Introduction to Soft Matter
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Lecture 32
N Maxwell Model

Welcome back to one more lecture on Introduction to Soft Matter.

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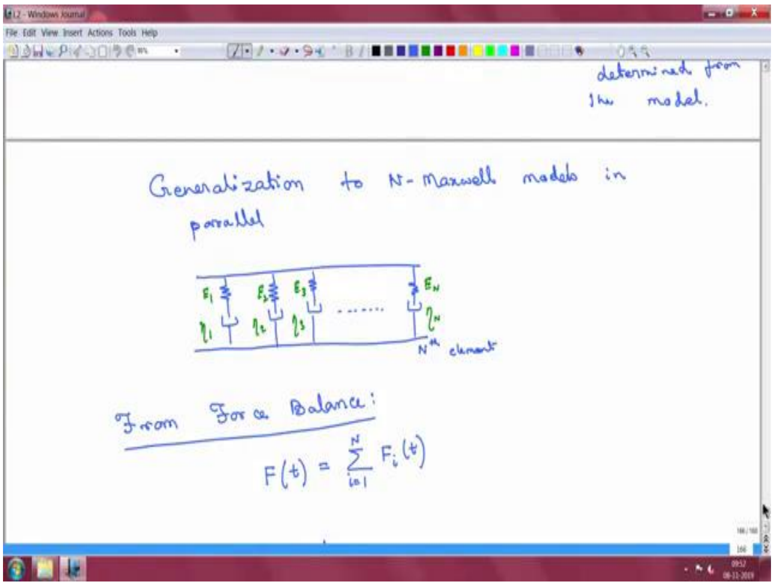

Last time we were discussing, the specific case of the generalization to N-Maxwell models in parallel. And we had left of where we had discussed that the force balance equation would look very similar to what we had before.

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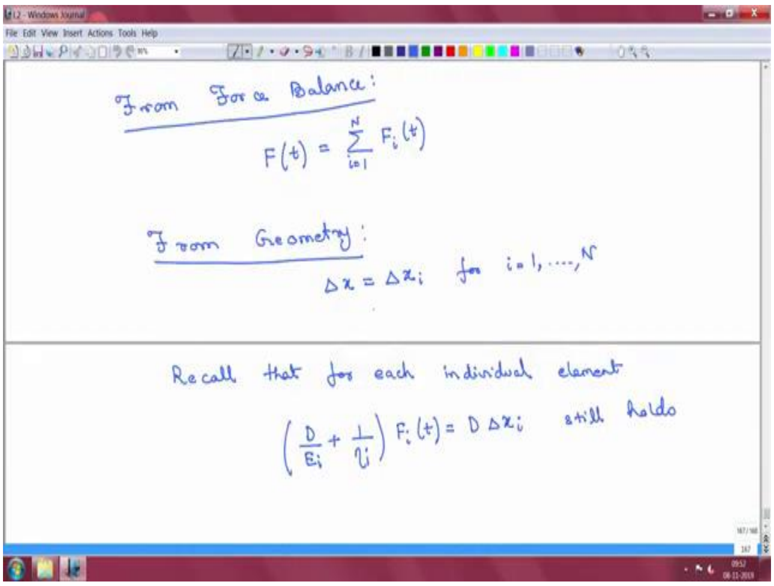

And we had also remarked that the geometric constraint remains identical to what it would remain in the case of elements in parallel, which is something that you would expect obviously. And then we had also remark that for each of the individual elements the original equation that we had derived that still holds. Given that, you are using the right force on the, in this particular equation, and the right, the corresponding deformation in the body.

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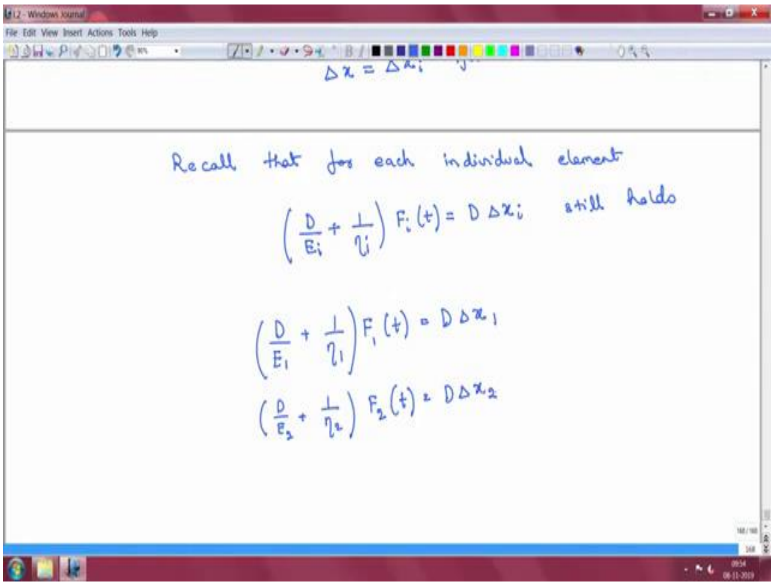

