Introduction to Soft Matter

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Lecture 08

Linearity

Welcome back to one more lecture on Introduction to Soft Matter. So, last time what we were looking at was the idea of linearity of scaling. And we discussed three important methods by which you can check linearity of scaling. And we looked at linearity, the implications of linearity of scaling for both the creep phenomena and the stress relaxation phenomena.

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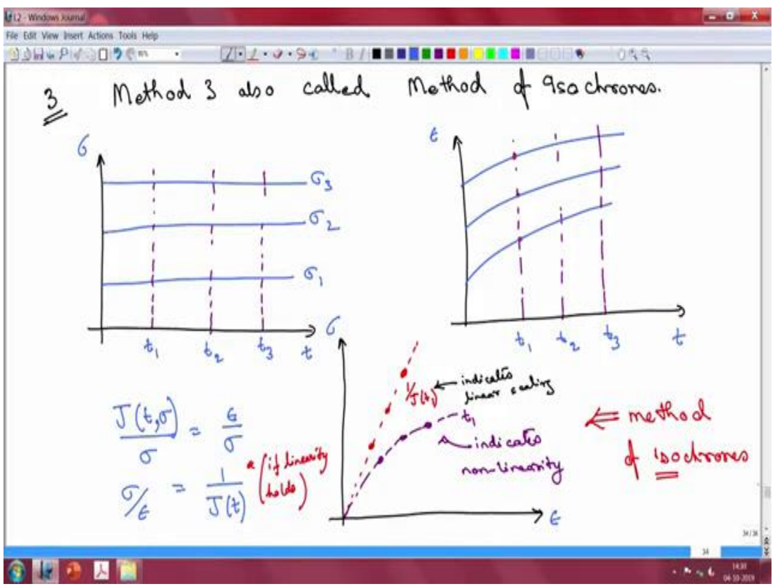

So, remember we had discussed that three different methods. The method of isochrones. So, this method of isochrones that we had applied to the creep phenomena, let us go ahead and do the same thing for the stress relaxation phenomena. So, instead of discussing all three methods, we are just going to discuss that one particular method.

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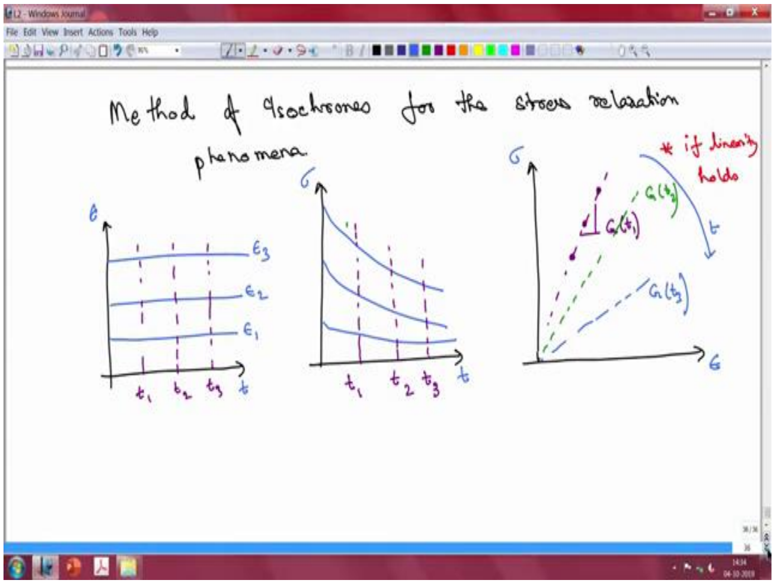

So, let us look at method of isochrones for the stress relaxation phenomena. So, here in the experiment, what you are doing is. First draw three graphs. So, here you had started off an experiment by having this strain as they put conditions right. Epsilon 1, Epsilon 2 and 3. And then you are measuring the stress as a function of time. And once again since you are limited by the rate at which you are sampling data.

Let us say you are taking data at three different points. You can take it. Obviously you are not going to take only three points. But for the simplicity of discussion. Let us say t1, t2 and t3 are your sampling times. Your curves look something like this. So, here again your, your sampling, your sampling, your data. At the same times. So, your input and output are being sampled at the same time and what you are going to do is you are going to plot. So, what are you doing here? You are plotting here stress versus strain relationship.

