Introduction to Soft Matter
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Lecture No. 30
Jeffery's Model

So welcome back to one more lecture on Introduction to Soft Matter. Last time we were discussing the solutions to the cases of the three-parameter model and it is different solutions.

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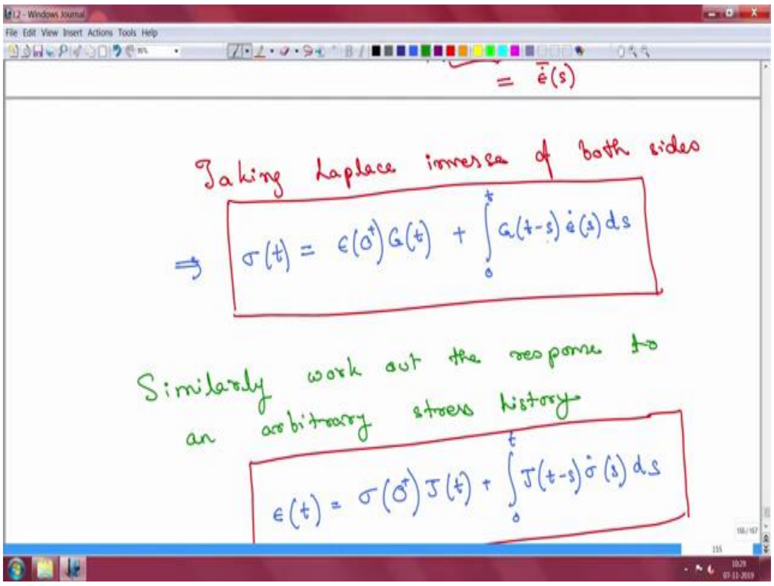

One of the things that I had like just like to point out before I start today's class is that the last expression that we wrote, and this is the expression of the Laplace in, when we took the Laplace inverse, we got expressions in the time domain. Now see this, I am using a variable s here also, but this is just so that I am consistent with what I have done before.

This is just this s here represents dummy variable for the integration, you can replace it with any other, you will replace it to the tau, you can replace it with any other variable you want and expression will still remain the same. So, this s here in the time domain should not be confused with the s of the Laplace inverse function.

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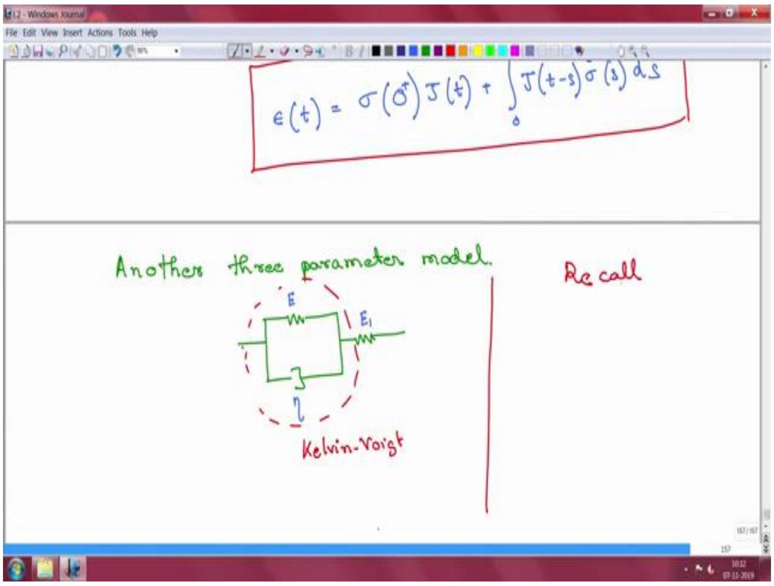

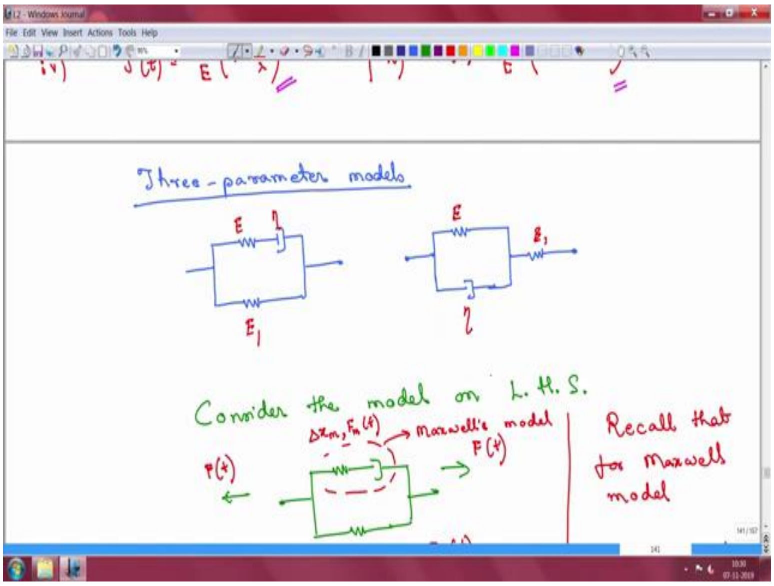

So, what we want to now do is we want to look at one more model. And so, let us take a look at we had to done one. So, we started off. So, the three parameter models, so, we had in the beginning we solved, we introduced two different three parameter models and we solved for this particular case. So, let us now solve for the other one.

