Introduction to Soft Matters
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Lecture No 36
Objectivity

So, welcome back to one more lecture on Introduction to Soft Matter. Last time we were discussing issues concerning constitutive modeling and in particular discussing the principle of objectivity.

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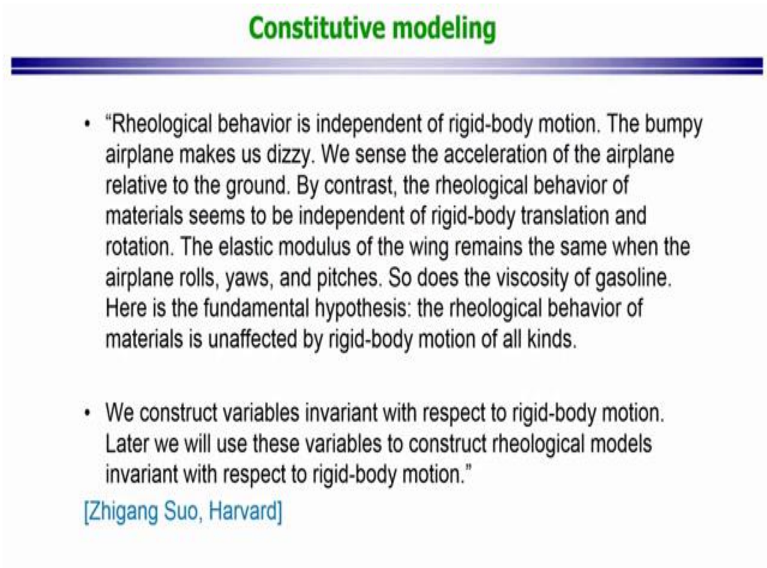

So, the one of the last things, that we were reading out was this line by processer Suo, where he says that, "We construct variables in variant with respect to rigid body motion, later we will use this variables to construct Rheological models invariant with respect to rigid body motion."

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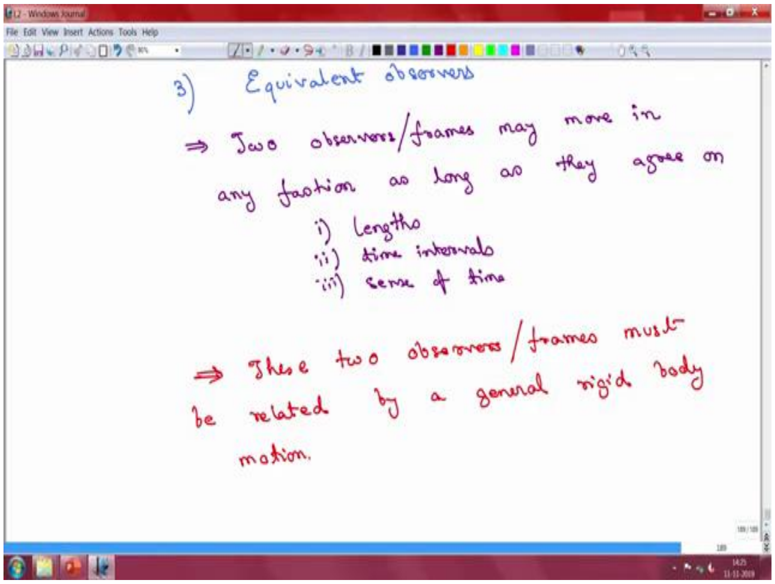

And here we have just stated that equivalent observers, two observers, who are basically related by a general rigid body motion.