And you want to see how that looks. So, linearity holds. You will again in this case by a similar argument to last time you find a straight line. So, so if linearity holds what you will have is a set of data points that will give you a straight line. And this slope is now given by G of t1. So, again let us just quickly mark this as if the linearity holds. Similarly, if you have at another time if you plot you will get it as G t2 and some other time you will get another straight line this is G t3. And the time Axis is this way. So, as time increases your graph slope decreases over time. It is easier to visualize this.

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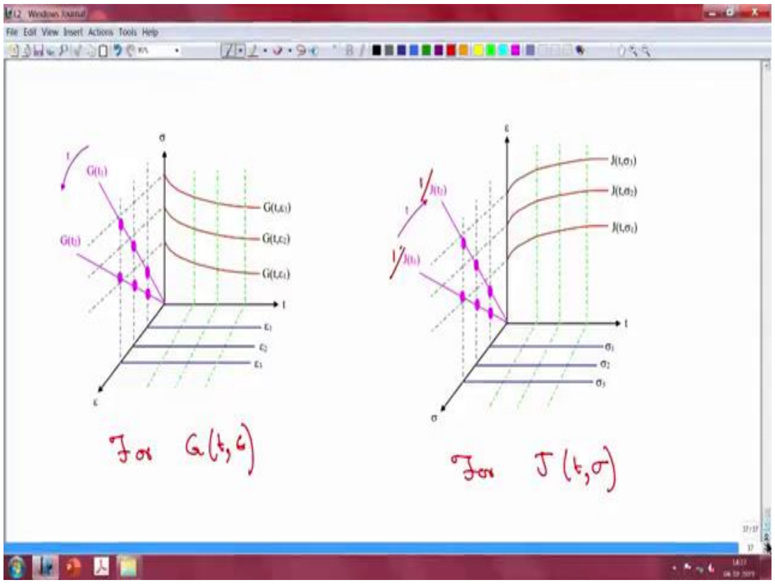

So, what we have done is for this phenomena, we have made very nice three dimensional graph. Which explains what we are doing. So, what we have done is we have made a three dimensional plot showing the same thing that we just discussed. In the bottom the epsilon t plane is the input plane and that is being sampled. Your sigma t plane is the output plane and that is where you are recording your output and that is also being sampled.

And then you are try to get rid of the time access altogether and you are just plotting it. There is Sigma Epsilon graphs and this is how they will look like. Obviously, you have more than three points, right? I mean, that is just for the simplicity. Otherwise the figure is already crowded, otherwise it will become too crowded. But three points are not enough to determine a straight line. So, you need more points there. So this is and if linearity does not hold, then you will not get these straight lines. So, this is your for G of t, comma epsilon.

Similarly, you can do the same thing for the case of the creep phenomenon. And in this case, what do you end up again is your bottom plane the sigma t plane. That is your input plane. And then you are measuring your output on the epsilon t plane. And depending on. So, here by the way, just let us I will add that this the slope is 1 by. Slope is 1 by G t1, 1 by G t2. So, this is for. For the two cases.

So, with this we have discussed the first point that we had raised last time, which was the linearity of scaling. And we had said that the material is linear. If it satisfies two aspects, which is another aspect was the superposition of responses. So, what do we mean there? So, we need to. So, the mean itself gives you a good idea of what the superposition of separate responses probably means.

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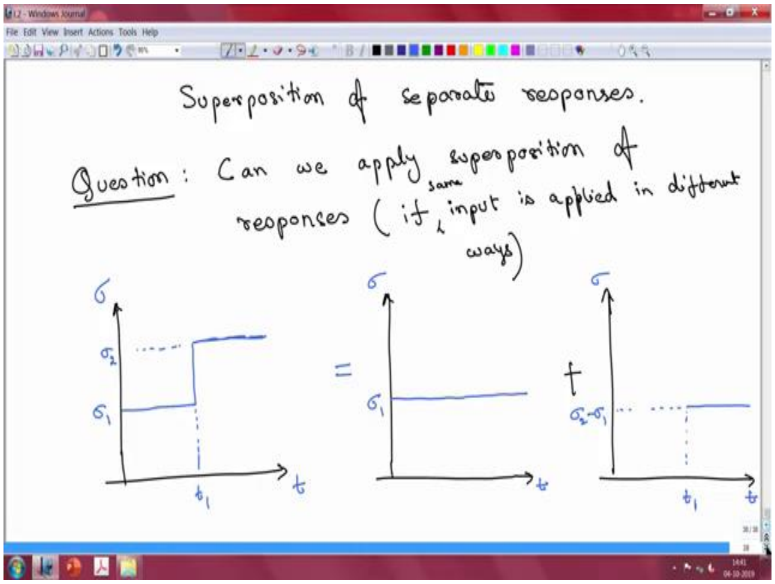