So, I am just going to redraw this. So, the model looks like this. So now let us say this is E, this is my dashpot characterized by eta, this is my another spring which is E1. And here, so the obvious question one needs to ask before we will try to solve this is will this have a solid like behaviour or fluid like behaviour.

Now, this is a Kelvin Voigt model. So, we have put a Kelvin Voigt in series with another spring and that is what it is. So, the Kelvin Voigt, we know already has a solid like response because for a finite force, the displacement of the system is already bounded, it is going to be finite and the same holds true for spring.

So, even when you put these two together, where for a finite force, the displacement in both of them are going to be bounded. So, this represents a solid model. So, this, so this is appropriate for viscoelastic solid. So, this is, so this is a Kelvin Voigt Meyer model. So, I am sorry, I forgot Meyer, but, so this is a Kelvin Voigt, Meyer name should also be there. So, now let us recall what was the governing equation for the Kelvin Voigt model?

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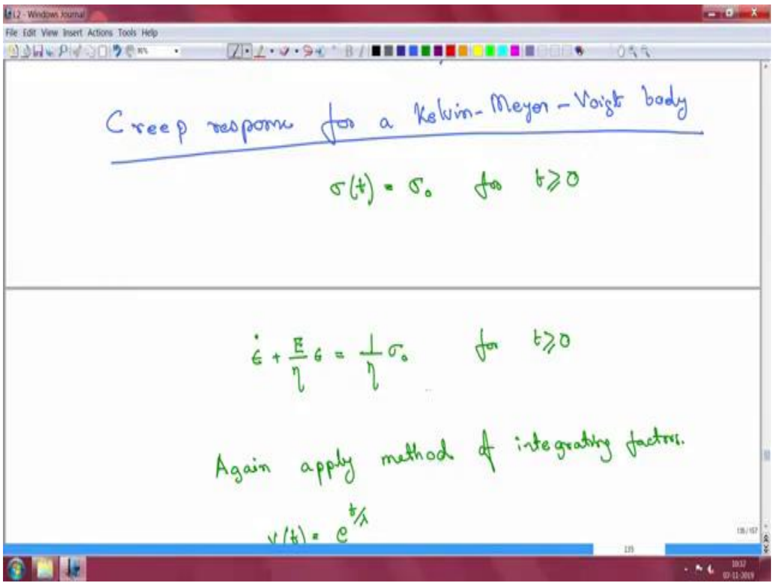

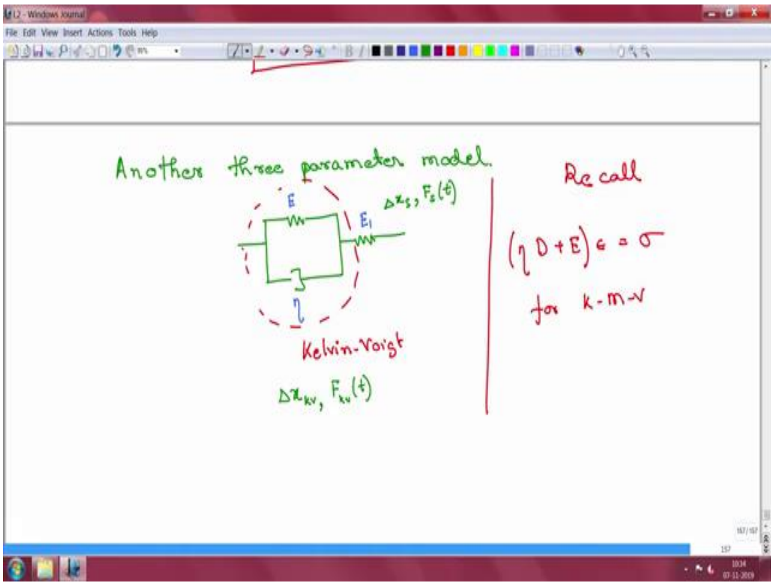

So, this will quickly go back, scroll back to where we are discussing the Kelvin Voigt and we had gotten this particular expression. So, now, just like the previous case where we introduced the operator D. I can write this as that for the part of the Kelvin Voigt this expression holds true, a Kelvin Voigt mayor. So, now, let us analyse this in the system such that, let us the displacement that is seen by the first body is going to be delta X, XKV and the force it experiences KV function of time. The spring on the other hand is going to see an extension of delta Xs and a force of Fs.

