Introduction to Soft Matter
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Lecture 07
Creep and stress relaxation functions

So, welcome back to another lecture on Introduction to Soft Matter. So, last time what we were discussing was the idea of creep, the creep phenomena and the stress relaxation phenomena. And what we saw was that if you are an experimentalist and you are going to the lab and you are doing some experiments, you will probably end up with different people who, if they go to the lab they will end up with different creep curves.

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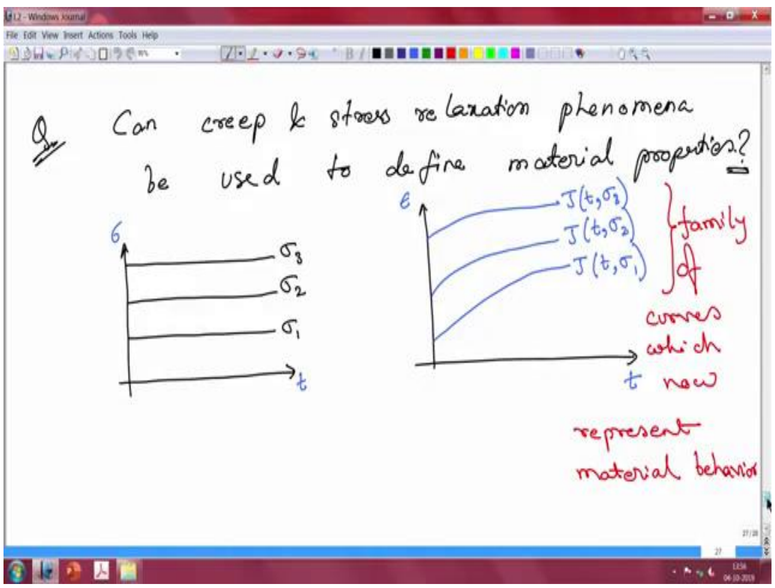

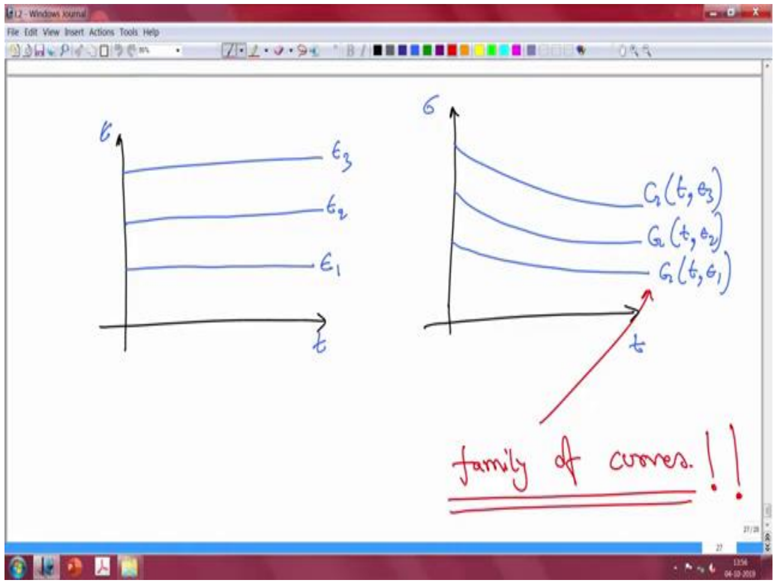

And this family of curves is what represents the material property now. And then if you similarly do set of experiments for the stress, (relaxa) the stress relaxation phenomena, then you will similarly see a set of curves, which are again, representing some kind of material property.

So, this is very interesting and very helpful, but we want to ask that question just like in previous in solid mechanics, what you might have seen that there is a single curve, which can represent the entire system. So, we want to ask that same question all over again. Is that can we simply this situation little bit? And can we end up with a system where we can define the material properties by a single curve rather than a family of curves?

