Introduction to Soft Matter
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Lecture 37
Objectivity

So, welcome back everybody to another lecture Introduction to Soft Matter. As we are drawing to a close, we were discussing the idea of Objectivity. Why we are discussing that it will be hopefully clear by the end of this class, okay.

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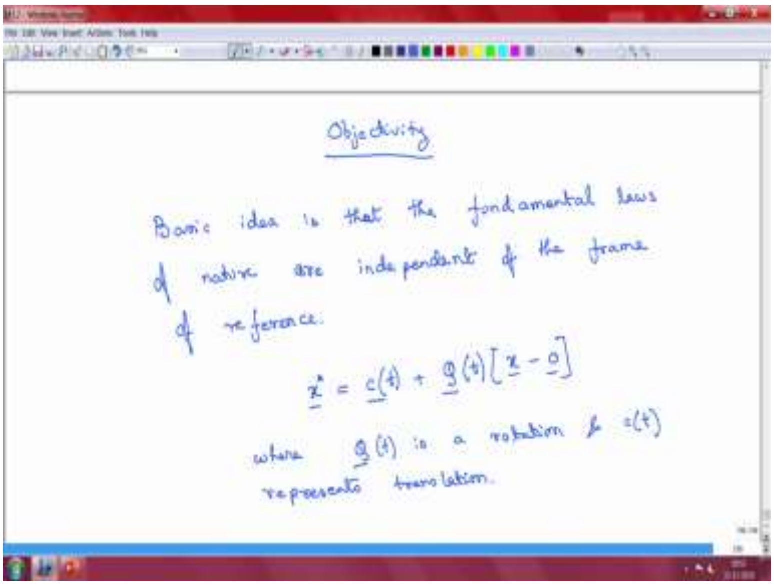

So, we were discussing this idea of what is called frame invariance objectivity. And the basic idea is that the fundamental laws of nature should be independent of frame of reference. So, the basic idea is that the fundamental laws of nature, here we imply laws of physics. Laws of nature are independent of the frame of reference.

And we said that another 2 frames are equivalent if they are related to each other. They understand how to calculate, 2 frames are equivalent if they have an agreement on the ideas of length, time and the sense of time.

And 2 different frames, which are related to each other by a rigid body motion can be given by this kind of a transformation, where this is the these are x star represents the transformed variable or the transformed components, let us say of a position and c is a translation given by a rotation and here you have the older position or the values that we found in the older system (where) Qt rotation and ct represents translation.

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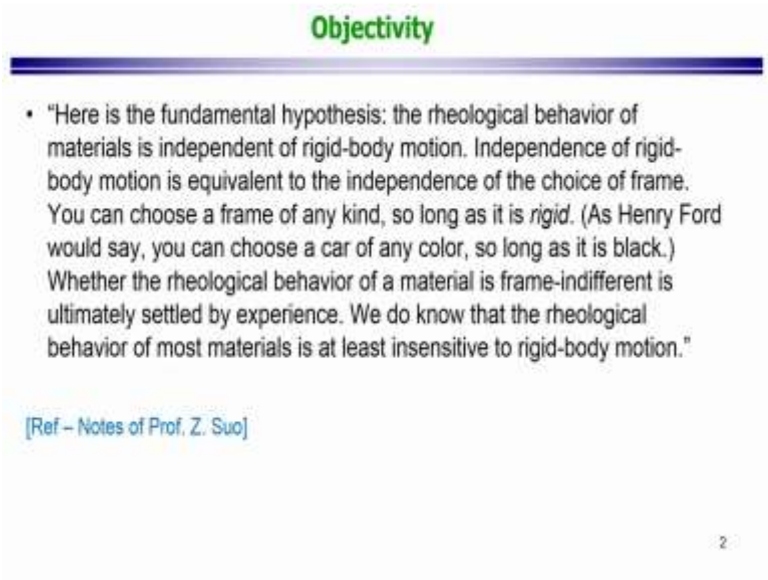

So, we came up with idea of objectivity then, and I will just review a very nice statement. This is again following up with what we discussed last time. These are again from the notes of Professor Suo and he frames it very nicely where he says that here the fundamental hypothesis, here is the final fundamental hypothesis. The rheological behavior of materials is independent of rigid body motion.