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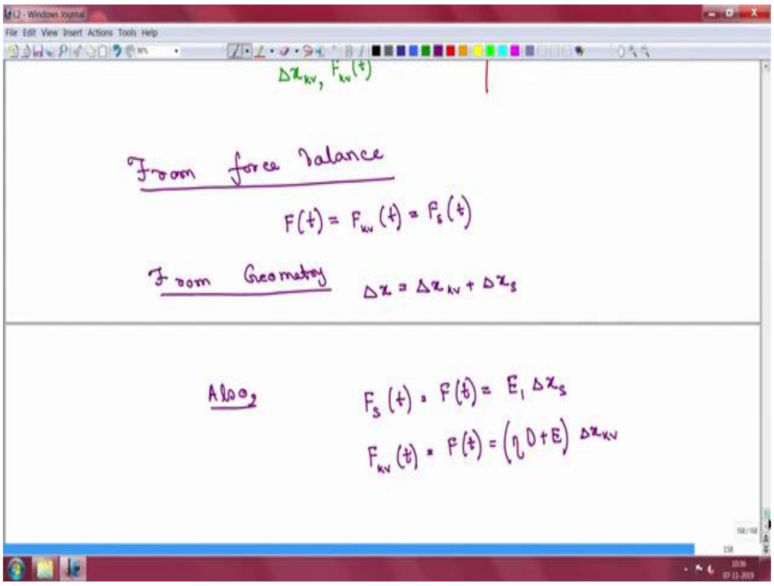

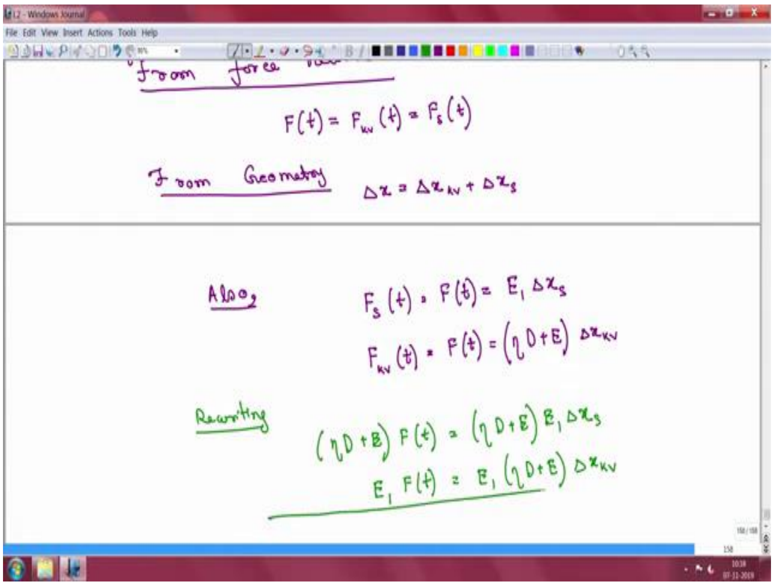

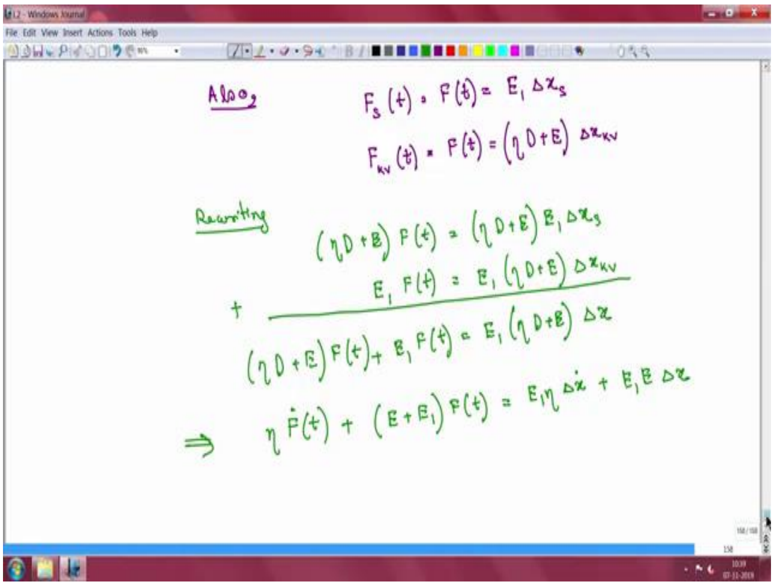

Now, these two, so I am treating them. So, I am going to treat this entire thing here as one system. So now I am just going to write a force balance. So, for force balance, so from force balance, and remember, these are all masses quantities. The springs, the dashpots, are all massless. So now, I am going to apply the force balance once again and in this particular case, in the case of the series system, you will see that the forces are all the same.