What we are going look at is super position of separate responses. So, the question we are trying to ask here is actually very simple. So, if you do parts of the experiment separately can you superpose the responses also? So, can we apply, can we apply superposition of responses, always, responses if input or rather we just put in a bracket if input is same input is applied in different ways. So, what do we mean by that?

We mean, so let us illustrate this with an example. We are doing let us say a stress response test and you have you your input. Initially is at sigma 1. And then later on jumps to another value. This value now is sigma 2 at some time t1. This can be thought of as being equivalent to two cases. We thought of we can actually make it in many ways. But one of the ways in which we can break up this input is that you have an input of sigma 1 for all times. And then at suddenly time t1 you apply another input signal. Which is now sigma 2 and sigma 1.

So, if I were to draw the responses or the response to my graph on the left hand side. Then can I say the response of the left hand side will also be equivalent to the two separate responses I did together simply, that is what the superposition of responses is asking you.

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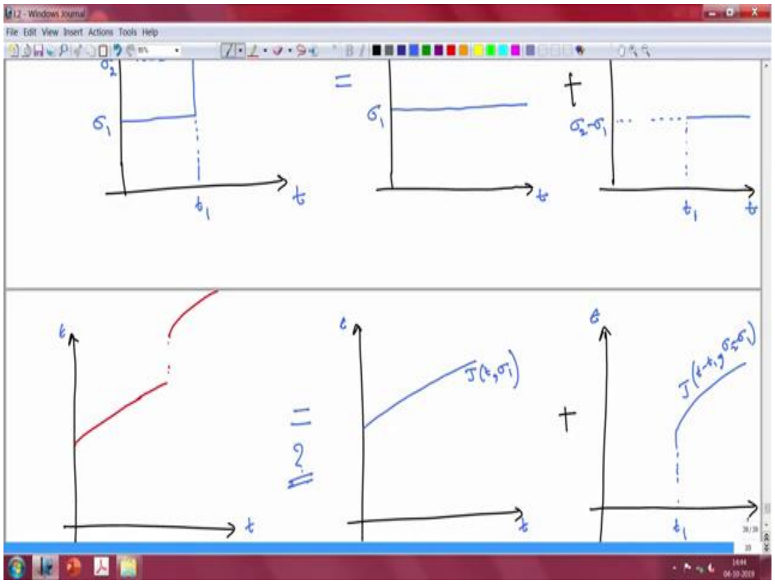

So, this is input plane. What it is asking you is whether I will just again draw three graphs. In first case you have the input to the signal Sigma 1. So, this is your J t, sigma 1, right. And in the second case is to quickly go back. I mean I just right like this. So, we will see that the second signal is Sigma 2 and Sigma 1 and is being applied at a time t1. So, till t1 what is your response is going to be 0. We have discussed that point earlier. Remember that. So, this is a t1.

So, now you are going to have a new response. So, the response to the separate function is simply J sigma 2 minus Sigma 1, comma t minus t1. Sorry. I think I flipped it. This is actually t minus t1, sigma 2 minus sigma 1. So, what is the question of super position of responses is very simple. It is saying that if this were your original input signal is the one in the left hand side, then can I simply add these two signals on my right hand side and get the response to this more complicated signal?

So, your graph will look something like you will have. And then there will be a break and then you will have this other curve. So, can we write this or not? And this is your question, so superposition of responses, superposition of separate responses says that no matter what kind of a complicated input signal you might wish to provide. If you break it up into its constituents, or break it up into separate signals. Then the final output is just the sum of the separate responses.