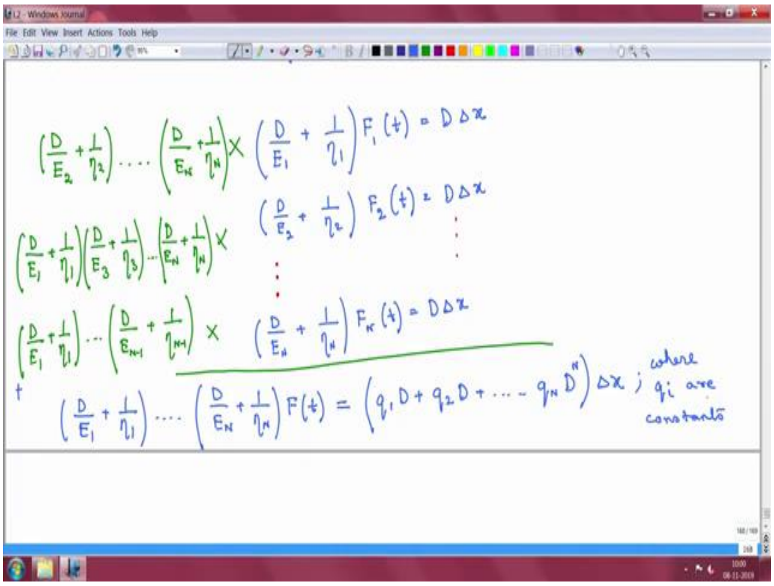

So, let us try to see how we can solve these, the set of equations. So, I am just going to rewrite this and I am going to say that this basically corresponds to a set of differential equations such that you have, so this is for first element than you would have one for the second, I am just expanding the equation that I wrote before.

So this process will go on, till I start writing the equation for the Nth element. So we had said capital N number of elements are there. So here the last term look something like this, sorry this is not small n this is capital N. So these are the set of equations, so you have n equations that we have written. Now from the geometry constraint we had said that the deformations are going to be the same in all cases.

So what we can do is we can just delete this, we can drop the subscript from all this different terms, because they are all the same. So I am just going to do that, I am just go on and delete all the different subscripts. But for the force the subscripts cannot be dropped. So I have to figure out some other way in which, some other way has to be figured out.

And the way to do this would be I want to add all the different forces, I just want the same multiplier with all of them, but the multipliers are different. So I can resolve this problem by multiplying both sides of these two equations by this combination. So see you already have D by E2, so I am going to multiply this with a set such that terms here and you multiply both sides of this equation.

So, I will be writing only one side but it is implied that it is also on this side. Basically this multiplier is something that we have to multiply both sides of equation here. So similarly what you do for equation 2, equation 2 you already have this E2. So you start with then the term D, E1 and then you drop the second term, so you multiply it with 3 and like this and you keep on going till you reach the nth multiplier.

And you can multiply this entire equation with that. So similarly by the time you reach the nth equation but you going to have is you are going to multiply it with 1 and this will keep on going, but here you will stop at the N minus 1th multiplier. And in this please recall remember that because the equations are so cumbersome I am not writing this multiplier both sides but they are on both sides.

So, when you multiply this you will finally end up with the same multiplier for all the Fs. And that multiplier would be, so adding all this different terms here and what I will end up having is and this dot dot dot signifies that all the other terms are there and this goes on till N number of terms. And then we have F, so all this will add up and this will give me F and on this side you will have a lot of different terms.

Now what you will end up having is you are going to multiply every delta X with some D and this kind of multiplier. D is an operator but when you are factoring, when you are multiplying this you can also use it as almost as if it were algebraic quantity. So you will see that what will happen is for them for delta X you will have all the quantities or the term here we will be such that they will powers of D.

So this with different coefficients, so what you will end up having is a polynomial n D such that where q1, qi's are constants, some constant which have to be determined. So you can do this cumbersome addition and you can determine these constant, so these constants are fully determines, but we are not working out here.

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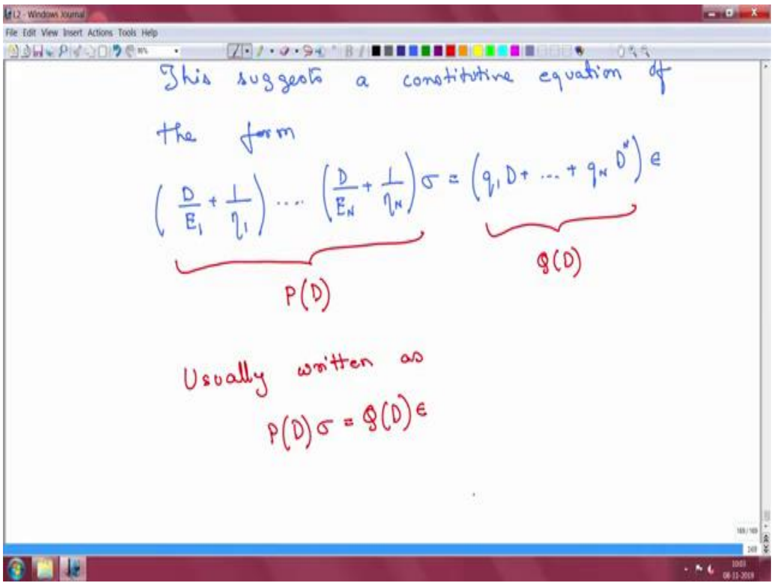

So this suggests, suggests a constative equation of the form, D by E1 is 1 by 1, multiplied all this different multipliers till the nth multiplier, sigma equal to this q1 D plus and this goes on, there are again n terms here, sorry this delta x will be replace with epsilon. Now note one thing that this system here, this is also result in powers of N, powers of D all the way till N but it will this polynomial will also have a constant term.