And then from geometry we get a different relationship for the displacements and that displacement is delta X is equal to. So, the entire system displacement is given by the sum of the individual displacements. And the relationships that we have are so, also given the individual constitutive relationships, you have that the force in the spring is equal to, is the same as the force in the system is equal to E1 times delta XS.

And in the Kelvin Voigt body you have the force, if I want to write that, this is again equal to the entire forces experience by the system. And now this is equal to eta times D plus E times delta XKV. Now, please remember that this D is the differential operator, so, it is a DVT here. So, I want a relationship between F here, the left-hand side in the force balance equation and the delta X which is again on the left-hand side of the geometric relationship,.

So, ideally, I will be able to get delta X, simply by adding these two, so to add these two. Now I do not want to take this into the denominator because this is actually a differential operator. So, what I want to do rather is I want to be able to keep it in the numerator itself and then add this up. To do that there is something very simple we can do, which is we introduce under the multiplicand.

So, I am just going to erase this because I just want to keep the order. So, we can multiply both sides or actually let me, let me leave it like that. So to add this, what we are going to do is we are going to multiply, so I am going to rewrite times of FT equal to rewriting the first expression, you want delta Xs. And in the second one I, for the second equation here, I multiplied that with E1. So, if I multiply E1 this is still FT, and we have E1, D plus E delta XKV.

So now this is in a form such that you can simply add these two expressions, so you just add and what you end up with is eta D plus E terms of FT plus E1 FT. And when you add these two together, then it has the same constant the delta X is and delta XKV at the same constants that just comes outside delta X. So, if I were to use this operator then we end up having, so, this is the same as writing F eta times F dot T plus E plus E1 of FT that is equal to E1 eta delta x dot plus.

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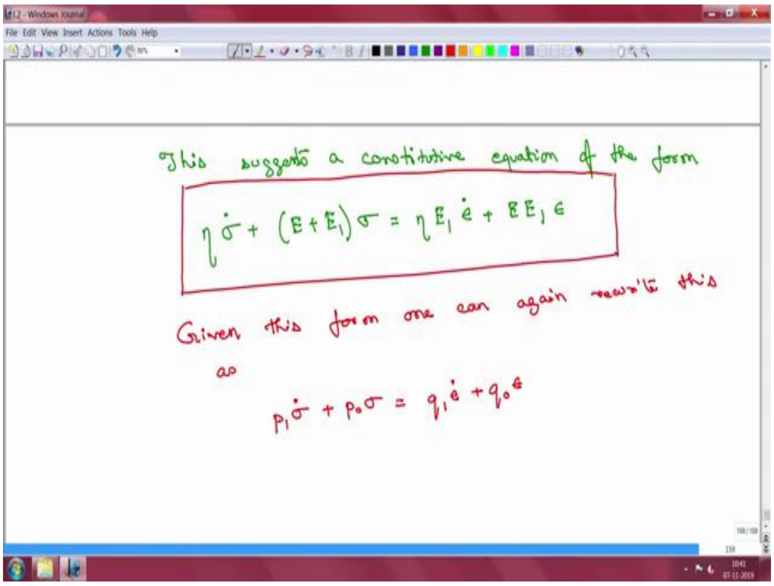