Seems like a very simple idea it is and you would expect that the separate operation of separate responses should always be true, but there is no such rule which requires that to happen, just like in the case of linearity of scaling there is you just want to hope that your material is going to obey that because otherwise life is going to be difficult.

So, even in this case you just hope that the super position of linear responses holds and sorry super position of separate responses holds and that linearity of scaling also holds and then life is easier. Let us just see one more case. So, let us see one more example.

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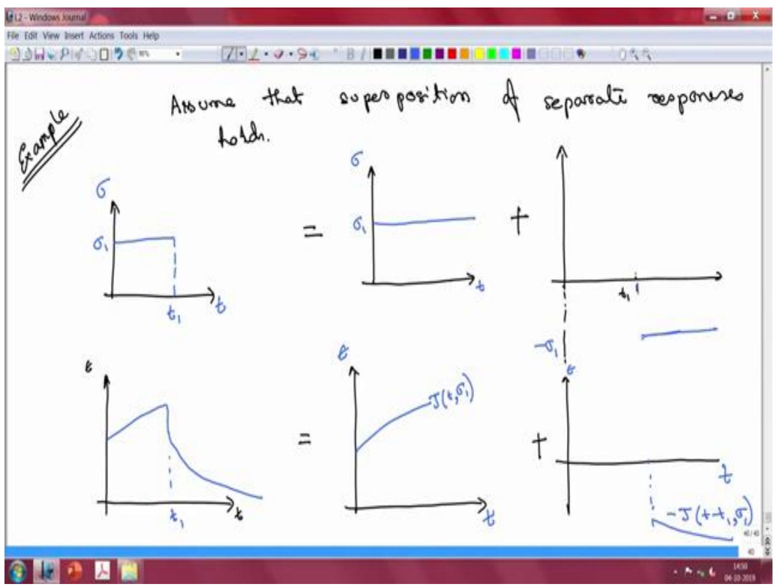

So, let us say this is maybe I will just use a new page. So, let us take a look at an example problem. So, we will assume that the superposition. So, assume will begin it assume that superposition of separate responses holds. So, what we want to do on the input side, let us say we have a situation like this to break this up. We have a signal which starts at some value sigma 1. These are horizontal lines by the way it is a horizontal line and then a t1 it goes to 0.

And you have been asked to calculate what is a response to this. I deliberately chosen an example that you might recollect we have already know the answer to this, but we let us break up this problem. So, let us say that you decided to break it up into two separate signals. One which is that sigma 1 and it remains, this signal remains at Sigma 1 forever.

And then this is added to another signal. Let us say this is your time t1. And at t1 you impose the signal of minus sigma 1. So, we see that the, this equality is correct. So, now what happens? The question is what is going to happen here? This is here. So, second graph I just put the origin at a slightly higher position because I can draw. So, this so we know the response to this. And that is simply the curve. J t, comma sigma 1. So, let us say that is available to you.

For the other case. You now also have a signal, a response which is negative of, of these two. So, your final response should be sum of these two. If linearity holds. So, what you will end up here, this is let say t1. And a graph like this, have you seen this graph before? We have seen this when we are discussing the creep and the creep relax, creep release test. So, this is your behaviour of your classical. The typical response of a viscoelastic material.

Now, I have only broken up. So, quick point before we draw an important conclusion and that is I have only broken it up into two different signals here, okay. But you there are many different multitude. You can break this signal up in this fashion in many ways. And the superposition of separate response is if it is to hold, must hold for all the different cases that you can envisage. So, you must understand that what I am drawing here are just examples. But it must hold for all different cases.

