Introduction to Soft Matter
Professor Aloke Kumar
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru
Lecture 38
Sinusoidal oscillations

So, welcome back to one more lecture on Introduction to Soft Matter. As we had discussed, we were discussing last time. We were discussing objectivity and its impact on the Cauchy stress tensor. And we had seen that given a general any general frame invariant tensor, let us say t, the time derivative of t is not going to be objective.

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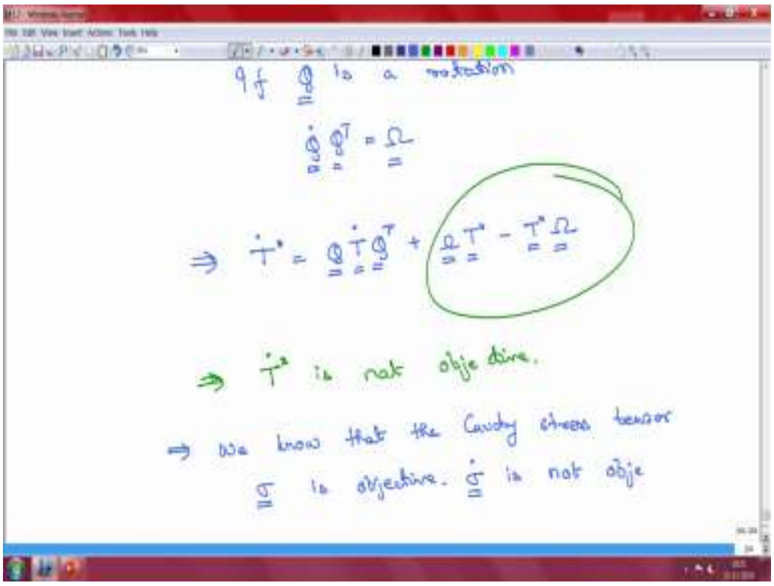