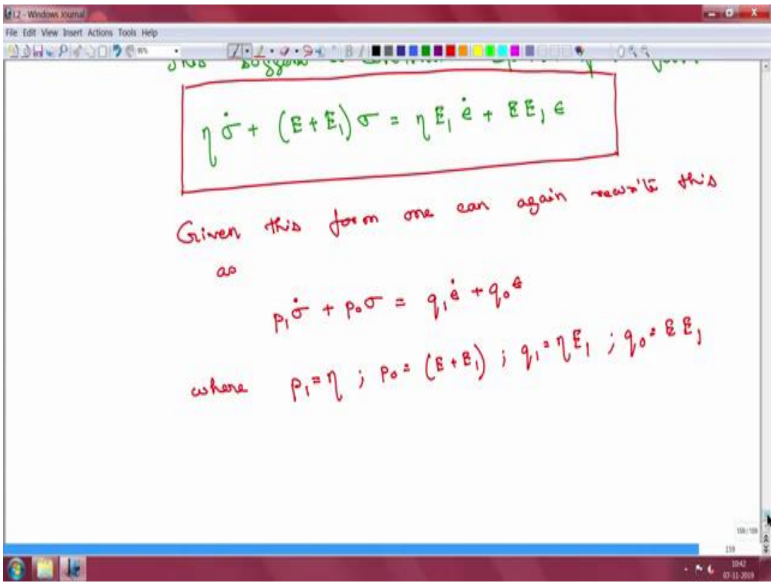

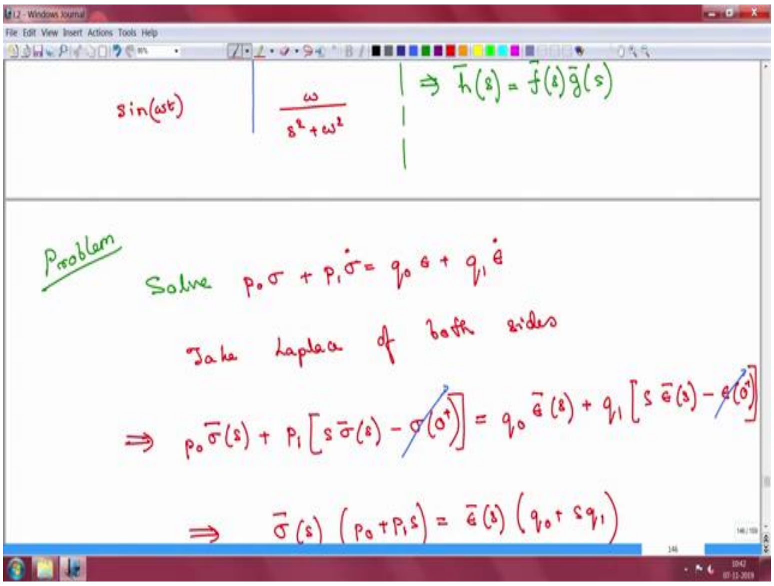

So, now, this suggests. So, this suggests, this is now the relationship between the force and the displacement. So, this will suggest, so this suggests a constitutive equation, equation of the form and this is the answer I was looking for. Now the important thing I want you to note here is that this is, this is again a linear ODE and the linear ODE has the derivative of stress, stress itself on the left-hand side.

And on the -hand side, you again have a derivative of the strain and the strain itself. And this is exactly in the same form that we had the other expression. So, this again can now be written as, so given this form, one can again write it, again rewrite this expression, this as being P1 sigma naught, sigma dot plus P naught sigma equal to Q1 epsilon dot plus Q naught epsilon.

The only difference now is that the values of the coefficients are changed. So, I have to now specify what the values are. So, this value will be P1 equal to eta, your P0 is E1, E plus E1, Q1 is equal to eta E1 and Q0 is EE1. So, now you probably can now appreciate why we were solving the entire thing in the form of these coefficients previously, because where we are solving this, so I am just going to quickly go back to the Voigt solution.

So, we were trying to solve this particular expression in the previous case, and when we did the Laplace there is nothing here which assumes anything about the model itself. The model builds in the way that the values of P1 P naught et cetera. are set. So, to now get the solution of this particular model, all you need to do is to insert the values of the P1, the constants in the solution.

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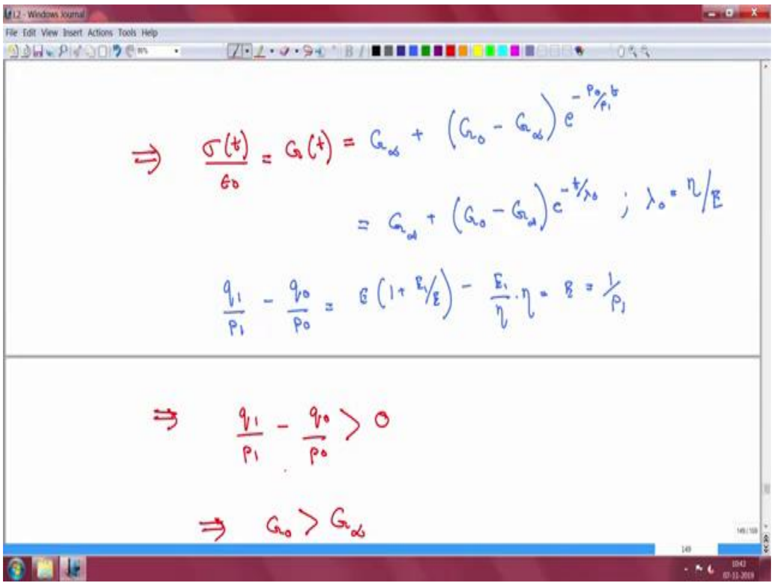