So this is also effectively a polynomial in for the operated D with the highest power being capital N again in this case. So the highest powers on both sides for the operated D are the same. So what we can say is that this is actually some polynomial P of D and this is some polynomial q. So this is sort of abbreviation for this entire polynomial.

And this set of equations is usually written as, so usually we say written in short form as a polynomial in D in the operator D multiplied by sigma the polynomial of which Q represents a polynomial and D for the for your right hand side multiply by epsilon. So this is interesting and now you can probably see the logic behind why we were writing some of the equations earlier in this particular form.

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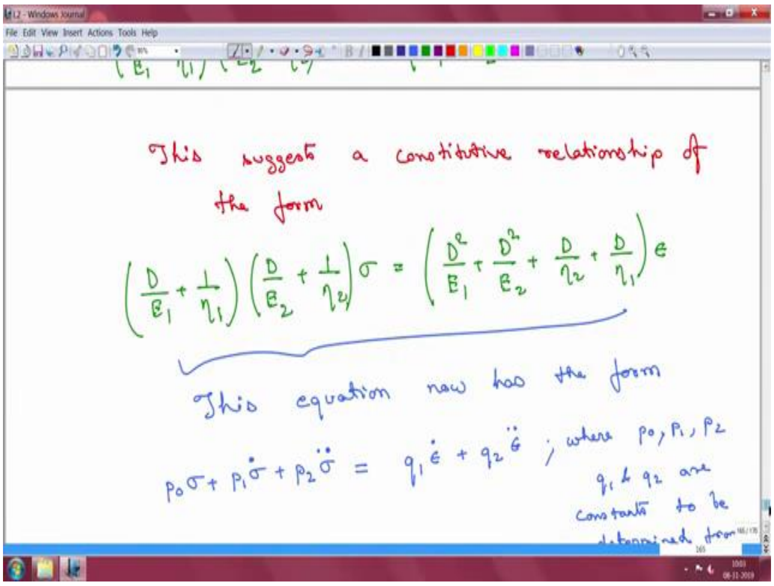

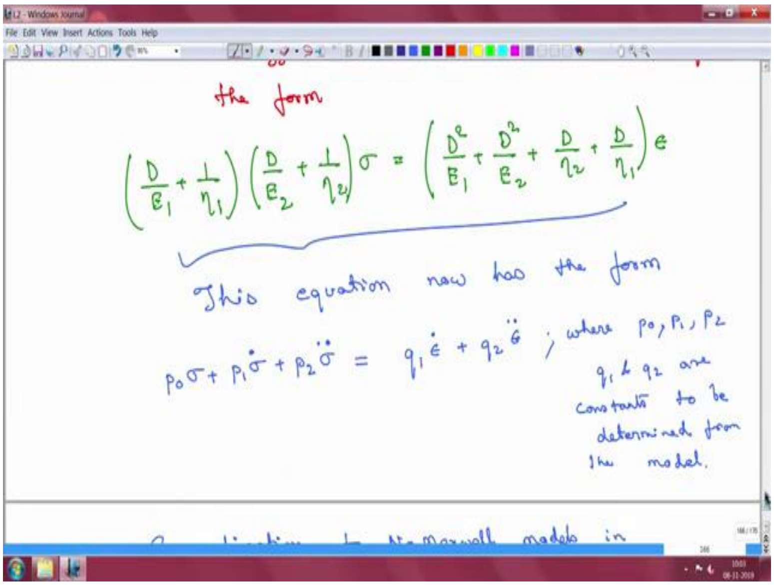

Remember we were write trying to always write it in some P0, P1, P2 and the rational for that is because once we generalized these basically you end up having a polynomial on this side and the polynomial on the other side. What the polynomial will look like? Depends on the particular model whether the powers or will be the same is not guaranteed it again depends on the particular model that you have choose whether or not there will be a constant terms also or again depend on the particular model.

So in this particular case for the N-Maxwell model we have a situation where we have polynomials in D on both sides where and the powers are the exactly the same. That was not the case if you recall for the Jeffrey's fluid where you had a mismatch in the higher powers and the powers of operator D in the two cases.