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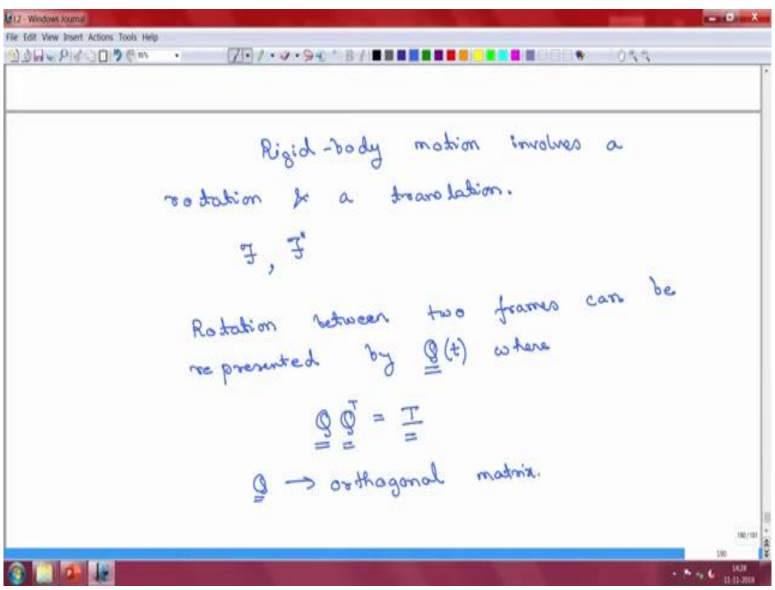

$\underset{=}{Q} \rightarrow$ orthogonal matrix.

$$\underline{x}^* = \underline{c}(t) + \underset{=}{Q}(t)\,(\underline{x} - \underline{o})$$

No loss of generality is incurred by assuming that $\underline{o}$ is fixed.

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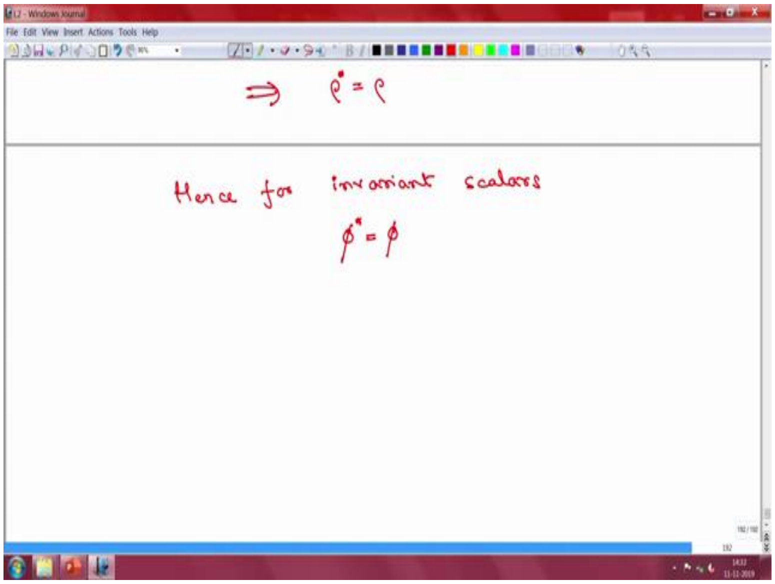
Let a measured field ϕ be ϕ^* in the frame $\mathcal{F}^*$.

Scalar fields (such as density) are invariant to a change of frame

$$\Rightarrow \quad \rho^* = \rho$$



So, rigid body motion, you can show that, I am not going to show it here. But it is, it can be shown that rigid body motion, basically involves, a rotation and a translation. So, let us say you have two frames F is your current frame and we will denote all quantities in another frame by F star and there these two frames are equivalent observers and they are related to each other by a rigid body motion.

Now, rotation between two frames, rotation between two frames can be represented by a matrix, so three dimensional, if we have 3-dimensional space, then the rotation matrix will also be 3 cross 3. So, the rotation between two frames can be represented by a matrix, let us call it Q and I just put double bar here to indicate that this is a matrix t, where Q into Q transpose gives us the identity matrix. So, Q is also often called an orthogonal matrix.