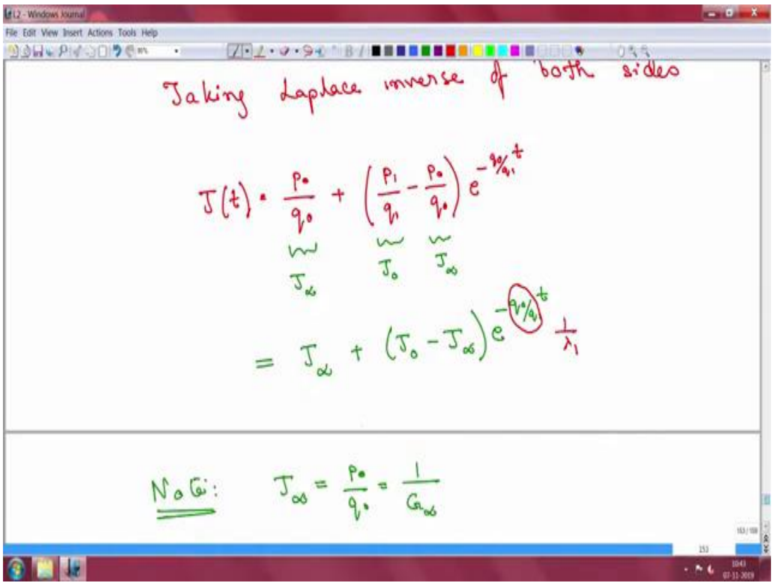

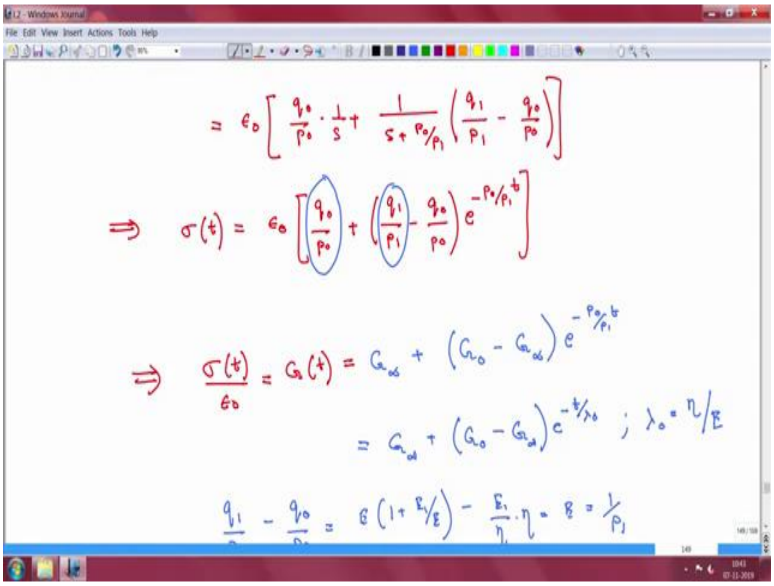

So, the solution for the, for the stress relaxation remains the same and so, does this expression events the same, among all our expressions actually remain the same. This one quick thing and that is remember that for this model to be physically realistic, when we had derived the stress function, the stress relaxation.

Then we had here a constant term and then we had this other term in the bracket which was multiplied by an exponentially decaying term. Now, for this to be physically realistic this particular fraction has to be greater than one, sorry greater than 0 for it to make sense, physically realistic system.

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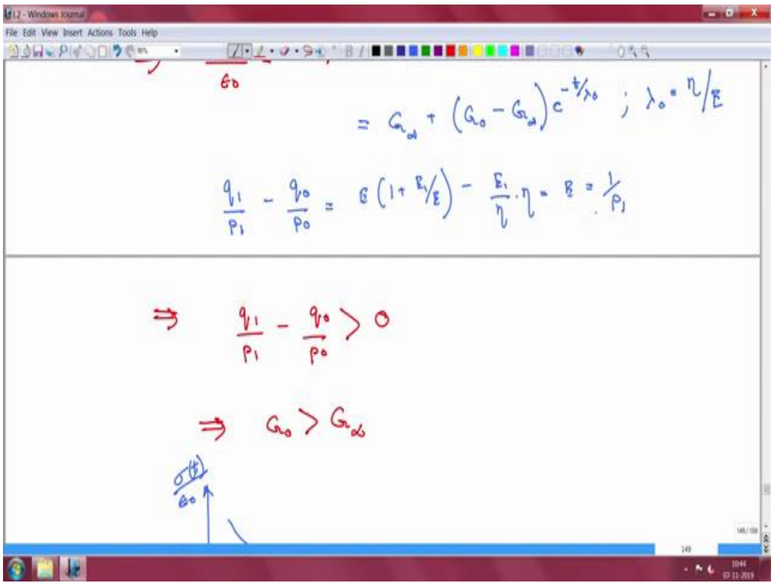

So, we had insisted that Q1 by P1 minus Q naught by P naught should be greater than zero, which it was true for this particular case because we saw that. So, we evaluated Q1 by P1 minus Q naught by P naught. And we got, we saw that this expression is actually equal to E and it is greater than 0.