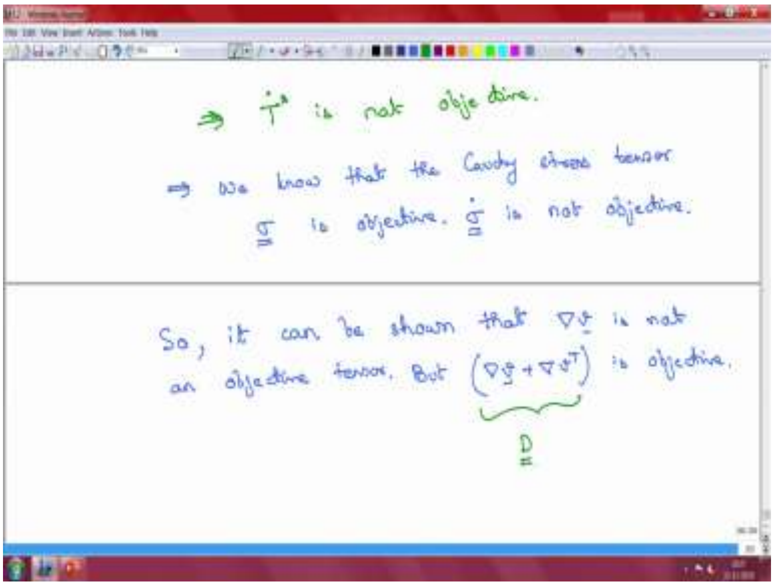

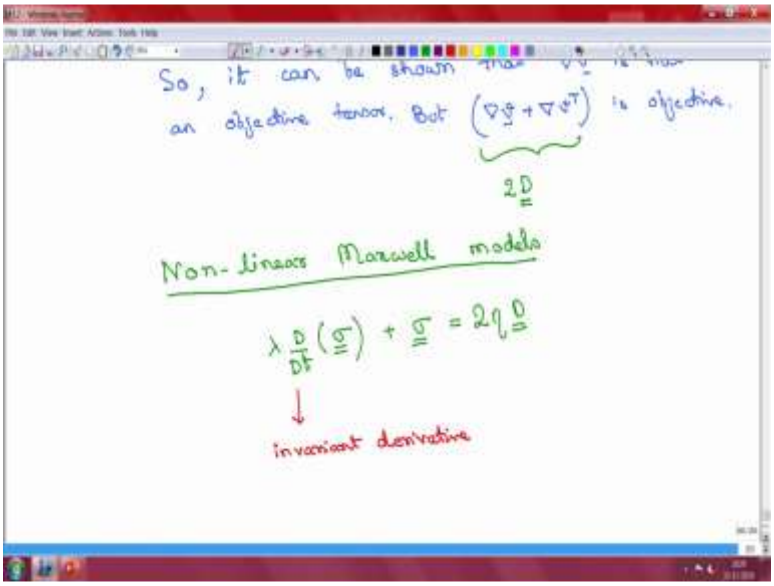

So, here basically the reason we were doing this is because we had already said, so we know that the Cauchy stress tensor is objective. But what we proved earlier, it implies that the time derivative of this is not objective, is not objective.

Similarly, so we also saw that the original governing equation had an epsilon dot term and what happens is that the gradient of the velocity field is also not objective. We are going to skip some of those calculations. So, I will just mention that and we are going to today just look at the consequences of all this. So, it can be shown that the gradient of velocity is not an objective tensor but v plus, so the gradient of v plus, the gradient of v transpose is objective.

So, in the actually the Navier Stokes equation, the gradient of v this quantity usually, almost all books would call this capital D. The Navier Stokes equation is given in form of this capital D. And now you can probably appreciate why because the gradient you cannot just take the gradient of velocity because that is not an objective field.

There are other it also comes because of other reasons which converge to the same idea. But this quantity is objective and to create a good working a constitutive relationship let us say for Eulerian fluids or for an Eulerian system, you do want to use D.

So, what happens is that the Maxwell model can be generalized to other forms to what are called non-linear Maxwell models. So, non-linear Maxwell models and you can generalize them just by using the expression that we had used before okay, so we had some lambda, which is a relaxation time scalar characteristic time associated with this.

And then we had a derivative, so some D Dt here, so I am just going to write DDT right now of the stress tensor. So, now we are generalizing at c, so it is no longer 1 dimensional. So, that is why, we have to use the tensorial form plus the stress tensor itself into now, sorry, this is 2 D. So, this is 2, sorry 2 eta times D. So, instead of epsilon dot, we have to use this particular term.

So, this now has to be some invariant vector, I am sorry invariant derivative. And this is where the ideas of different types of invariant derivatives come in. And actually there are many-many different types of invariant derivatives. The purpose of this class is not to go into details and depths of invariant derivatives, remember that this is just an introductory class. So, we are sort of leaving going to leave you at a point such that this is where from where a more advanced course can begin.

So, the invariant derivative, there are many different forms and one of them for example, as an upper convected derivative also called the Oldroyd derivative.

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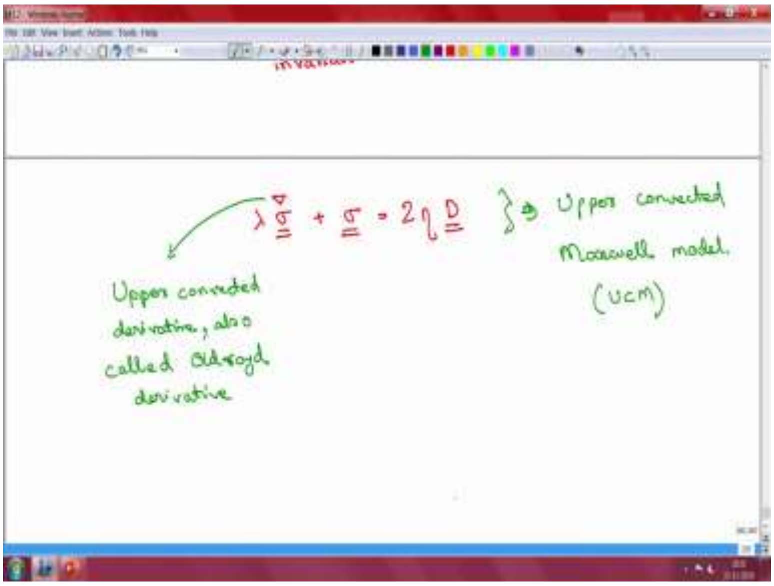

