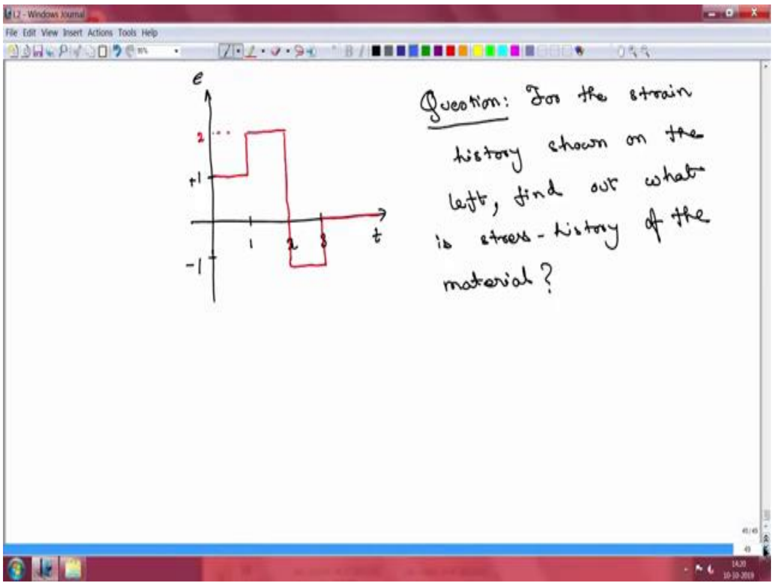

And now the same material is being subjected to this strain history, which is drawn on, on the left and you are being asked to find out the stress history for this case. So, we will stop here today. And we will continue to see in the next class what we will see is we will work out the solution to this particular case and a couple of more examples, problems. Okay. So, what we learned in today's lecture was, we looked at aging materials. And we went over the difference between a non-aging material and aging material.

And then we saw that the elastic body, the classical elastic, and the classical viscous fluid behavior, allows us to use two constructs the spring and the dashpot to model the macroscopic response of the viscoelastic materials. And then we are working out this example problem and we will see the solution in the next class for this. So, we will end the class here.