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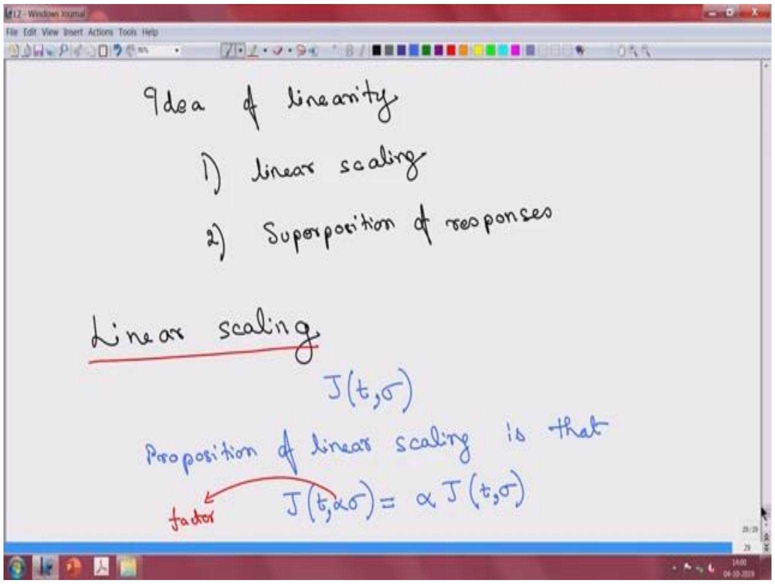

So, here, there is a important concept that is going to be important and that is a concept of linearity. So, we are going to introduce this concept and this so I am just going to say that before that what we will do is so the idea of linearity involves two concepts, one is the idea of, second, sorry, is the idea of linear scaling. And then the other is and the other is called the super position of responses. These two ideas has different ideas.

So, let us first take up the idea of linear scaling. So, first, we will attack this problem, the idea of linear scaling. So, we ended up so we have this function, which is a function of two variables, time and stress. So, the question we will ask here is can we reduce this to a function of a single variable? So, here, the proposition of linear scaling is, the proposition of linear scaling is that when you multiply the stress input by a factor of alpha.

So, this is so let us say this is a factor, multiplying factor. Then your response changes proportionally. So, what will what should happen here is that it should become alpha times of J, t comma sigma. So, is this so by the way, this is a proposition, this need not hold true. That is why we are saying that it is a proposition.

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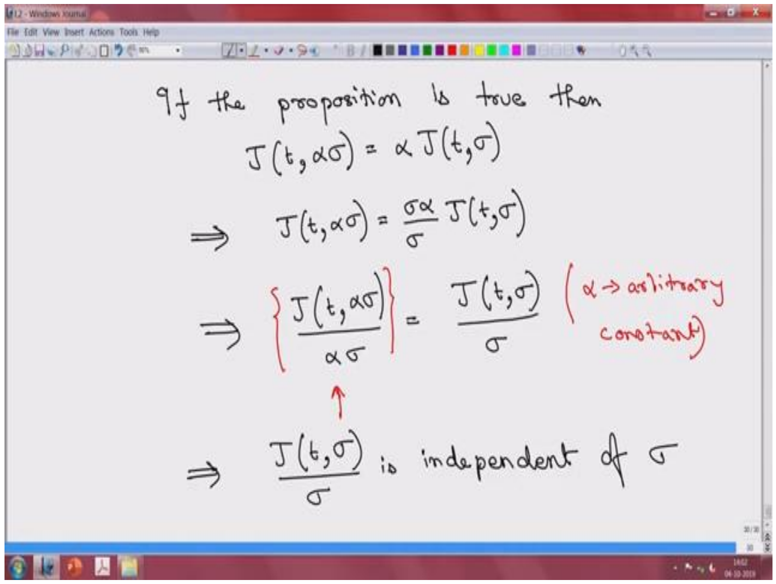

So, if this holds true, so if, so if the proposition is true, then so we had written J times of T alpha sigma to alpha times, which implies that J times is equal to, if I am just going to multiply here, both of them sorry, denominator should be sigma. I am just going to introduce this. So, now, if I bring alpha sigma over onto this side. You might have seen this kind of relationship before.

So, we have a function of two variables and what we are claiming is that when you divide so you see, this function J is being divided by the value of the second variable. And what this claims is, so this alpha is an arbitrary constant. So alpha, let me just make a quick note, alpha is an arbitrary constant. So, for this to hold, this, this thing, this functional form should be independent of sigma. So this implies that J, t of sigma by sigma is independent of sigma, or it is basically a function of only one variable a time.