So, now if you have in the rotated frame you can represent, let us say, the new location for a particle, let say you have a particle located X and you have another frame, which is rotated and translated with respect to the previous one. Then the, what is the new position for that same point in the rotated frame, in the rotated and translated frame?

So, we can actually write, this relationship that X star, which is the position of the same point in the rotated frame can be given as some translation we will just called by C and t just shows that it can be a function of time plus the position of this point with respect to some origin multiplied by

Q, where Q is the rotation matrix. And here no loss of generality is incurred and I just make a note here.

So, no loss of generality is incurred by assuming that O is fixed. So, let us say, that you have a measured field phi, in the previous, in the reference frame F and that field now becomes phi star in the new field in the new frame. So, let a measured field phi be phi star in the frame F star. And let this be a scalar quantity for the time being, if it is a scalar quantity, then we say that the scalar fields are invariant, if they retain the same value and they have change a frame.

So, we will say the scalar fields, such as density, for example are invariant to a change of frame, which implies that, in the rotated frame and the translated frame, if you measure in the F star frame your measuring a density, which is phi star, that should be the same as the density that you are measuring in a another frame, in an equivalent frame. So, from this we can make our first rule for scalars and that rule says that hence for invariant scalars phi star equal to phi.

So, this is an important point, because even though, this is a very simple idea, you must understand or we must have a situation or a constitutive relationship, where the change of frame cannot affect simple quantities like viscosity. So, whether you are measuring the viscosity of water on Planet Earth or Mars or somewhere else, that it is a material property, it cannot change with just because you have changed your different special location and maybe at a different point of time, you cannot change that.

Again as I said, this is an axiom. So, this is something that we have believe is true and we believe, and we build our equations and our theory based on this. So, for invariant scalars, this simple relationship holds, but this relationship will obviously not remain as simple for other quantities.

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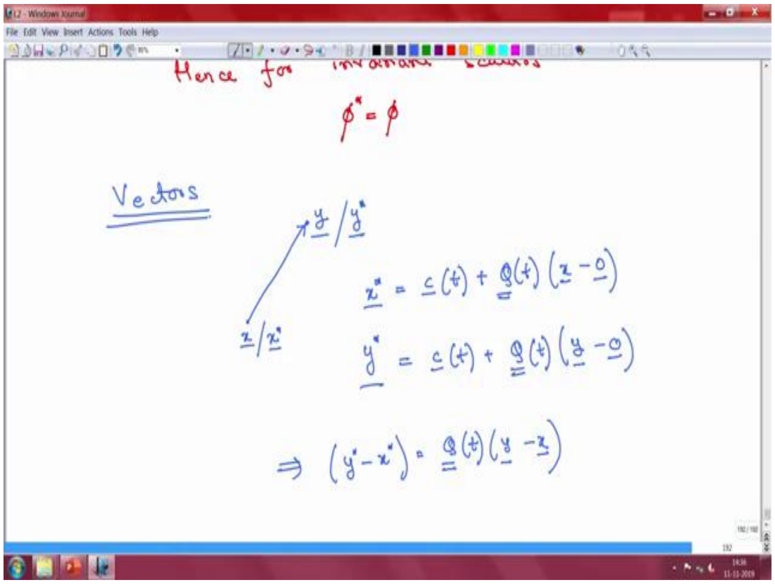

So, let us now discuss the case of vectors. So, let us take our old idea of vector, which is that you have two points and this is the displacement vector between the two points, the point here is given by some X these is a coordinates in one frame, reference frame, when you change to a different reference frame the coordinates also changed. So, you have now, this coordinates in the new frame.

Similarly, this is another point let us say this is Y and in the rotated frame in the other frame, you have a rigid body translated and rotated frame, you have Y star as the new value. Now, displacement we have to understand is something, that is truly frame in variant, because otherwise how will you even discuss lengths, if displacements are not the same in all for all observers, how can you even agree upon a length.