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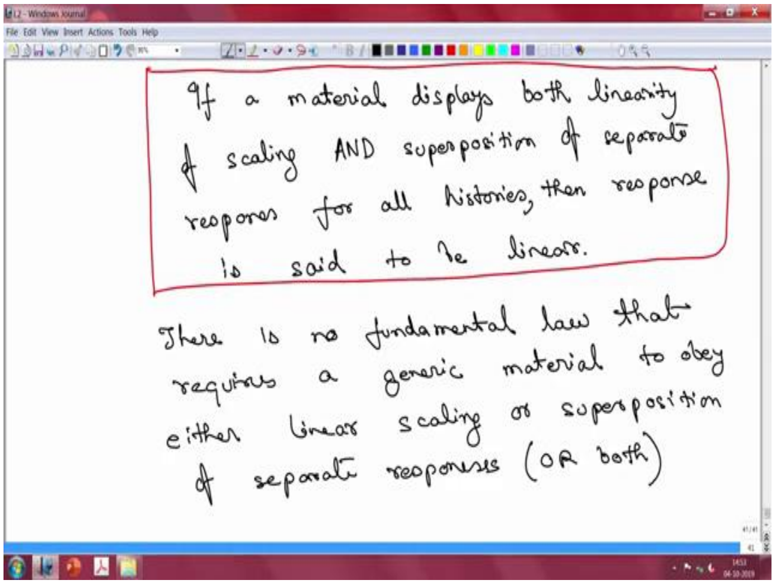

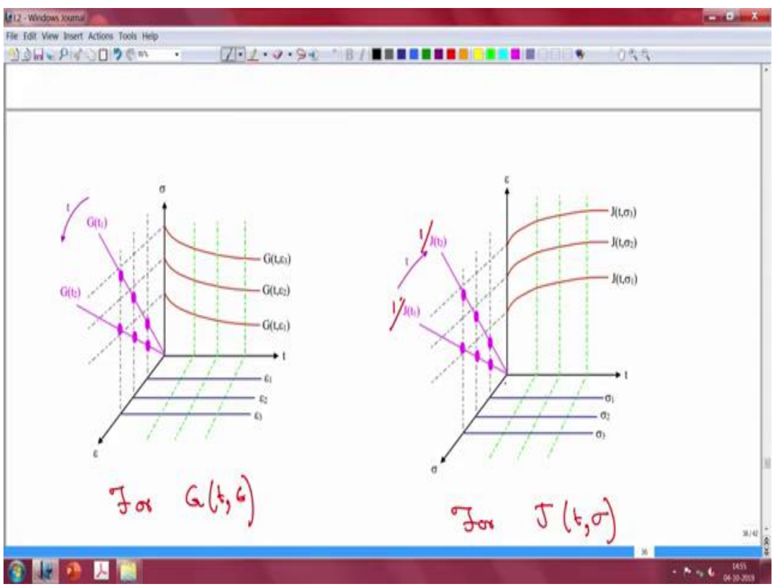

So, now we can. We will say that if, if a material displays both linearity of scaling and superposition of separate responses for all histories. Then response is said to be linear. This is important criteria for us. And we will also note that there is no fundamental law that requires a generic material to obey either linear scaling or superposition of responses, superposition of separate responses.

Or both obviously, if there is no law which requires it to hold for either one, then it is even more impossible that it is going to hold for the other one. So, this is a very important idea and we can see that if a material has to behave in a nonlinear fashion now it has many different ways in which it can behave.

So, we already saw that the linearity of scaling can have at least two different aspects. One, where there is a critical stress above which it behaves as a linear or non-linear. It is also time of application of the stress above and below which it can be linear or nonlinear. Now, the obvious question at this stage is that if materials, so why are we saying. Why is this question of super position arising? Is it because materials have memory. So, is the materials have memory can, we can ask them that they might hold information about the creation also how they were made.

And if that is the case. Then you might have an edge that you can associate with the material when it was created. There is one more point which is in all our previous discussion. Remember whenever we are discussing all these different graphs et cetera what we are always assuming is that when you are applying a certain amount of stress or a certain amount of strain to the material, the material is in some native form or some, exist in some datum condition such that there is no previous stress history that it remembers.

Because if there is a previous stress history that it is remembering then that would also come into picture here. So, when we say t equal to 0, we are assuming that the material is stress free or strain free. There is no residual stress in the material.

So, the material does not have any edge. You can just come in and take the material maybe it is lying on your experimental shelf and you can just start doing that experiment. So, that is another very important condition.