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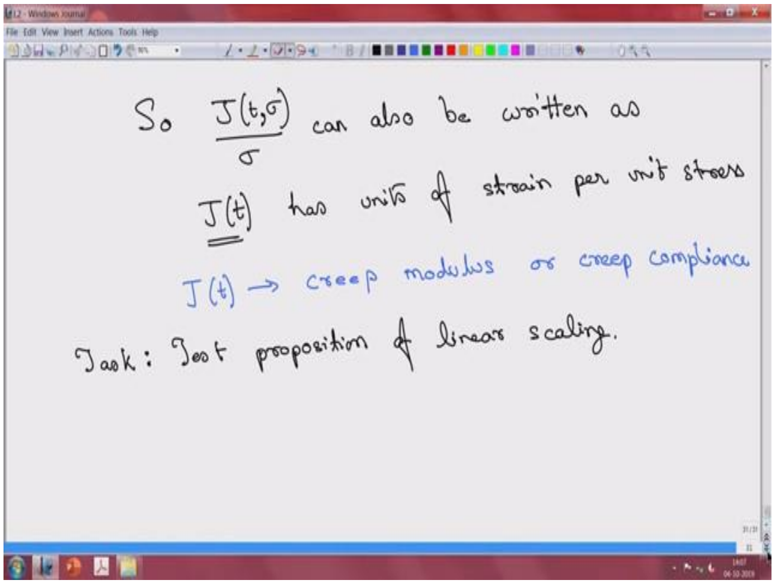

So J, T comma sigma by sigma can also be written as some function which is just a function of time. I have chosen the same variable, J, to represent the function just to keep it easy. And this now has so this J, T has the units of the inverse of the stress. The strain divided by stress, the strain is of strain per unit stress. So, the strain is damaged unless basically the inverse of stress.

So, basically, this now can represent some kind of material property. And where this linearity of scaling holds, there we can define J, T and this J, T is now called the creep modulus, or creep compliance. So, we will just go with the word modulus here.

So, this is quite interesting that we started with a function of two variables, but we ended up by applying a proposition and we are saying that if the proposition holds, if the proposition holds, then you can define a creep modulus, which is now just a function of time. So, see, how different this is from your original classical elastic solid in the sense that you had to apply another proposition to get here, to find a creep modulus which is not a family of curves but just single curve. But it is still the modulus is still a function of time.

So, which means that for a viscoelastic material, the modulus, even if it satisfies that proposition of linear scaling, it will still have a modulus which is going to be a function of time. This is a key difference with your classical elastic solid. It is also a key difference with your classical viscose fluid in the sense that it is now being defined by a parameter, which is a function of time.

So, now, now that we have this proposition, so we are if you had noticed that we are discussing this entire course from the perspective of an experimentalist. We said that there is something called a viscoelastic material, but how do we test this? How do we know what a viscoelastic material is? So, we propose six different experimental tests and those experimental tests are something you can do in the laboratory. When you do it, then you can figure out where or how viscoelastic the material is.

So, we are looking at this entire thing from that perspective. So, let us carry on with that. So, now that we have a proposition and we are seeing that the proposition is probably going to be very helpful in our case, we must also device experimental methods for checking the proposition. So, let us say, testing this, so the next task, so the task at hand is test proposition of linear scaling. So, let us start with method 1. I will just, let me just go to a new fresh page that is.

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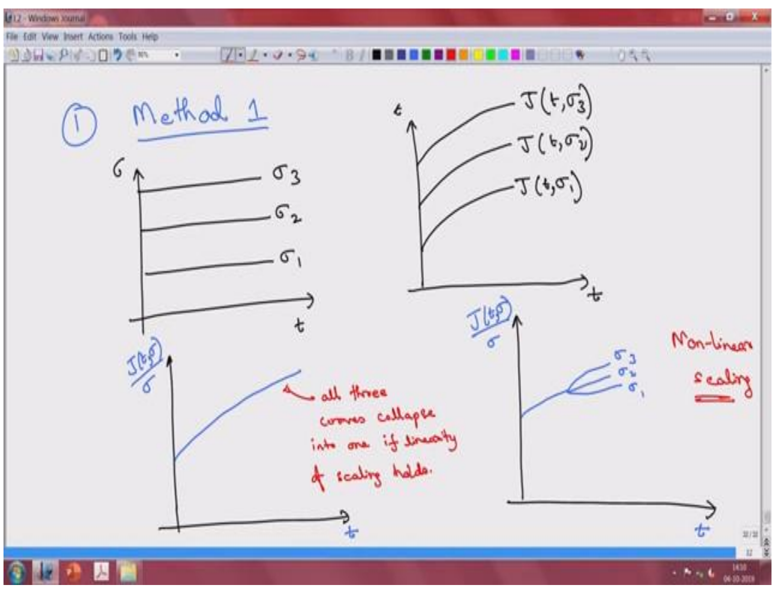

Okay. So, test 1 or method 1, I will just say this is method 1. So, what were we doing earlier, where we had a function, a family of curves? We were applying different values of stress and then we are recording the response. And when we are doing the, when we are recording the response, what we found is we end up with these family of curves. So, if we want to test linearity, rather than plotting the strain here, we should probably plot a different type of curve.