Independence of rigid body motion is equivalent to the independence of the choice of frame. You can choose any frame of any kind as long as it is rigid. And then he inserts this funny anecdote where supposedly Henry Ford is famous to have said that you can choose a car of any color as long as it is black, right.

So you can choose a frame of reference as long as it is rigid. And this idea that the fundamental this behavior is independent of rigid body motion is a hypothesis. So, whether this rheological behavior of a material is truly independent or not, is ultimately settled by experience. We do not know the rheological behavior of, we do know that the rheological behavior of most materials is at least insensitive to rigid body motion.

And here we are all discussing velocities et cetera, much smaller than the speed of light. So, we are not discussing any relativistic, we are not taking into consideration relativity here.

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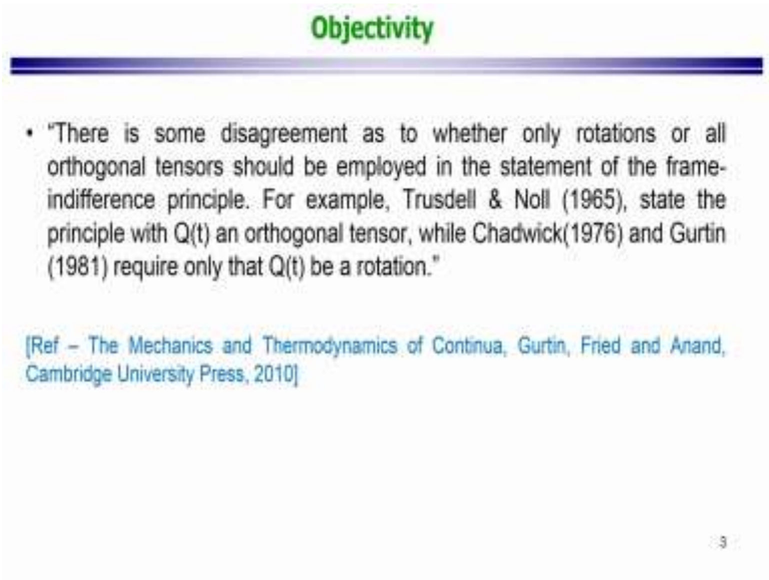

And, I also discussed that this idea of Qt which is a transformation which gives you rigid body motion where Q is a rotation. And then we also used the idea that Q, Q transpose is a identity matrix. Now this is from the book, the mechanics and thermodynamics of Continua Gurtin, written by 3 authors Gurtin, Fried and Anand.

And they say that there is some disagreement as to whether only rotations or all orthogonal tensors should be employed in the statement of frame indifference. For example, Trusdell and Noll, 1965 stayed the principal with Qt an orthogonal tensor. While Chadwick, 1976 and Gurtin require only that Qt be a rotation.

And then they finally go on to clarify that, according to another work, we believe that Qt should only be a rotation. And here it is important to just point out that you can have an orthogonal transformation which is not a rotation, so you can have a reflection. So, you can have Qt equal to minus 1, the determinant of Qt and that would reflect that would be a reflection. And that is not to be taken into account in this sense.

Now, this is only an introductory course, so we are only brushing up or lightly touching upon some very-very complicated topics. And I would like to point out that these topics are basically something that are in the realm of advanced tensor mathematics and advanced continuum mechanics. So, we are not dealing with, we are not going through the proofs, very-very rigorous proofs of everything.

We are just lightly taking up some of the simpler ideas that are inherent in this and you are more than welcome. Here the idea there is a specific reason why we are discussing objectivity and you will figure that out by the end of this class.

But if you want to understand this in more detail you would have to take a different lecture or a series of lectures which goes through as mathematically very rigorous. Okay so, let us return back to our notes.