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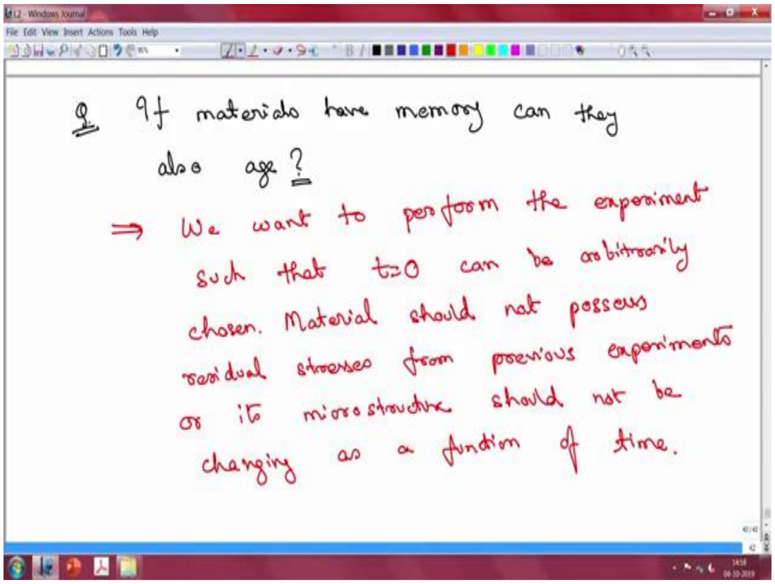

So, so here our question is and the implication of that is whether t be equal to 0 can be chosen arbitrarily or not. So, we are trying to wonder, if materials have memory can they also age? So, there is again no fundamental law that says that they cannot age, okay. But we want to select only material we want to work with materials whose response to the step stress does not depend on when the experiment is done.

So, we want to choose. What we want to do is we want to perform the experiment. Such that t equal to 0 can be arbitrarily chosen. So, materials should not have, should not possess residual stresses from previous experiments or its microstructure, its essential microstructure should not be changing as a function of time. This need not always hold true.

So, there are many materials for which this may or may not hold true or for materials this proposition that they should not have a memory of the previous. You know in a sense their previous life they should not remember or they should not remember when they were created. This is something that you can only determine from an experiment whether this is true or not. So, what you can to do simply, okay. And we will discuss.

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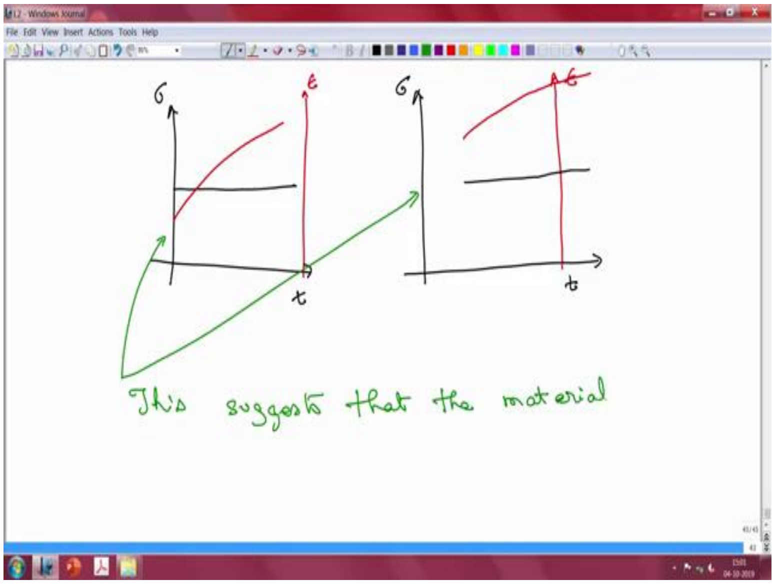

So, let us say you are in the lab. I am going to draw. I am just going to make my life simple. So, let us say this is sigma, this is the time axis and this is epsilon. So, let us say you apply a step stress test and you choose some time. Maybe that is the time you enter the lab and you are measuring your strain value and you will get a curve like this. You let. So, let us say you have 500 ml of that solution of the Scholastic fluid. And then you let your friend come in at a slightly later time and you divided a material into two halves, one for your experiment and for friends experiment.

So, he starts his experiment at a slightly later time but in his case. So, he applies the same step stress test, but here for him graph see this intercept the Y intercept in the two cases I have changed so drawing. The response of the same material at two different times and this suggests so this change here. This suggests that the material has aged. So, today what we discussed or we discuss the issue of the linear scaling. And then we also discussed the issue of the superposition of separate responses. And we saw what it actually implies, okay.