So, this equation can be is often written as lambda and then there is a triangle over it, which indicates an upper convected derivative and this indicates an upper convected. Thus giving us what is called, so this equation here is also known as the upper convected maximum model, maybe I will just write it separately. This is the upper convected derivative also called the Oldroyd derivative.

So, this triangle here, this marks upper converted derivative also called Oldroyd derivative. And this model is now my upper convected Maxwell model also often called UCM. Now, there are many different types of invariant derivatives and different forms will lead you to lower convected Maxwell models are also there, then there are other forms as well.

And if you try to do this for the Jeffrey's fluid, then you end up with a form which becomes what is also very famously the Oldroyd B fluid model. But the primary idea remains that you need objective tensors in order to be able to generalize, okay.

So, now, you can probably appreciate why we discuss this advanced idea of objectivity in this course and that is because in order to go from the simple 1 dimensional models into generalizations, you need derivatives which are appropriate the objective derivatives. So, this with this what we will do is we will end our discussion on the various Maxwell models and the constitutive equations.

Because from here we start on to something basically this is where we end for this particular part because from here on if you have to go on that you need to more advanced course. And I have not assumed that you have you have a very clear understanding of tensorial mathematics in order to go there. So, we are not proving some of these.

So, a lot of these things we touched very lightly and just so that you have a complete understanding of where this goes and what are some of the when you read manuscripts et cetera, you will probably come across the upper convected models Oldroyd B these terms you will probably come across in a quite often, so that is why we sort of end this particular section here, so that you have an understanding of that, okay.

So, that now with that brings us to the last part of our course, which is going to be the response to sinusoidal oscillations. So, if you remember we had discussed that viscoelasticity, there were 6 different tests, right, 2 of the last points the 5^{th} and the 6^{th} point concerned the work done by the material when it undergoes deformation and then the response to sinusoidal stress history or strain history. So, we will look at some of the some important results in that context.

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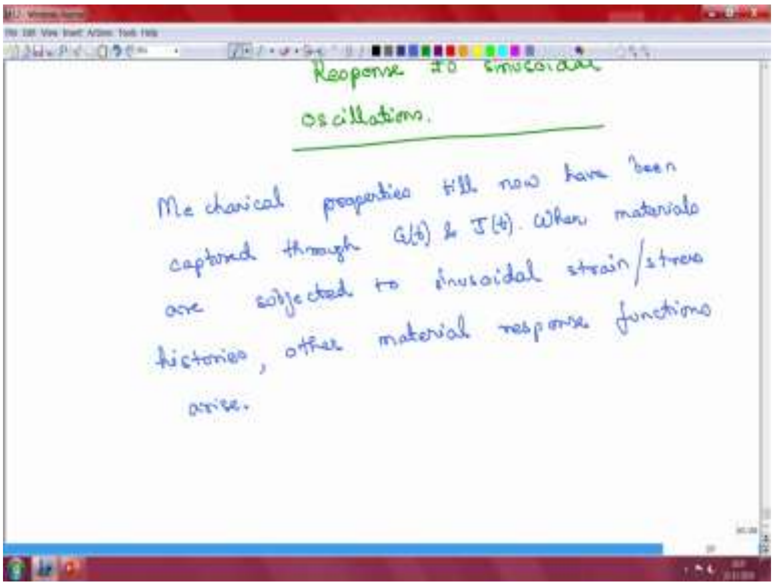