So, displacement is the, so the position vector of a point may not be independent of the frame, because that is tied down where your measuring it from. But the displacement vector between two points that is truly a frame invariant. So, what we call separation is frame invariant, so we had just returned above the equation for the change of reference frame.

So, let us write down what X star is in terms of X, so we had said that, X star is equal to some C t plus Q X bar minus O bar. Similarly, I can write it down for Y, the same equation I am going to replace X for Y now. So, what is the, so we know that Y, this displacement vector is going to be invariant. So, if I subtract the two, I end up getting. So, this is important here now, so this, since we know that the displacement vector is invariant, we know that if we have to calculate the invariant, if this form is the form of invariance.

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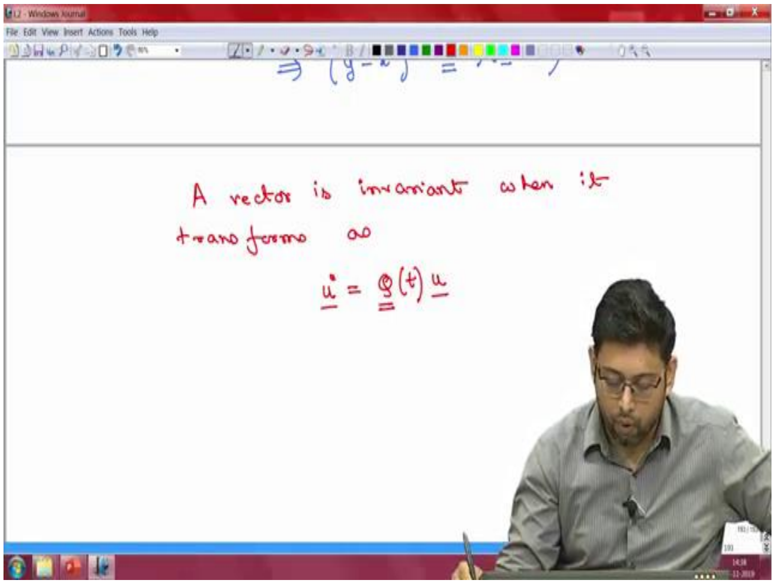

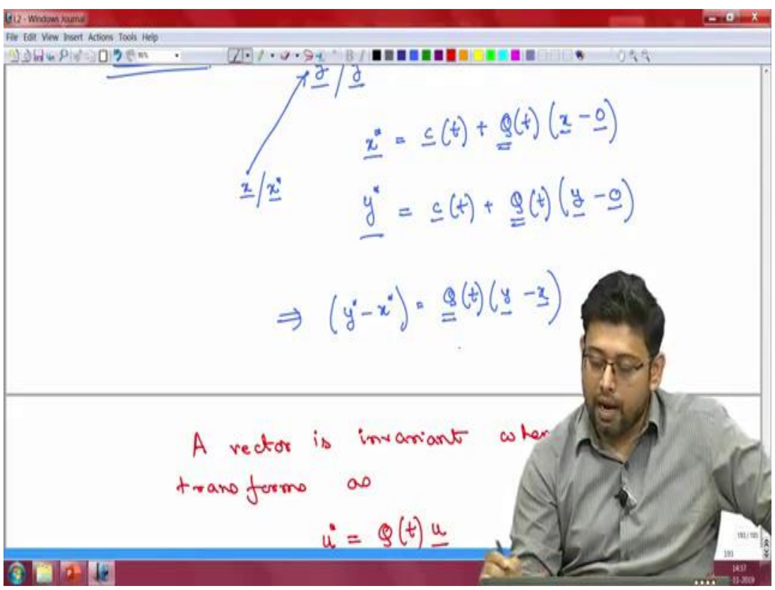

So, we now what we are going to do is? We are going to say looking at this is that a vector is invariant when it transform as. So, let us say you have a vector u, when you transforms as this rule. Now, there is one thing that I just like to point out and that is sometimes an issue between different books and different presentations by different authors, that here what we are trying to do is, this Y star et cetera. this relates to the position to the coordinate values.