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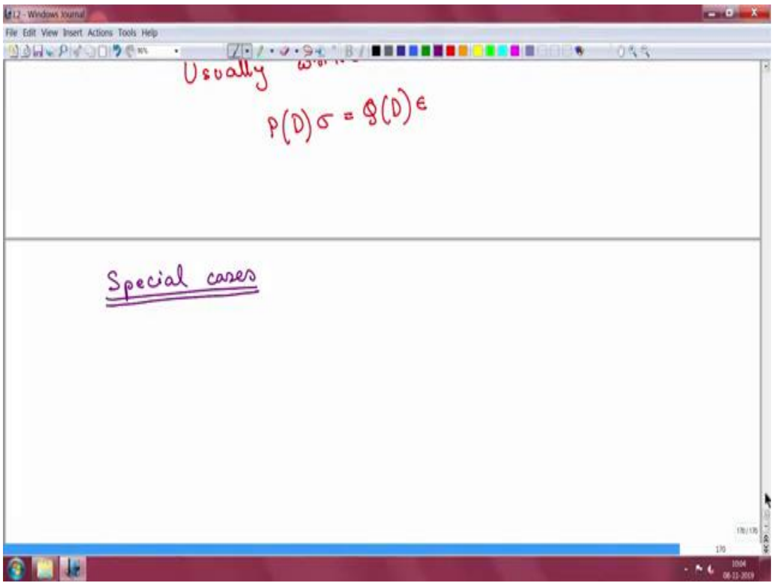

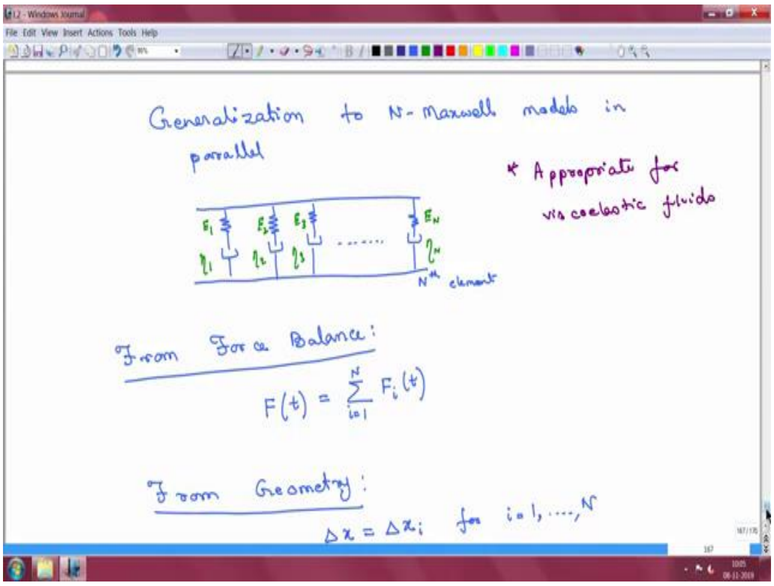

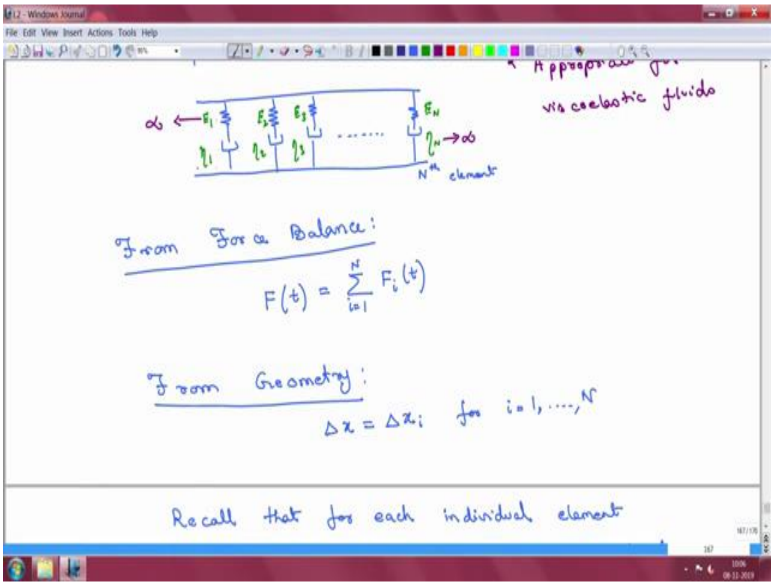

So, so now let us try to take some a special cases, so let us analyse this further. So this entire N-Maxwell model is going to behave as if this is appropriate for a viscoelastic fluid. But what happens so this is by the way so I just make a note here I think I said that in last time appropriate for viscoelastic fluids. Special cases so what if you want to, I will just quickly go back the diagram so if you want to have solid like response. What you would do here?

One thing you could do is take, for example, the last or any of the dashpots. And make this quantity tend to infinity. So if I take the last spring and I make the viscosity there tend to infinity basically what we have effectively is a spring. So all of the deformations will be taken by the spring and it will be determined from there.

Similarly, if you want to get rid of the initial instantaneous elastic response then the, if you take one of this springs and you can take any of them because I already take N here, I am taking another case where if you take this term and you make it tend to infinity then your initial elastic response will be gone and it will be determined by the viscous (())(13:15).