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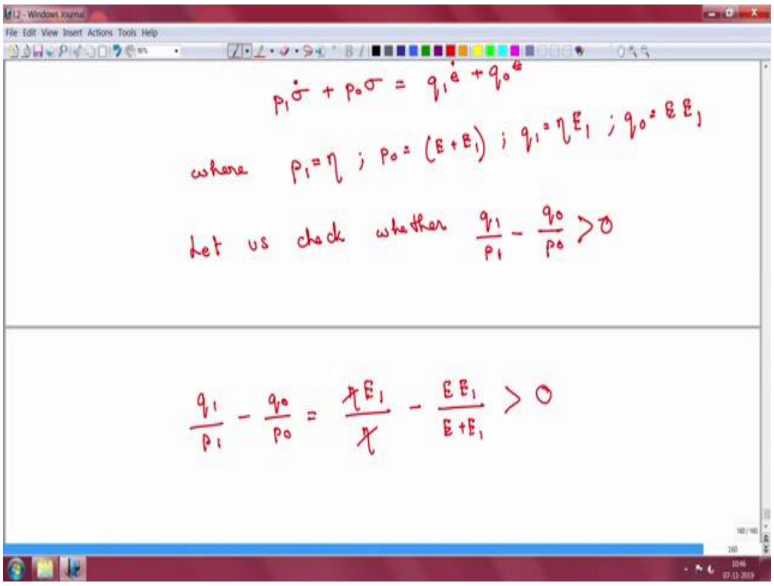

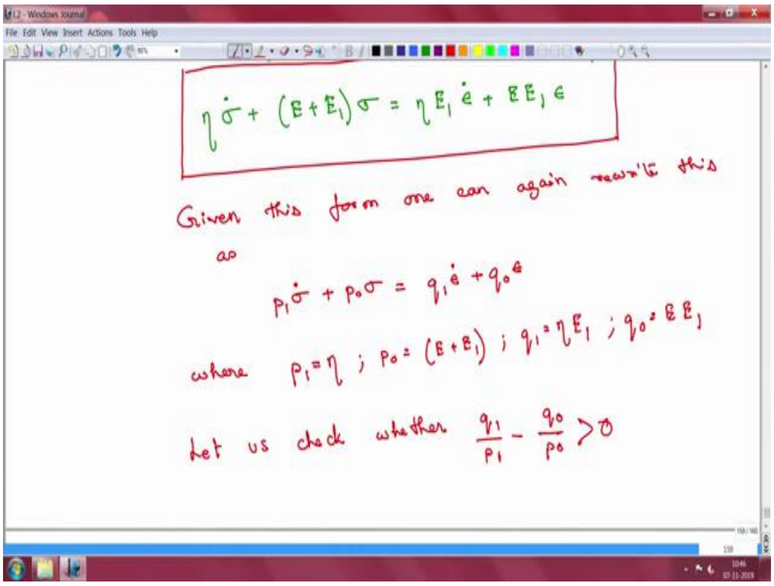

So, let us just do that here one more time. So, let us just ensure that it this is a physically realistic model and you will get solutions that are going to be realistic. So, let us check whether Q1 minus P1 minus Q naught by P naught is greater than 0. So, now Q1 by P1 minus Q naught by P naught is equal to it eta E1 by eta minus Q naught, this is E, E1 divided by P naught just E plus E1.

So, this is equal to E1 minus 1 minus E by this one which is, so this quantity here. So, this is a fraction which is so, just a second. So, this quantity is now E1 minus E1 multiplied by a fraction, and this fraction is less than one because this is, if you look at the functional form of the numerator denominator, this is less than one, so this quantity is greater than zero. So, in a sense we are saved, we do not have to worry our functional form is such that that the physically realistic angle is retained.

So, this particular system has the solution and that same solutions that you have seen before can be applied in this case. So we have done two, three parameter models, and both of which we saw are actually good for representing the case of viscoelastic solids. Now, it should be obvious to you that when we are doing the three parameter models that there will be one more obvious three parameter model where you would not have a spring but you have a dashpot.