So, if you have we can think of it as may be a column vector with three points for X Y and Z coordinates. So, these are actually the values itself and it does not, it is not the abstract quantity factor itself. So, I am taking, I am adopting a scheme, that is or I am adopting a formulation that is taken by Phan Thein and some of the other authors, some other authors like Suo they say that it is better to write in terms of the components. So, I just wanted to point that out.

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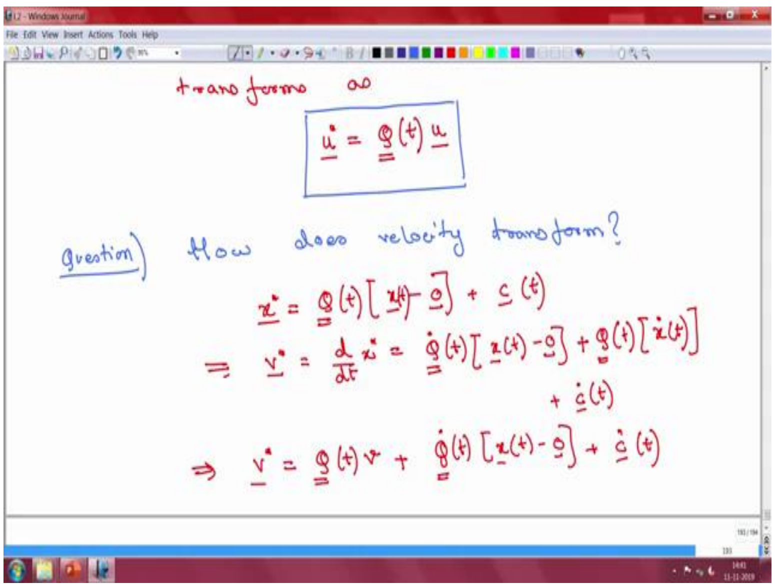

So, good, displacement is invariant, but what about velocity, is it also and variant? So, let us look at, let us ask also simple question, the question is by the way this Q is question. So, I may be write down question, that is we have already another Q. So, the question is how does velocity transform? So, we know that, we have previously written that, expression, we let us just rewrite this.

And this is also a function of time, we have to discuss the velocity. So, velocity, your transform velocity, is nothing but the derivative of this, the d dt of X star and if you take the derivative and here this is remember that this Q can also be a function of time and you have X also have a function of times. So, you have to apply chain rule.

So, first time may be we will just take the derivative of Q so we have Q dot, and then the second time we take the derivative of X. So, we have X dot and O is fixed as we had just said previously, there is no loss of generality in assuming that is just fixed and you have C dot t. So, this implies, if you just clean out this expression with this quantity is the velocity, in the other frame. So, you have transpose, this is a velocity plus so I am just rearranging the terms here and you will probably can understand why I am doing that.

And we had just said that a vector is invariant if it transform as this equation previously. So, this was our important equation. But we see that when we apply to velocity, you now have this an extra set of contribution here. So, velocity is not, is not frame invariant that is something you should expect also, if you think about it velocity is a very important frame dependent, frame sensitive quantity.