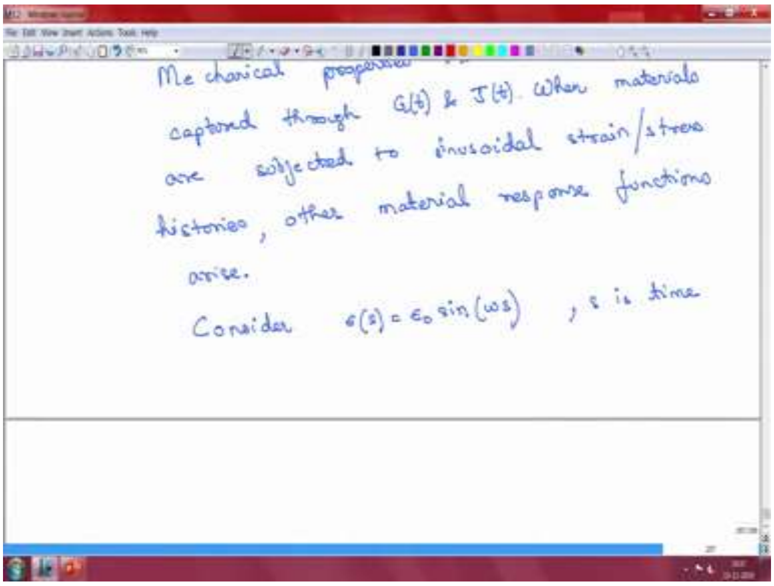

So, the last point or the last important chapter of our course is going to be response to sinusoidal oscillations. And we will see the mathematics here now becomes very simple, there is, we are not going to have any tensors etc. We just want to deal with integrals and again start to behave as the quantities involved are simple scalars.

So, you might recall that the mechanical properties in the beginning of this class we represented through 2 important functions, the stress relaxation function and the creep compliance, right. So, mechanical properties were till now have been captured through 2 very important functions Gt and Jt and these were material properties.

Obviously, all these were discussed for linear viscoelastic, right. Now, when what happens when materials are subjected to sinusoidal stress histories or strain histories, so when materials are subjected to sinusoidal strain or stress histories other materials response functions become important material response functions arise.

And these functions are related to Jt and Gt but their forms become very different from the initial looks. So, what we are going to consider is consider us an input of a sinusoidal strain, so consider a sinusoidal strain in time which is given by this very simple function epsilon naught sine of omega s and s is some, s is time. So, basically it starts from 0 and goes until infinity.

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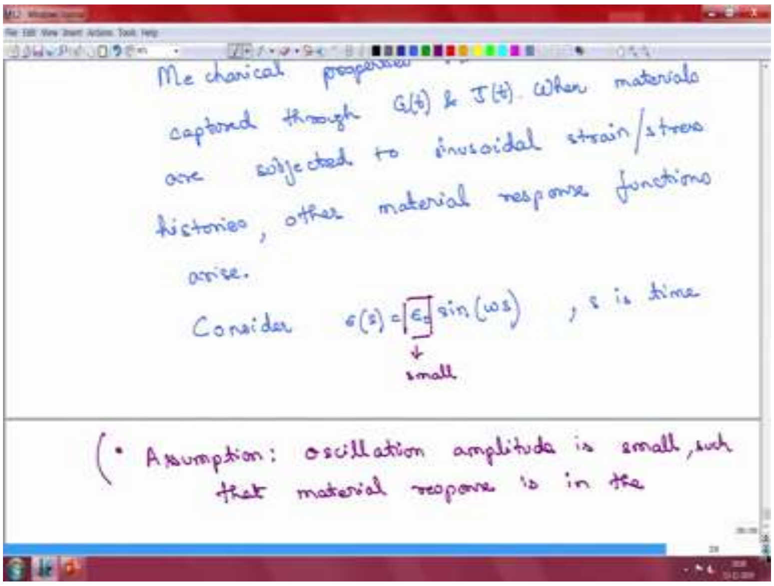