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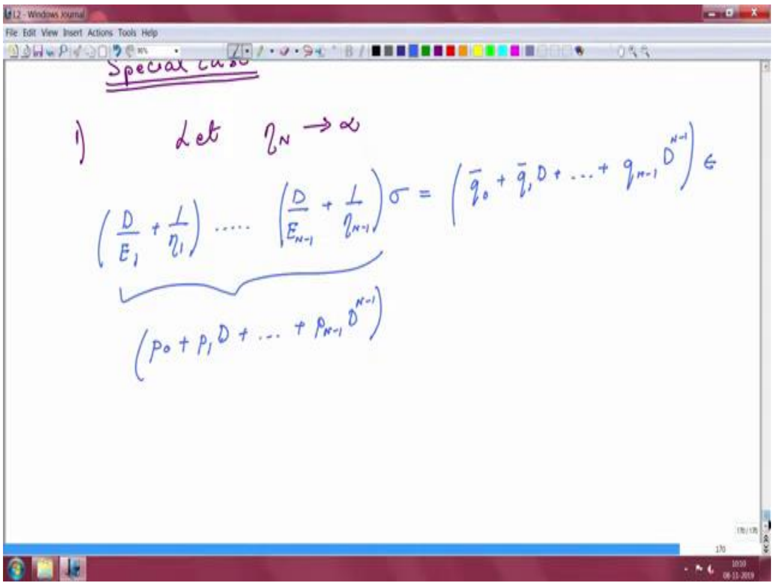

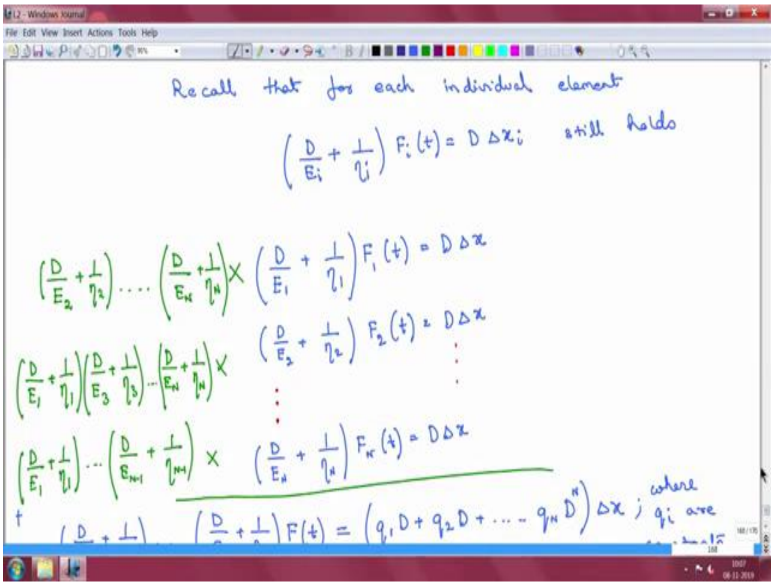

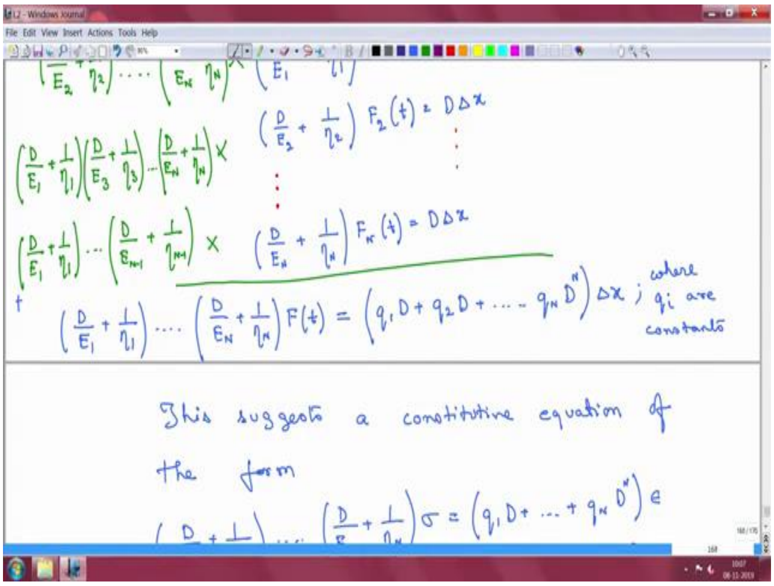

So there are different cases we can do so special case. So let us say, I am not sure how many cases we will be able to discuss. So let say the special case. So let eta N tend to infinity. So let us see, what really ends up happening. So if you take eta N to infinity you can redo this entire calculation for that case or you can take this final equation and you impose the condition that eta N is tending to infinity.

So one of the things that will happen is, basically for the last term here this quantity will disappear 1 by eta N and you will have the derivate, so the differential operator acting on a F and on this side you will have again at the differential operator acting straight on delta X. A little naïve way of thinking would be the differential operators can be actually cancelled but that is not the case what basically it implies is; now you have the simple algebraic relationship between the forces.