Let us now plot this functional form that we had seen. So, now, if it is just a function of time and not a function of the stress values, then what do you think will happen to these three different curves? They will collapse into one. So, you will end up getting only one curve. The same so we will just note it make a note of this.

All three curves collapse into one if linearity of scaling holds. Please also make a note that I am saying linearity of scaling every time and not just linearity, because the idea of linearity has two this another component to it. So, you will end up getting only one curve. But what if this linearity does not hold? What will then your world look like?

So, in that case if you plot this, then your maybe in the beginning the three will be similar, but after some time they can start diverging. So, maybe this is sigma 1, maybe this is your sigma 2, and this is 3. So, this is how, if you have a non-linear scaling situation, then your graph looks different. So, this was method 1, which was easy.

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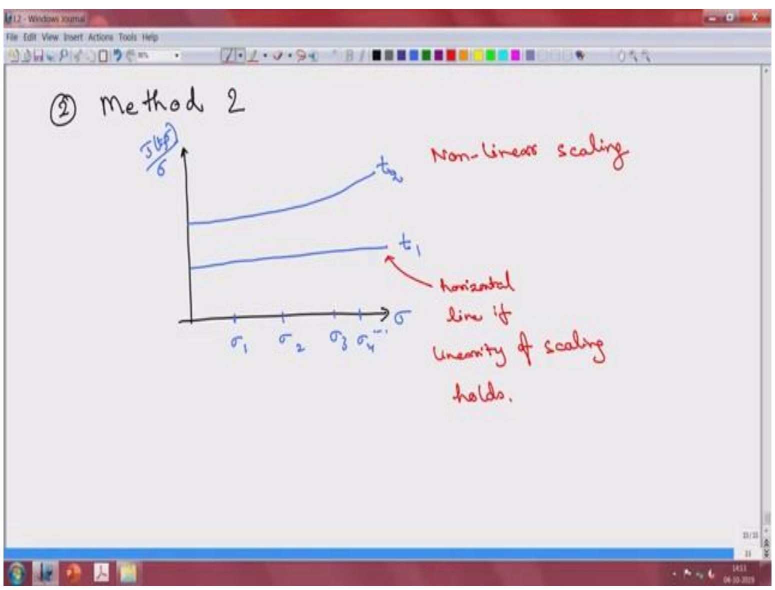

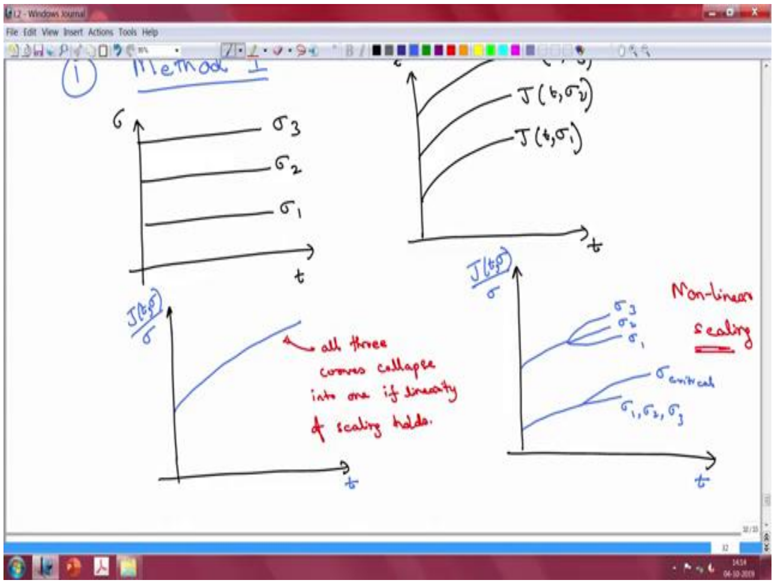

There is another method, and here so when we say this variable is independent of sigma, then why do not we plot this, so why do not we make instead now a plot? On this axis, let us put this. And here this is your stress axis. So, if you are doing these experiments at different levels, etc, etc. Then what you will find is, this is only going to be a function of time.