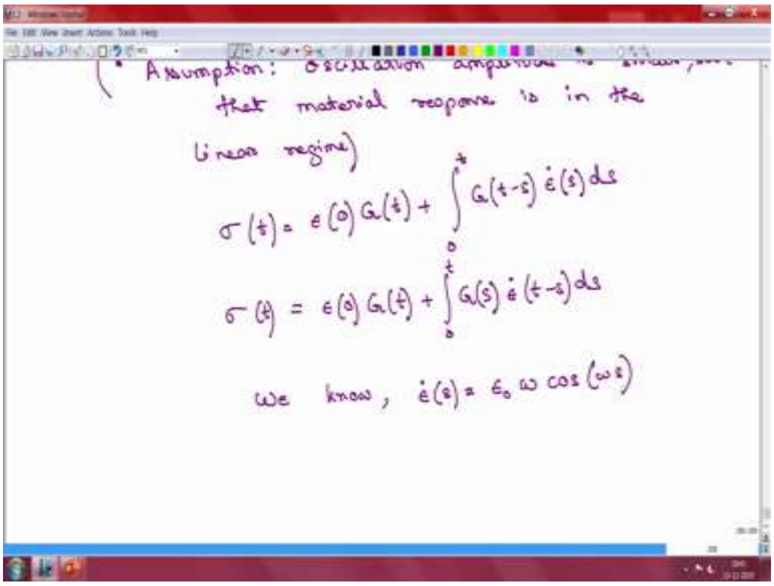

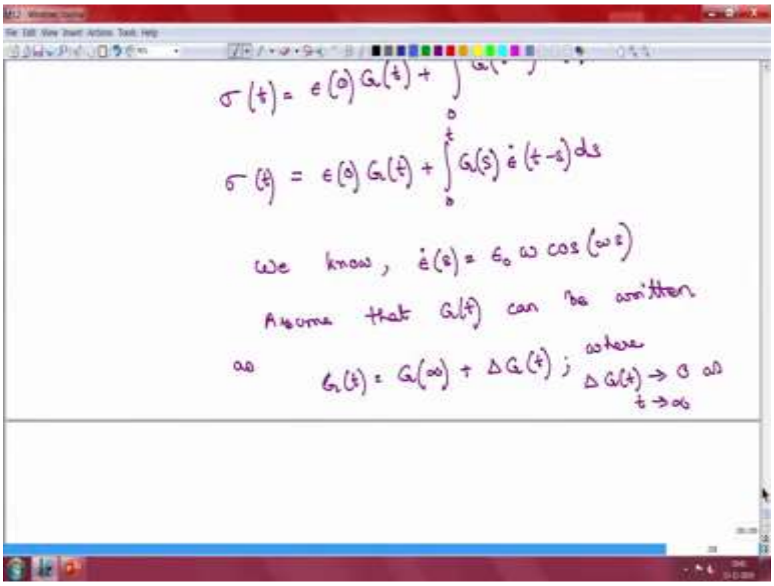

We are going to further assume that the oscillations are so, we are going to assume, so the assumption here that is going to be required for any further calculation is that the oscillation amplitude is small enough. So, basically this epsilon naught term this is small enough and when we say small enough we mean that such that the material response is still in the linear category.

So, oscillation amplitude is small such that material response is the linear. So, now we have to calculate now that we have a sinusoidal strain, what is that we are looking for? We are obviously looking for the output stress. So, we had already first mentioned the stress as we had seen the different forms of stress, the generalized forms and that for example, we had seen that this it can be written as epsilon 0, epsilon at 0 into Gt plus an integral.

This was the convolution integral. So, they actually go from 0 to t. And then you can have this G in the convolution form, right. So, and because this is a convolution, we could also switch these. So, it turns out that this is not, this form becomes a little bit more difficult to work with. It is easier if you work with this particular. So, we are going to work with this form or we are just going to switch these 2, we are going to say this is G of s.

So, now we know since we have the form of this input strain, we also have the form of the derivative, right. So, we know,(okay we know, we do not have to write that) will just write down that epsilon dot s is epsilon naught into omega into cosine of and before we proceed further, we are going to make one more assumption. So, before we proceed, we will assume that there this G has a particular form.