So you can actually drop one of these operates, when you drop one of these operates. So this term gets drop. If you, if this term gets dropped then the power on this side will go down by one. So at least first thing I can do is I can rewrite the left hand side and we can say that your left hand side would become and you have all the other terms. But this is now going to end at, N minus 1.

And what happen on this other side, the highest power if you try to redo this what you will is the highest power of the differential operator on the other side also decreases by 1 so it is N minus 1. But now you will also have a constant term because one of these, so the way the multiplication and the addition will occur is then you will have a term that does not have the differential operator.

And your because you had a another E and you still have that if you multiply this number will change so essentially what we have here on the other side if you rework is will you have a new set of coefficients and a new polynomial. And I am just going to use keep on using q but I may be just put a bar here to distinguish it from the previous q, sorry, this will be now be 0 and then you will have q1 the and all the way and now the highest power will end at N minus 1.

So this is interesting and one reason this is interesting is because this side also has highest powers of N, so maybe I can even write this one as some P0 plus P1, D plus P N minus 1, D N minus 1. So this is very a nice ODE because the polynomials on the both sides are very similar, it is only that the co-efficient are now going to be different. The powers are the same on both sides the highest powers you have all the different terms and then you also have this other constant term.

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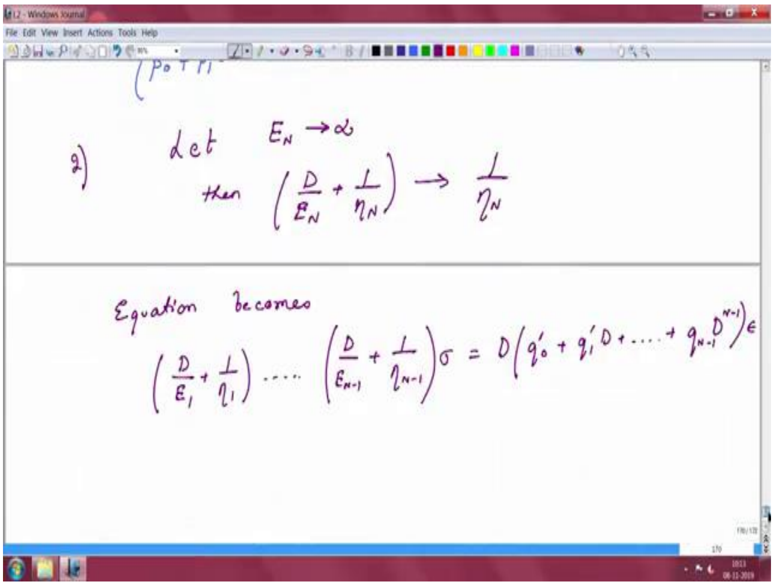

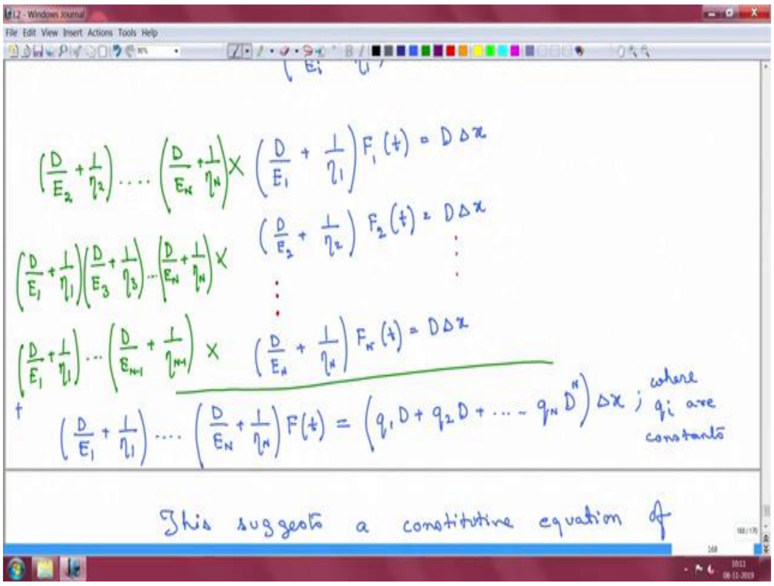

So another case, if you want, so let instead of the viscosity tending to infinity what happen is the elasticity tend to infinity. Be the elasticity tends to infinity and the last term here this particular quantity now disappears and you have 1 by eta N, FN and that will be equal to this derivative. So basically if you add up all these, then we will again have 1 powerless on the left hand side.

So then your this multiplier basically tends to this term become negligible. So your equation become if you rework this whole thing the equation now becomes, if you retain the factored form again you will end up with the last differential operator will be of the N minus 1th now. This is multiplied by F and on this side you will retain more or less what you had. So you can actually rewrite that as, so the coefficients will now be different than the previous case the original case.

So maybe I just put a dash here and I am just writing this so that here it look as if it has the same powers. Or I mean here you can write it in terms of force or the strain and both we know that they are equivalent so we can do that to this is strain, this is stress and that is strain. So you will that here now there is a mismatch of the highest power, on the left hand your highest power of the operator D is N minus 1, whereas on this side you have N.