So, at a given time, if you had to plot the response so this is at maybe T1, it is a straight line by the way. So, I mean, I just wanted to draw horizontal line. So, horizontal line, if linearity holds, linearity of scaling holds. But maybe at a later time, it is not linear. So, in that case, you will end up getting something which will not be constant. So, maybe this is at time, T2, and this is your case of the non-linear scaling.

Why did we discuss two different methods? Just quickly, let us go back for a second and understand that when we say linearity of scaling, it is a very restrictive condition. You are asking whether a material will satisfy this condition or not. And when it does, the thing the graphs will look simple and the world will become a little bit simpler for us. But if it does not hold, if the non-linearity is there, then the non-linearity can appear in multiple ways.

It can be that the non-linearity occurs at certain stress levels or there is a critical stress level over which non-linearity comes in, in which case your graph here. You might be doing an experiment such that for many different values, you will keep getting a linearity linear the linearity will hold, but above a certain critical value, non-linearity will set it.

The second method demonstrates that you might have linearity for an initial period of time. There might be, just to say maybe for 10 minutes of the test, so 10 minutes is very large though in some cases. So, maybe for 5 seconds, linearity holds and after that non-linearity comes in. So, the way in which non-linearity can come in, can come in multiple ways.

So, that is why we discussed two different methods. But in reality, there is one other method called the method of isochrones, which is very popular, and it is more suited for experimental the way we collect experimental data, it is more suited for that. So, let us discuss that too.

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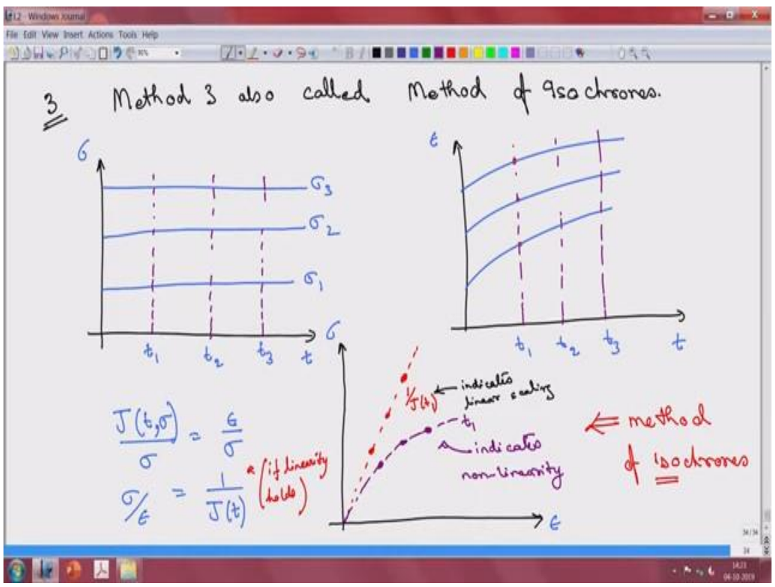

So, this is method 3, also called method of isochrones. Why is it called isochrones? Well, it will be very clear once we start discussing the material itself. By the way just to make a note, all these tests are isothermal in nature. So, we are not discussing temperature, so you might have noticed that temperature is something that we have not discussed till now and we do not intend to discuss this, this is a introductory course.

So, we are just trying to get familiar with some of the first important concepts of this and temperature is another important aspect, but we will not discuss this because this is a introductory course. We are holding that out.

So, in the method of isochrones and I would clarify why the name comes in a second (befor), once I explain. So, you are in the lab and you are going to collect data and you are applying input and you will collect, in sample, your input at certain frequency. So, let us say, we just simplify our life a little bit. So, let us say, you are applying sigma 1, somebody else is applying sigma 2, somebody else is applying sigma 3, these different conditions. And we are sampling this at different times. Actually, ek second, let me just say.

So, you are sampling your input at some t1 time, t2 time, t3 time. Similarly, this is the input in your experiment. Similarly, in the output what are you doing? You are measuring straight this function of time and you have these different curves. So, let us say you are sampling again at the same time, so you have a system which is sampling the input and the output at the same time. So, here.

So, now just like in the case of what we want to do is we want to construct a diagram where just like in the previous case of the solid mechanical systems, we had sigma and epsilon. So, we want to plot the stress strain relationship for these cases. So, what you will find is that you will end up getting three data points. I am just going to so if you have a linearity of scaling, if you sample and plot your responses at a given time.