So, we are going to assume that Gt can be written as, this Gt can be broken up as G of some G of infinity plus delta G, at some time, it is some constant. And this is another part which is varying with time. And we are going to further assume that G tends to infinity, why this assumption is required will become clear.

Okay, so you can see that this particular form, if you put G infinity equal to 0, then basically you are modeling of viscoelastic fluid and if G infinity is non-zero, then you are essentially modeling viscoelastic solid. So, you can tweak what G infinity is, so it can you can keep it as 0 or you can keep it as nonzero and that is not a problem. But, we will for the time being we are going to assume that this is a non-zero quantity.

So, now we have to rewrite this particular equation. So, we have to insert this form of the G here, we also have to insert the derivative of epsilon dot here. So, what does this equation become? So, our original equation now becomes it is just this is now going to become very cumbersome.

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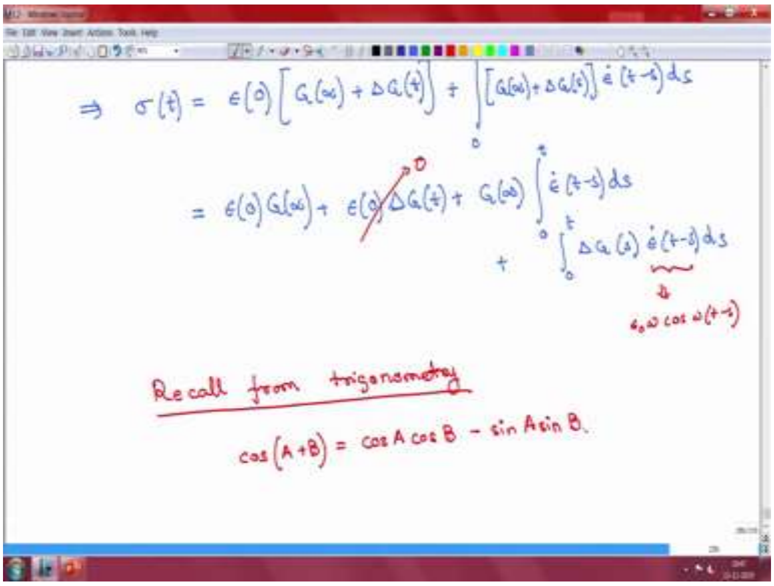