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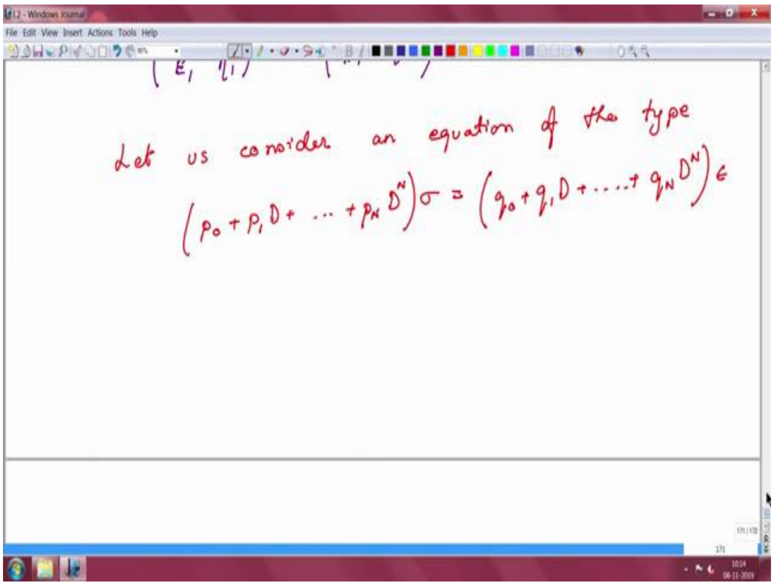

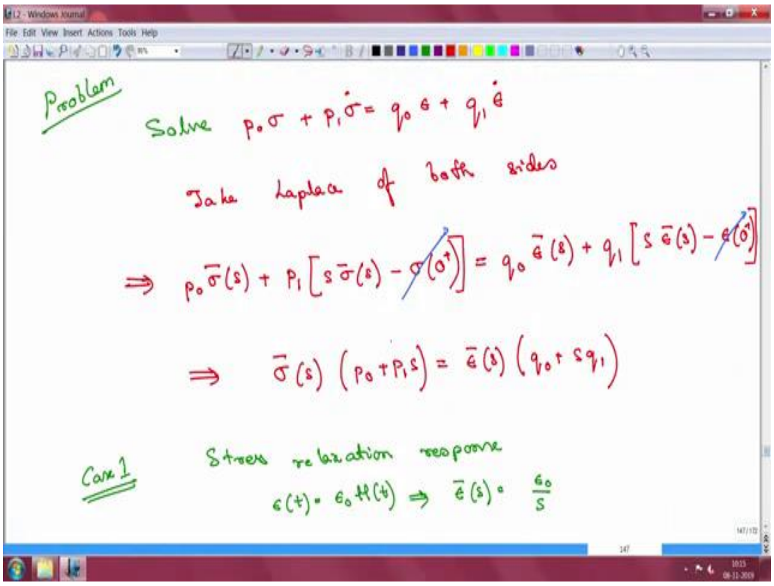

So let us take this type of a case, so let us consider, an equation of the type, P0 plus P1 D, P N sorry this is not legible, q0 plus q1 D plus all the way till qn DN into epsilon. So we are just considering this type of equation and we have to let say solve them. Now there is a, the jump condition issue will apply here and you have to redo the jump condition the same way that we have done it before.

And that is very big, quite cumbersome here so we will not go in details of how the jump conditions will work out but it will be very-very similar to what we had seen it before. And the way we will apply it is when we take Laplace of to both sides and that what we want to do just like the previous case. You remember if you recall, the last Laplacian.

So remember this particular problem when we are looking at solving this particular form and that why you see you can see that we are just recasting this previous equation in this particular way. When you take Laplace of both hand sides it is such, its, the initial conditions were such that they cancelled out.

In our and then you are the whole system become such that you are basically taking the Laplace of this particular equation without having to consider the initial conditions is almost equivalent to that, and so you here the derivative and whenever you have derivative you just have a equivalent S multiplied by that and that gives you the Laplacian.

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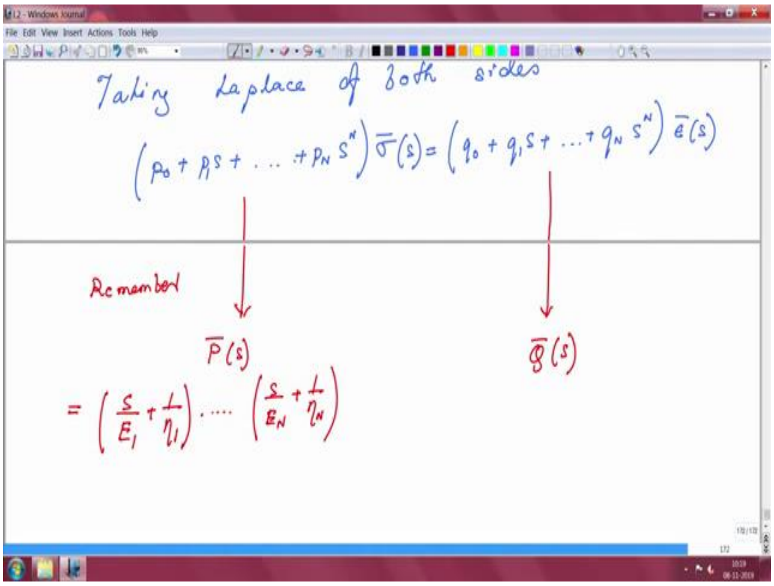