We know that the J, the response, so just quickly going back to that. So, what we had done is we said so sigma by epsilon in this case is your sorry, just a second, so this J, T comma sigma by sigma is basically your epsilon by sigma. So, now if this is just a function of time, if linearity holds, then when you are plotting epsilon by sigma or sigma by epsilon, whichever in this case are, then that is just if linearity holds, so linearity holds and plus for space so I did not write linearity of scaling.

So, at a given time, the slope is constant and you are plotting sigma by epsilon. So, if linearity holds, you should get a straight line where we have put all your data points. Such that the slope of this is given by 1 of J at t1. And similarly, for the other times. So, this method, so here, so this method is called the method of isochrones, because iso means the same, chrones is time. So, this is indicating that what you are doing is you are just sampling the different values at different times, sorry, the sampling values at the same time and we are drawing a curve through it.

So, this really indicate a linear scaling. So, this indicates so if you get a straight line, it indicates linear scaling. But what if you do not have a linear scaling? What will your graph look like?

Your graph will then look like, it will not be a straight line. So, if your graph okay, maybe I will just use a different color so that.

So, maybe in this case, if you are doing an (exp) if your linearity does not hold, maybe your graph looks something like this. And this is still the sampling data points from your time T1, so this is your T1 curve. So, this kind of a curve indicates non-linearity. And I apologize, this graph has become a bit crowded, but to discuss all these that is what we have to unfortunately, there is no other way. So, these are the three methods.

Now, quickly before we finish this part of the lecture, we discussed two important concepts. One was creep and the other was stress relaxation. So, what about stress relaxation? We have not discussed that yet. So, what happens there?

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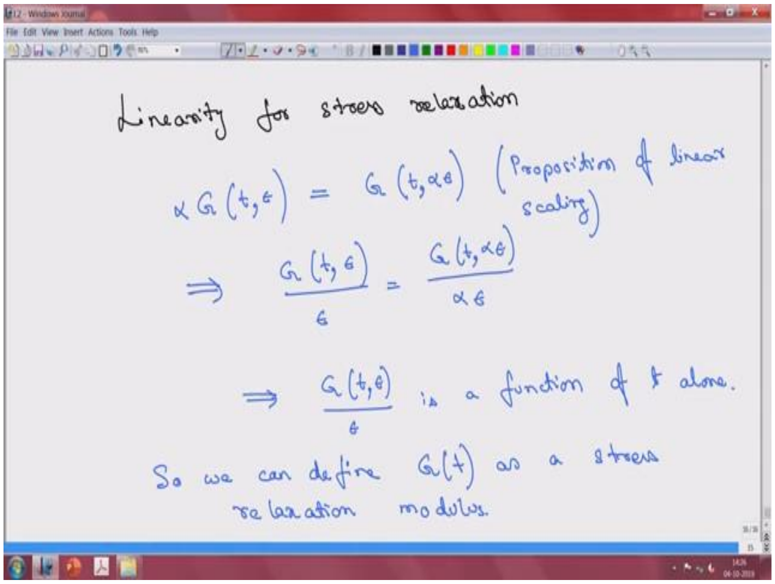

So, for stress relaxation, so linearity for stress relaxation. Let us. So, here we have G, which was a function of t times of epsilon and linearity the proposition will say that if you have another case where you multiply your epsilon by alpha, this should also scale. And just like we did it into the previous time, we see that this implies so this is proposition of linear scaling.

So, again we have the same or a similar equation that we end up with, which implies that if linear scaling holds then we have this G t by epsilon is a function of t alone. So, we can define G t as a stress relaxation modulus.

So, what we did today is we had started out earlier with the idea, with getting ourselves familiar with the ideas of the creep relaxation phenomena and the stress relaxation phenomena. And we took this idea a little bit further today and we introduce a proposition of the linear scaling and we saw what the linear scaling implies. We also looked at, if you want to test the idea of linear scaling, whether the proposition holds to or not, and we did not say by the way that the proposition has to hold true. It might or might not hold true. So, both cases are possible.

So, if both cases are possible, you must first test whether linearity holds. And to do that, we identified three different methods, method 1, method 2 and method 3, and the final there was a method of isochrones. As methods that you can use in an experimental setting to decide whether linearity, the proposition is really true or not.

And it also gives you an idea of when linearity breaks. So, those are three methods, are not just for you to test whether linearity holds, but they also give you an idea of when linearity breaks. So we will take up some of the other following concepts next time. Thanks. So, we will stop here today.