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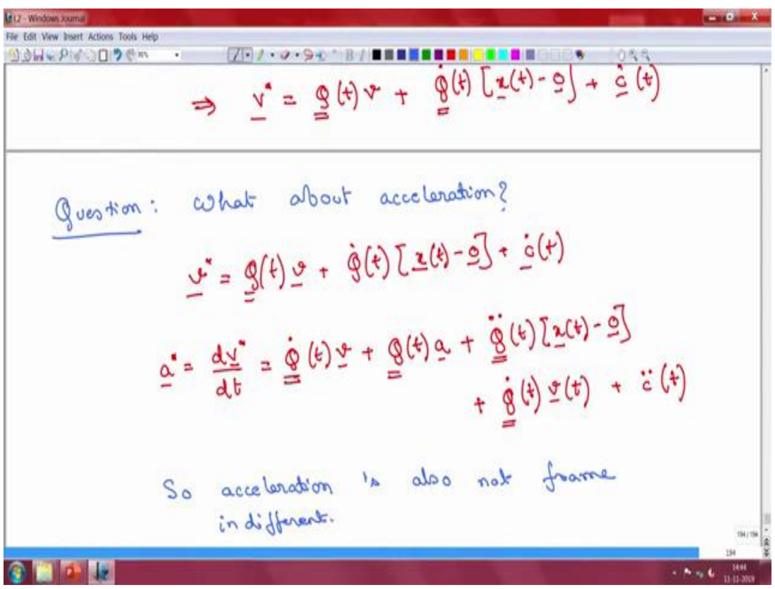

So, what about acceleration? So, we just may be do one more question. What about acceleration? So, we know that, we have already written that expression V star is equal to Q bar t V bar plus Q dot t. So, a star, which is now going to be the derivative of this where we are going to have to take the derivative again. So, let us see what we get? We get Q dot t into V plus Q dot.

So, this time, we differentiate the other term, we get acceleration, and differentiating the first one, and in the second case, second term I am differentiating the first one applying chain rule, plus sorry, by the m is this. So, we can see that contradict to our expectation had this been invariant, that only this particular term would have been there, but we have so many other terms. So, acceleration is also not frame in different.

So, we will make a note. So, acceleration is also so not frame indifferent, actually frame sensitive. So, we see that despite the fact that displacement is frame invariant, its time derivative is not. So, that also we should make us question as to are there other type of time derivatives, which might be, we will look at that later.

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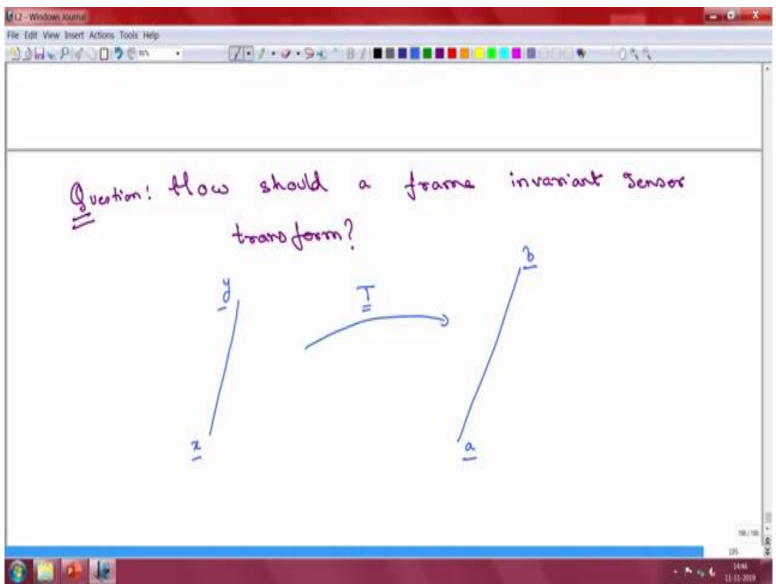

So, we have, we discussed scalars and we discussed vectors, how vectors should transform. So, the obvious question is how should tensors transform? So, we ask now, how should a frame in different, a frame invariant tensor transform? By the way you might have noticed that I have used a way a word frame indifferent here and somewhere I am using the word frame invariant.

Now, most authors from what I have seen they tend to use the two interchangeably at different times. Although sometimes the principle of frame invariance or indifference is stated as a different principle. So, how should of frame invariant tensor transform? So, what is a tensor by the way? We know what vectors are, but what are tensors? A tensor is something that can transform one vector into another vector.

So, let us say you have a vector X Y, then trans, the tensor T is a transformation of this vector into another vector. And we want to understand how frame invariant tensors should transform, a what should be the rule that should be governing that.