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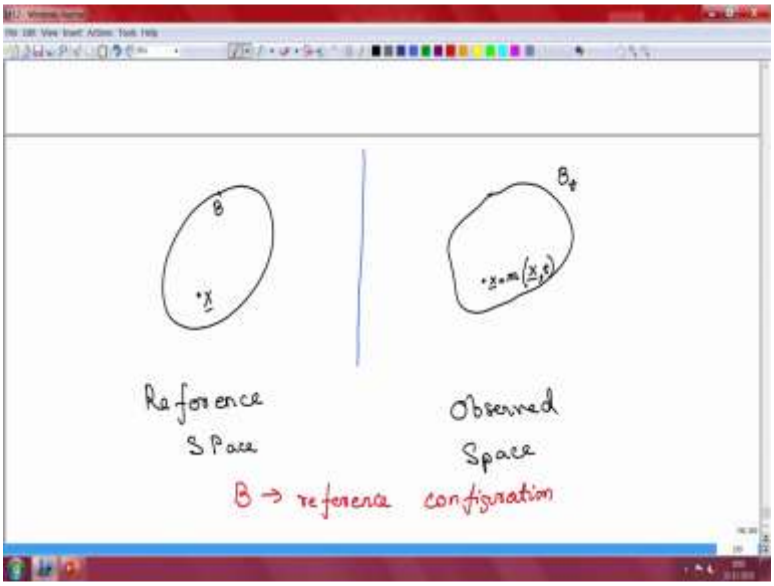