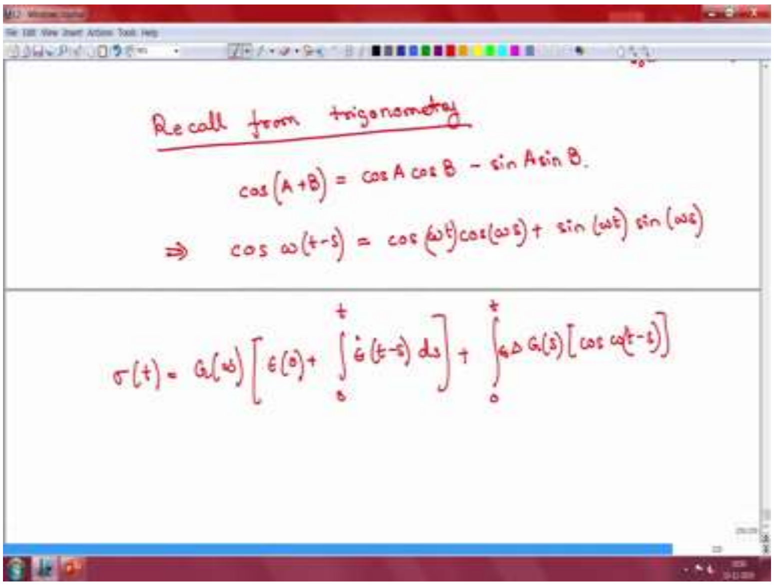

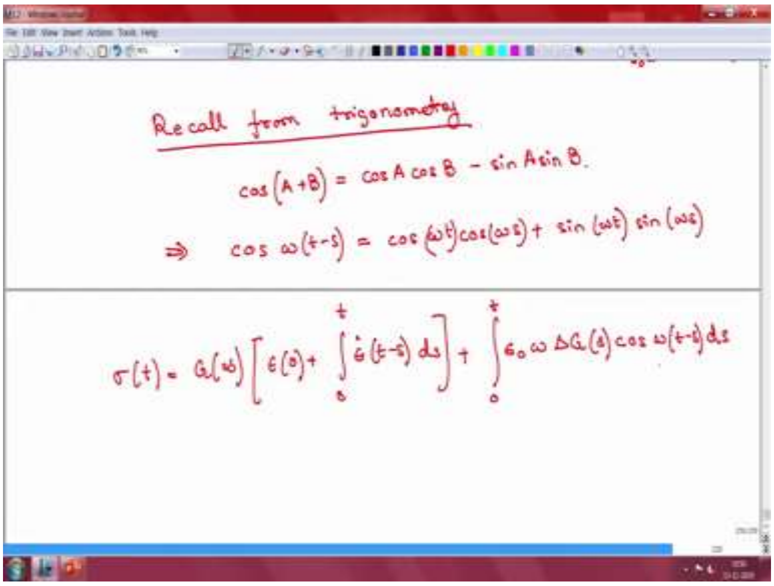

So, we have to keep track of all the different quantities we are going to write very carefully, okay. So, we have n (eps) first term which is an the term outside of the integral, let us write that. So, this way it say this is G infinity plus delta G and then you have the integral term, so 0 to t and here you have to write this is G infinity plus the delta G.

Now, inside the expression we are using s as the dummy variable for time integration over time. So, we are just now going to make it this and then we also have this epsilon dot t minus s ds, okay. So, we have 2 terms here, epsilon naught into G infinity then you have epsilon naught into delta Gt plus the term under this integration. So, we can break this up into 2 different integrals, 0 to t just a second.

Now, let us quickly consider this particular term, we have already provided a particular form of sinusoidal strain, right. This is epsilon naught this particular term is going to be 0, we have a sine function. So, this will just go away and then we have to simplify some of the other terms. So, since this equation is becoming a bit cumbersome, let us try to simplify each term by itself.

Now, here we have this term is going to be a cosine term. So, this is actually going to be equal to, if I use my previous expression, this is now going to be epsilon naught omega cosine of omega t minus s, right. So, I will have to expand this particular term over here. So, we have to recall from trigonometry that, if you had cos of A plus B that was equal to cosine A times cosine B minus sine A into sine B, what is it going to lead me to?

So, we can quickly see what this is going to do for us. And that is for in our case, we are going to have to deal with the cos of omega cosine of omega t minus s, right. So, what we are going to end up having is cosine, this is equivalent to cosine omega t into cosine of omega s plus sine of omega t into sine of omega s.

So, what I would like you to do now is take the expressions that we have found. And try to insert this over here. And one of the things we can easily see that what we are going to have here is this G infinity term. So, you have one integral where we have to expand, the other integral, this particular G infinity is common. So, you can add epsilon 0 plus this integral together, right.

So, I am just quickly going to write, the other this expression, and then I would like you to try and simplify this for the next class. So, you have sigma t is equal to G infinity plus, okay sorry this is you have epsilon 0 plus integral of 0 to t epsilon dot t minus s ds plus you have, integral of second delta Gs multiplied by this cosine of omega t minus s ds, 0 to t. Now there is a couple of things missing, which are going to be, it is going to be an epsilon naught.

Just a second, I am just going to erase this instead included that term earlier epsilon naught omega delta Gs cosine of sorry, there is so many terms. So, further, we have to keep on doing our algebra, we have to keep on inserting the terms and we are going to obviously, use this particular form of cos omega t minus s and put it here. So, we will do that in the next class. So, I will stop here for the class today. And then we can restart from where we left off in the next class okay, so will stop here, thank you.