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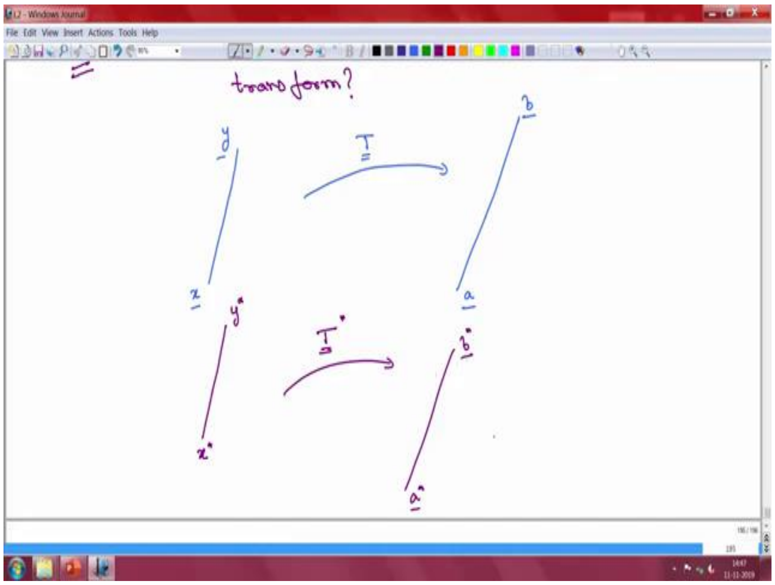

So, let us say in the other frame, this is X star, this is Y star, and you have another tensor now at different tensor and this will transform it into. So, that transform tensor should do the same job, this should take these displacement vectors and transform it to some other vector.

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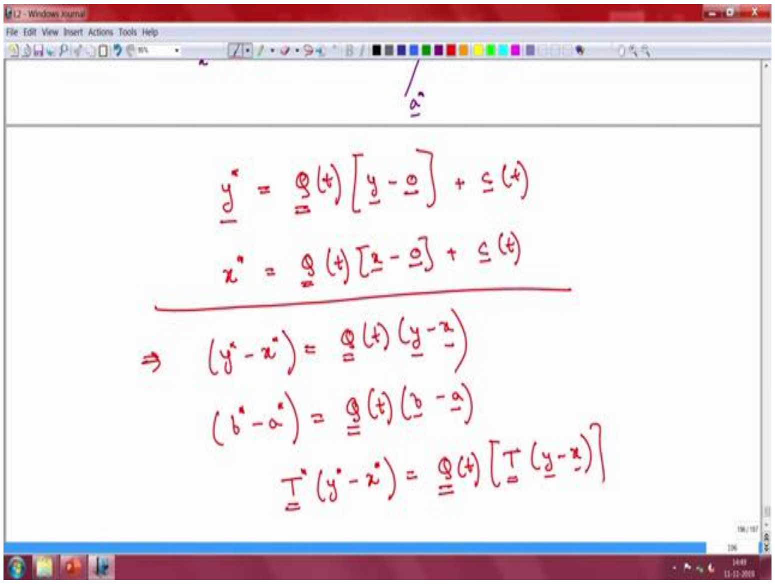