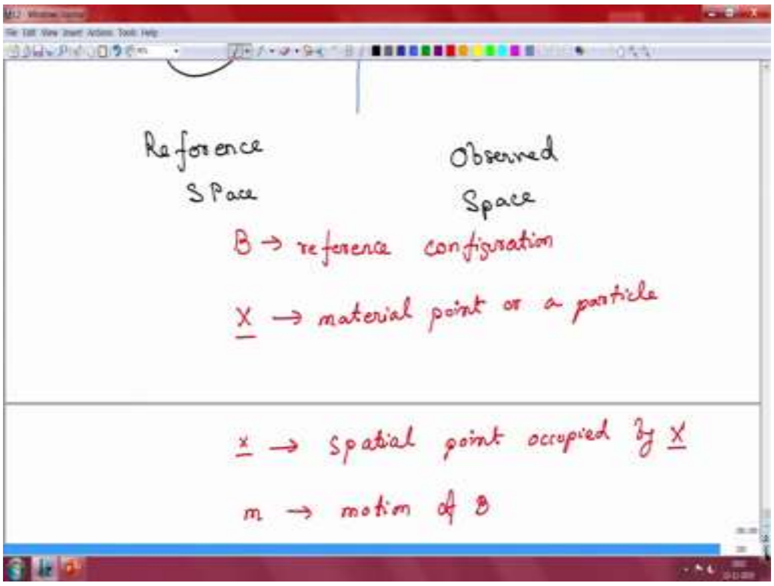

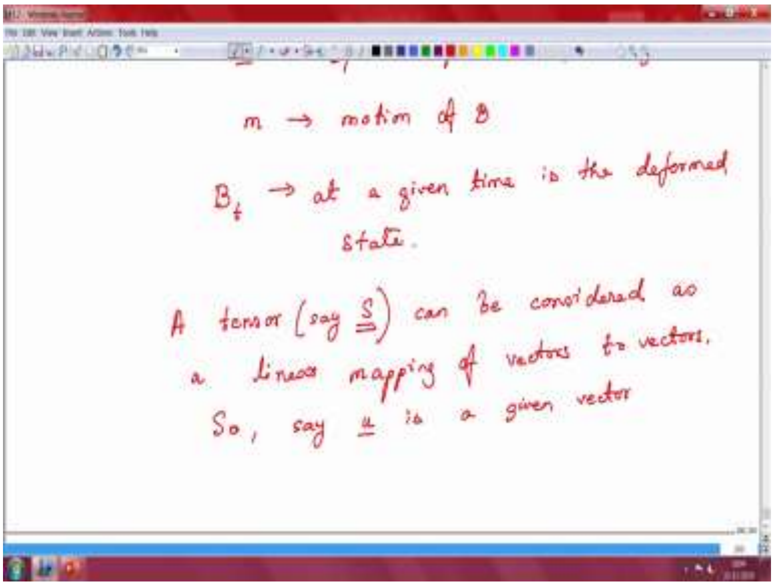

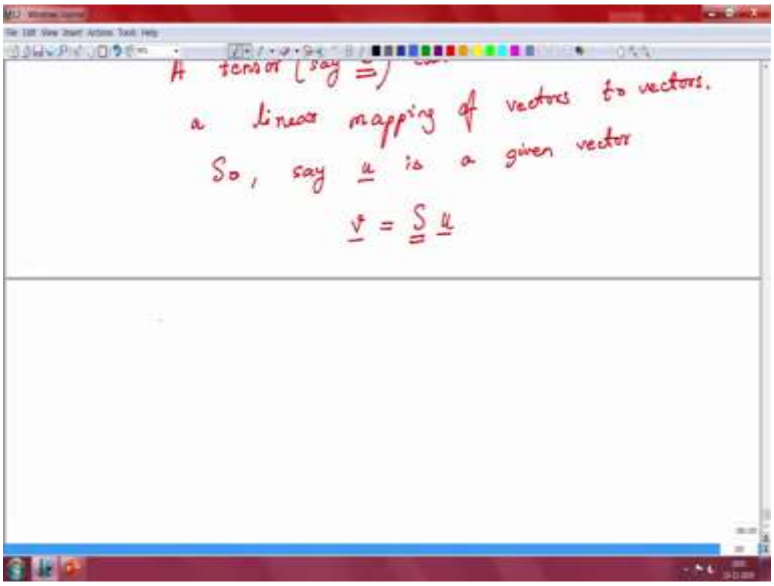

So, before we discuss further let us understand a few terms. So, in mechanics we assume that there is a body and this is something called the reference space. The reference space can be arbitrarily set but it is basically a situation from where you start comparing all your deformations that are going to occur later.

So, this body B has some material point in it, let us say X, capital X. And this later on is going to go through a series of deformations. So, this body B is now going to go through a series of deformations in time. So, that is why I put this index Bt, so that this is at a later point.

And then this particular point here is going to map to some other point now. And let us call that small x. And let us say that is equal to some function m of capital X and time. Okay so B, this is B is the reference configure, B is also called the reference configuration. The name itself should alert you about the purpose of this, this is the reference state with which we are going to calculate others and this capital X, this is also called a material point or a particle.

The small x is a spatial point, which is basically a mapping of this capital X to at another time in the same body. So, small x is also called a spatial point occupied by X bar. And m is such a function that it gives you the motion of the body. And at any given time, so, if you freeze the time, then Bt at a given time is the deformed state.

Now, we are doing all this because we want to introduce a very specific tensor which is called the Cauchy stress tensor. And I want to have a discussion on that and basically the relationship of the 1 D models that we introduced and more generalized models, so to do that we are going through a series of steps, okay.

So, now we are going to discuss since we are going to discuss stress tensor, I told you last time that a tensor is a linear mapping of vectors to vectors. So, let us say a tensor say I am just taking some variable S, and then I am putting this double bar here is a linear mapping can be, a tensor can be looked in many different ways, but it can also be considered can be considered as a linear mapping of vectors to vectors.

So, say u bar, so let us just say there is a given vector, so u dash is a given vector. And this S is a tensor that is giving me this transformation, then when S is going to act on u, it is going to result in another tensor, sorry another vector for me. Let us call that just v. So, by the way, these are just variables okay where v, S and u, we are just using some names at the moment to illustrate an example.

And we will see why this particular form is important. Now Cauchy's hypothesis, so since we are going to discuss the Cauchy's stress tensor, so now we have understood to some extent or in a very simplistic manner, we have an understanding of what tensors are.