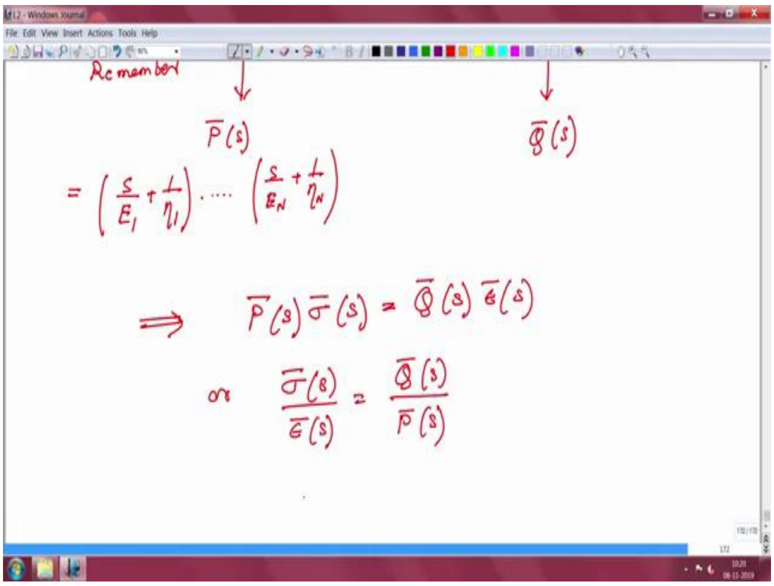

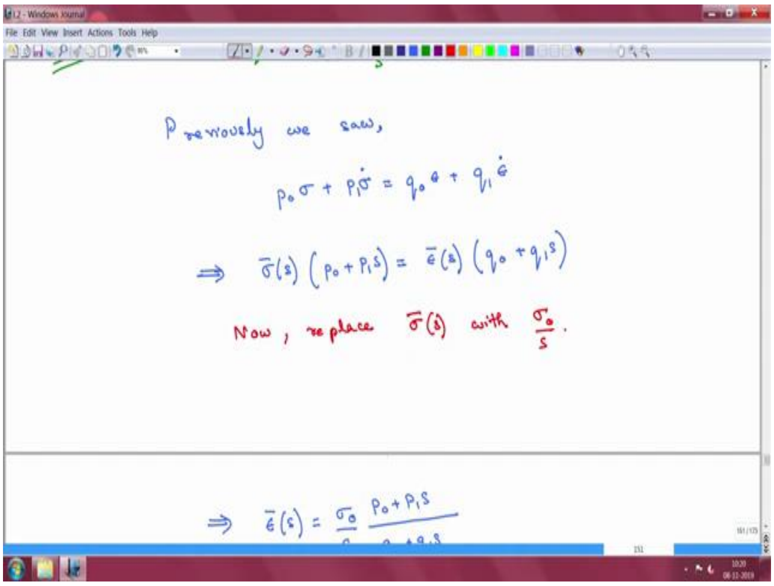

So even though we are not going in to the details of initial conditions in this particular case then you do take the Laplace of both on the both sides then you have, system such that you end up having all those initial conditions cancel on both sides. So when you do take the Laplace so let say taking Laplace both sides you have system says that you have P0 plus for every differential operator you basically have one S.

And this is going to be very nice for us because what you inessentially end up having is for the nth power of the differential operator you will have S to the power of N sigma, now this is in the Laplace domain so I will write sigma bar S and on this side you have q0 plus q1 again S the same way to the qN D sorry, not D that differential operator will become S to the power N and you will have epsilon bar S.

And this is such that this particular, so remember that the left hand side can be easily factored and this is basically or Laplacian of the polynomial now and I can represent it like this, and this is basically equal to S by E1 plus 1 by eta 1, S by EN, 1 by eta N and on the right hand side this is basically equivalent to, this is the Laplacian of the polynomial.

Now cube bar this particular polynomial may or may not easily factorable but it still a polynomial so it will have the n number of roots. So whether or not it is easy to find the roots it is a different question in this case the polynomial p can be easily factors so the roots are very well known.

So basically what you have is, so this form is going be to very helpful for us in solving for the different cases, so this form is very similar to what we had done before, so I will just quickly want to so you the similarity with what we had done before and that should help you understand how the solution process will work. So remember this is what we previously saw right in this particular case it was simple because the polynomial had only one power of the of the differential operator and then thing become simple for us.

And the same process that we have done can now be repeated for the case of the polynomial with the nth power. So what we do is we will end here and the next class we are going to take a look at a solution process and what are the different functional form so the stress relaxation function and the creep compliance is that something that we will see in the next class. So we will stop here today thanks.