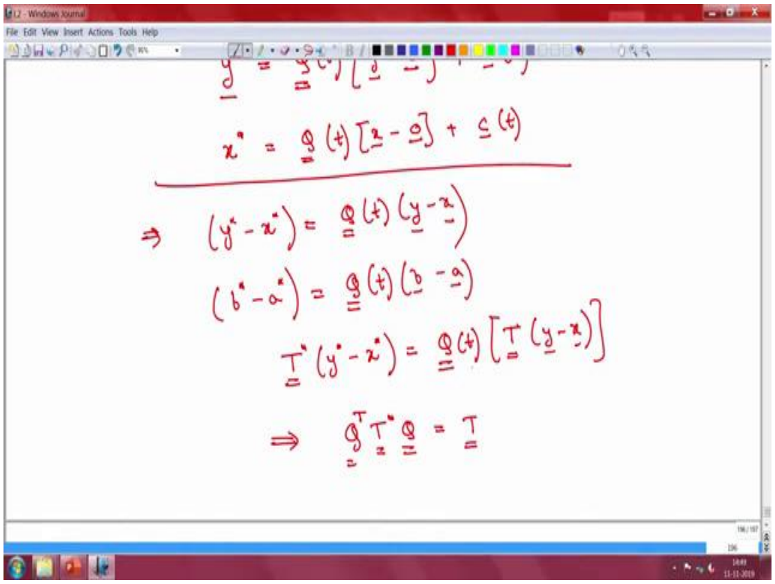

So, we wrote down before, let us write down the relationships that we have written before also, we let us say Y star now is sorry, is equal to Q times of Y, if you want to put. And then X star in this case is again. So, we are already just writing down the expressions that we had agreed upon earlier. So, when we take this, then we know that you have this is the same as the expression that we had previously written, which is going through that exercise one more time.

So, similarly, your b star minus a star will be the Q times of b minus a. And this represents now the transformed. So, this is the t tensor should act on this quantity and convert it to b star minus a star. So, T star into Y star minus X star should give me this quantity now, but this is T acting on Y minus X. So, and now I am going to skip a step here and I want you to sort of complete this, Q was the orthogonal matrix, in fact I can use that property and then I can assert that Q transpose T star into Q will lead me to T, that is one step I have skipped and I have deliberately skipped. So, that you can fill in the gaps here.

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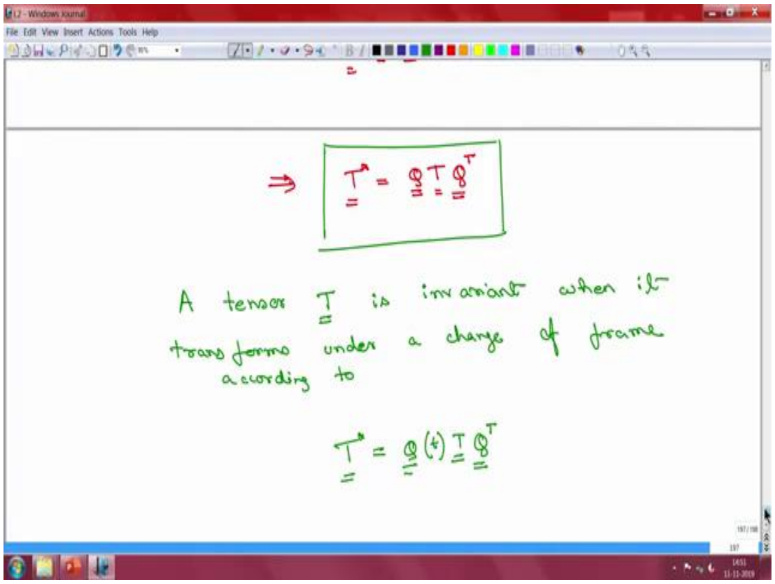

And this implies that your T star is equal to Q T into Q transpose and we are going to this as the rule to say. So, this we are going to do that attend, so we are going to say, so we are going to use this to make our rule. A tensor T is invariant when it transforms under a change of frame according to this relationship.

So, what we have been able to do is today we came up with 3 different relationships one for scalars, one for vectors and one for tensors. And we said that if they need to be frame invariant, then there is a certain transformation law that should govern them and we found out what that is. We also discovered that the velocity, that while displacement of separation is frame invariant velocity is not, neither is acceleration, they are frame sensitive terms.

So, now that we have established the rule for tensors we will look at some quantity, some tensors, and we try to establish whether they are in frame invariant. So, we will do that in the next class. So, we will stop here today. Thank you.