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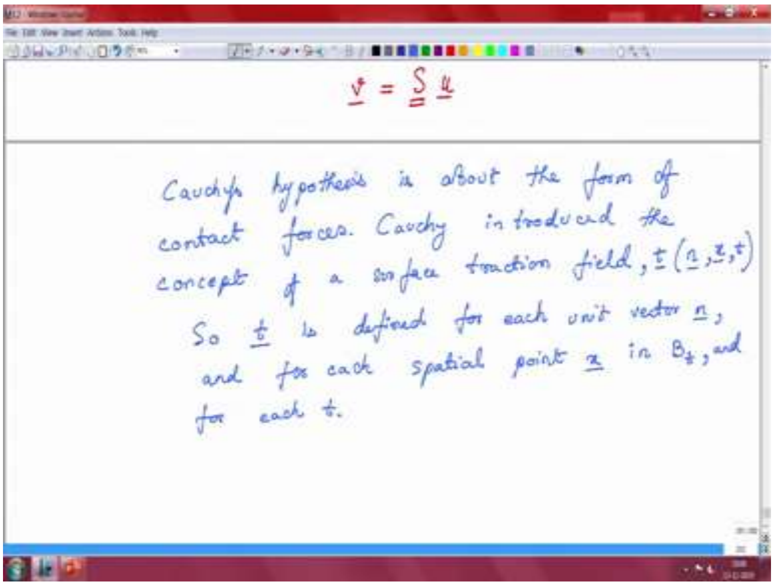

And Cauchys hypothesis concerns the form of what is called contact forces is about and he introduced, Cauchy introduced, what is called as a traction field. So Cauchy introduced the concept of a surface traction field. Let us say that is some t, is actually force that is associated with a given area, a small area which is an area we know that infinitesimal area is uniquely determined by its normal vector.

So, let us say n is the normal vector of that, x is the position at which it is being computed and t is the time, okay. So, I have already introduced a vector traction vector t. So, t is here also the time and we are but this t is a scalar. So, even though we have the same symbol, we can still distinguish between the 2.

So, this t is basically defined is, so t is defined for each unit normal vector and for each spatial point in B and for each and for all times, so maybe we just add that and forth. So, in a sense this traction vector represents the force per unit area exerted on the material.

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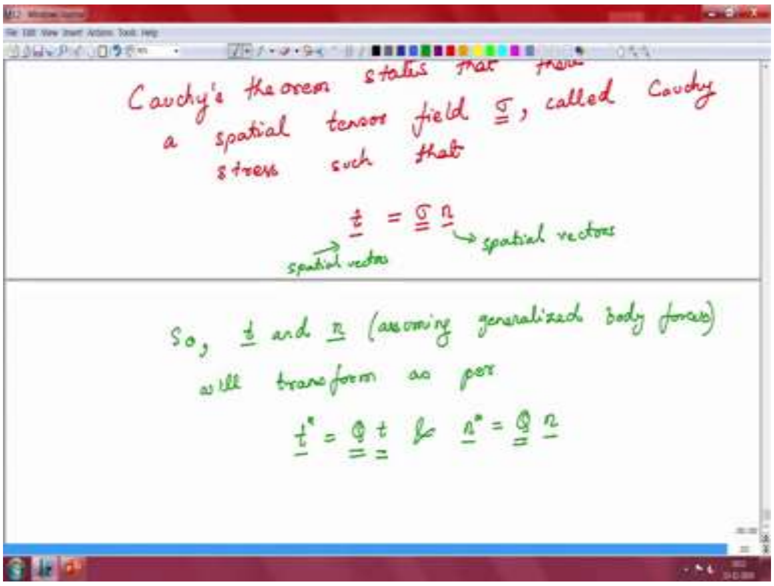

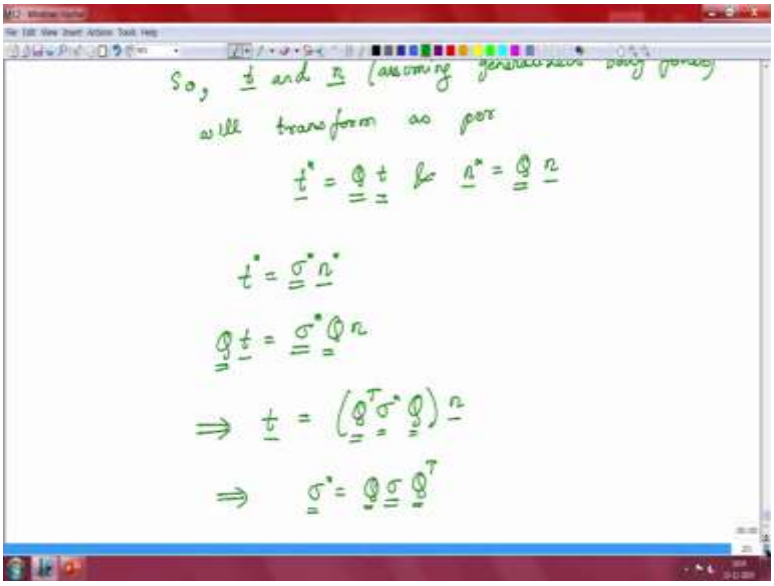

So, Cauchys theorem which concerns this and Cauchy's theorem states that there exists a spatial tensor field and this is actually the stress tensor field and we had already introduced the symbol sigma before right, so I am just going to say a sigma with a double bar.

So, this is the stress tensor field called Cauchy stress such that, so now you have a tensor field, right. So, this we know that a tensor has to act on some other vector to give me another vector. So, this tensor field is now going to act on, you can probably guess, on the normal vector. And what is it going to give me? It is going to give me this t bar.

So, these are spatial vectors because they are in the observed system. And these are against spatial vectors. Now, assuming that this, these forces are being calculated from the generalized body forces, you can show that these are going to transform in a way that they are invariant.

It is probably easy to understand why the normal vector is going to be invariant because the normal vector does not really depend, going to depend on what reference frame you are using. It exists irrespective of the particular reference frame you are going to take, because the only thing it is dependent upon is that infinitesimal area.

And similarly there is a way to discuss what is called as a generalized body force. And then you can show that this t bar is also the traction field is also a invariant form. So, they are going to transform as per the rules that we had already set before.

So, we will just quickly write that down. So, t bar and n bar and here in brackets, I will just write assuming generalized body forces were transform as per the following rules, so we had already said that this T bar in a rigid body rotation given a rigid body rotation and translation, the rule that governs this is going to be and for the normal vector similarly, this is the value in the transformed coordinates, this is now, the Q is the rotation.

So, you can show that, so from here you can show that the Cauchy stress tensor given the relationship that we have already found is frame indifferent. So, just let us write that. So, now your t star is equal to some transformed stress tensor multiplied by the transformed normal vector. And we already know what these 2 individually transform as, so I am just going to write that.

So, I am just replacing the previous equations, here. And this implies if you do one t bar is actually equal to Q transpose sigma star Q n, this just right. So, in the original coordinate system, so now from this, you can see that sigma star is and I am skipping one step because all you have to do is to equate the 2 and then multiply with the Q transpose and what you will get is this, okay.

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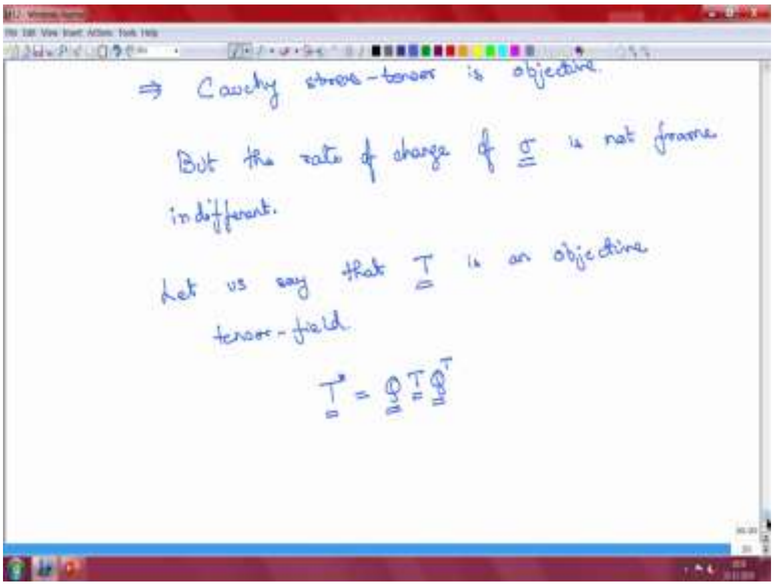

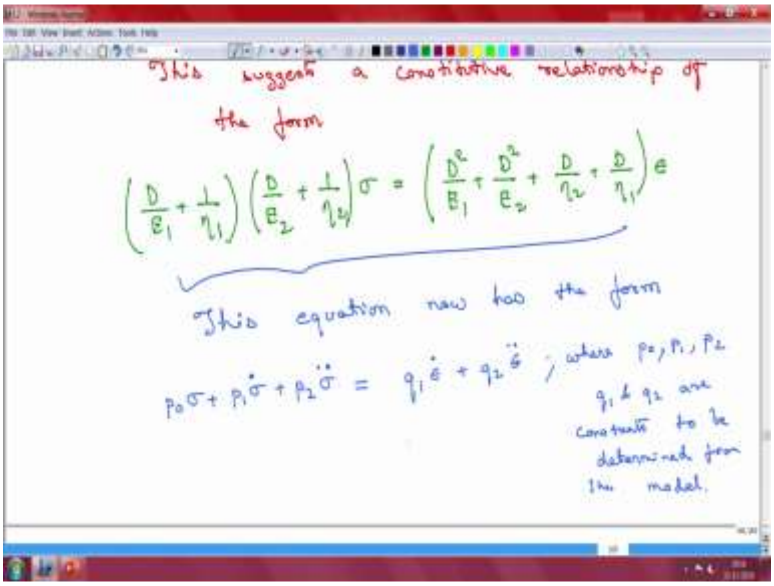

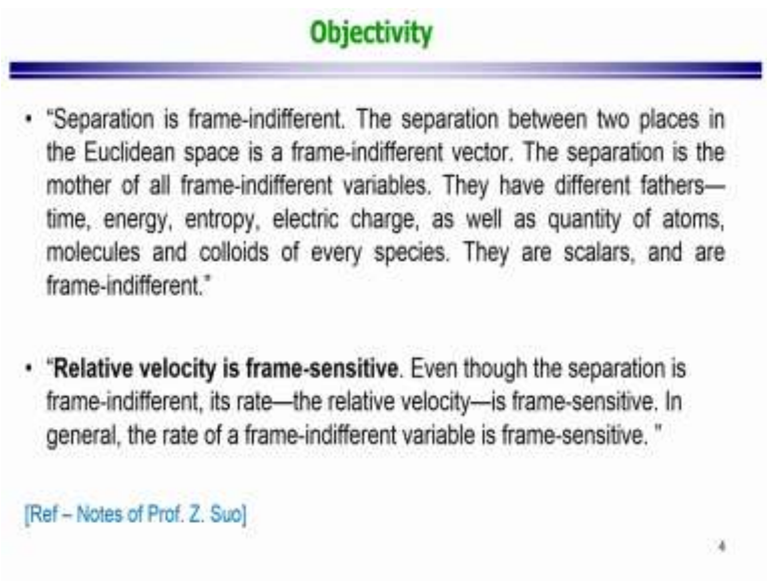

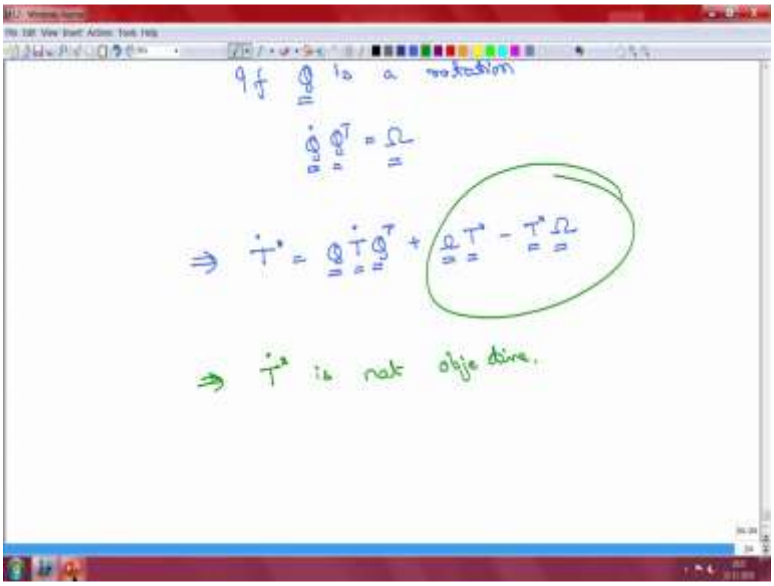

So, this has the implication that the Cauchy stress tensor is frame indifferent or is objective, okay and this is very nice, because why is this nice? PDEs, the ODEs that we are derived. So, for let us say for example, we had this particular equation, right here. Now this was a 1 dimensional system.

So these are just, we treated them almost as scalars and here this Cauchy's if you replace this with a Cauchy stress tensor then this forms the state invariant. But you also have a time derivative. So, is the time derivative going to be invariant, right?

We are trying to write an invariant form for the entire thing and on this side you have (the deformation), the strain rate fields. So, will that be frame invariant? Now, we had seen previously last time where we have discuss that the velocity. So, although the separation is an objective variable, but the velocity is not, right.

So, if you just want to quickly review that. So, we had seen last time that separation is framed indifferent right, that was one of our most important points of discussion and we had said that the separation between 2 places in the Euclidean space is frame indifferent vector.

The separation is a mother of all frame invariant variables indifferent variables. They have different fathers, time energy, entropy energy, electric charge, as well as quantity of atoms molecules and colloids of every species. They are scalars and frame indifferent. But relative velocity is frame sensitive. So, even though separation is frame indifferent, its rate the relative velocities frame sensitive. This is what we saw in the last class.

And in general, the rate of a frame indifferent variable is frame sensitive. So, we are just discussing the reason this last note is important is because we just said the Cauchy's stress tensor is objective. But then that rate of the Cauchy's stress tensor is not going to be objective, okay.

So, we are going to make a note of that but the rate of change, the time derivative rate of change and the rate of change here implies obviously with respect to time. The rate of change of this is not frame indifferent. And this can actually be proven quite well. So, we will quickly prove the generalized idea that if, so let us say.

So, let us say that t is an objective tensor field. So, t transforms as the, if you have, if you are going to calculate in a new frame, the components, you know that that is going to transform as this quantity, where Q is some rotation matrix. So, now, if you take a derivative of this, so let us say what happens if you take the derivative. So, since this is an equality I can take derivatives time derivative of both sides, so I will just indicate by a dot that we are taking the time derivative.

And here you can start taking time derivatives. And first I am going to take the derivative of t, I am just going to keep that on the left hand most corner and you will see why. But I also take that derivative of the others, do by chain rule. So, that becomes Q dot. Now, if Q is a rotation, then if Q is a rotation, then this particular tensor is another tensor, which is also called the spin tensor of the frame.

So, I can replace these Q dots basically that is what I am trying to do here. So, I can replace this Q dots. And what I will get in return is t dot star. You can probably already see that it is not that derivative is not going to be objective because then only this part, you should not have had this this section, this section is the one that is going to create the problem.

You only wanted this term if it were supposed to be objective, but these terms are going to be there and you cannot wish them away. And you can show that this is now going to become, you will have a Q dot, is no sorry, not Q dot this is the derivative of t. So, you have this term plus when you just do one more step, but you will find that this becomes and this extra term is not going to be, is non-zero, right.

So, we have this extra term and this is going to create a problem. So, basically this implies that t dot star is not it is not objective. So, we just verified the statement, last statement that we had seen and where we said that in general, the rate of a frame indifferent variable is frame sensitive. So, if you want to make a constitutive equation such that, so if you want to make a constitutive equation, which involves a Cauchy stress tensor, the Cauchy stress tensor is not going to give a problem because it is objective.

But the derivative of the Cauchy stress tensor which we saw happens occurs in the Maxwellian, the Kelvin Voigt Meyer bodies and its generalizations there is a derivative of stress is there, the time derivative of stress and that is going to be a small problem because it is not going to be objective. So, we will see how people resolve this in the next class. Okay, so we will stop here for today.