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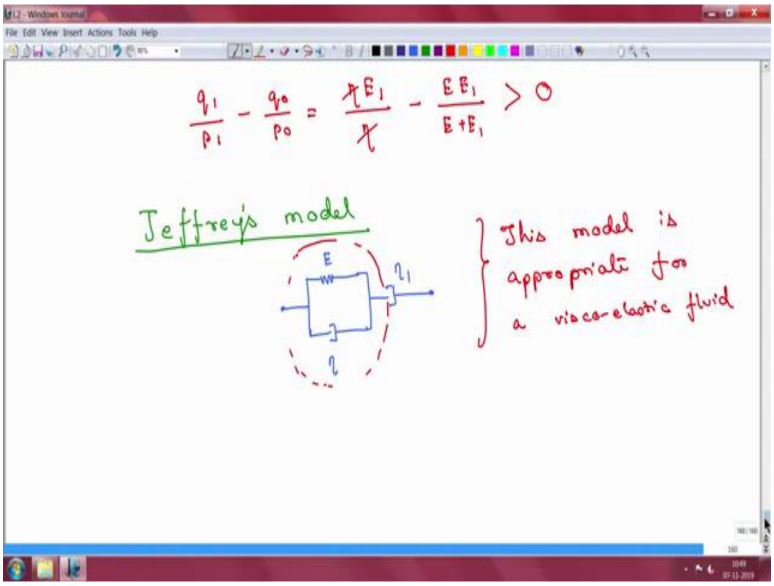

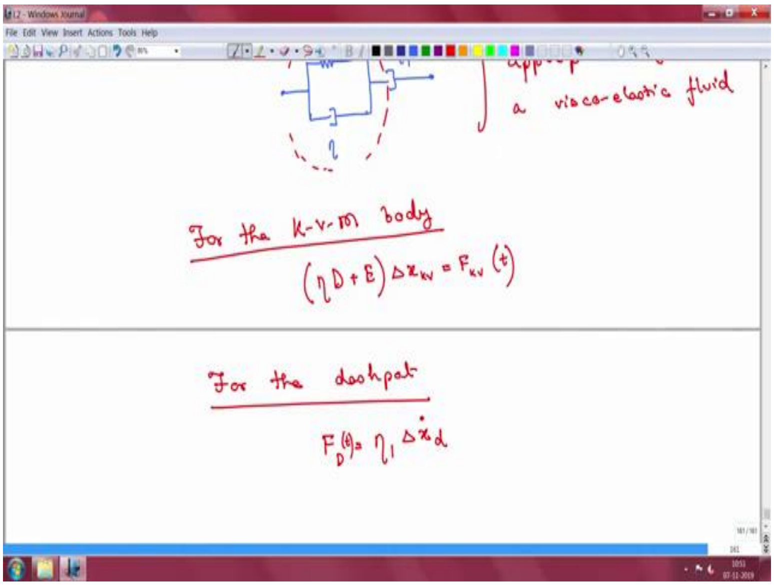

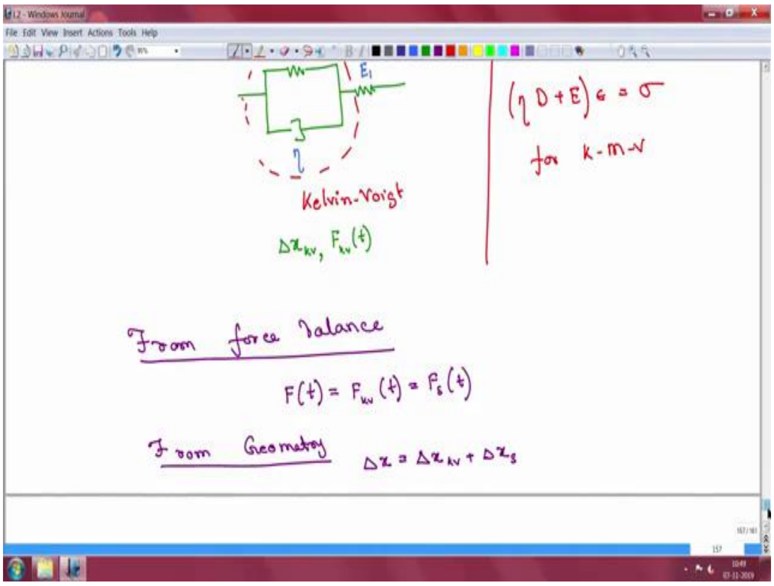

So, this model is also called what we have into going to introduce now is also called the Jeffrey's fluid. So the Jeffery's model, so in the Jeffrey's model, what you have is, you have a Kelvin Voigt system. So, you have an E, this is eta and I was going to say that this is eta and that is eta1, the same spirit that I have been. So, we are going to solve for this, but before we do that, we should ask ourselves one more time in as an intuitive idea whether this is going to be viscoelastic appropriate for viscoelastic solid or viscoelastic liquid.

Now, take a look at this particular system, this is obviously the Kelvin Voigt Meyer body here. So, when you apply a particular force, the displacement in this is going to be bounded. And, but for the case of the dashpot you cannot ensure that, so even for small finite forces, the dashpot will you keep on increasing or keep on displacing. So, this entire model will have a system level displacement, where this is, the displacement will keep on occurring even for small forces. And that is reminiscent of a solid, sorry for a fluid.

So, this particular Jeffrey's model is appropriate for viscoelastic fluids and that is why this is also often known as Jeffrey's fluid. So, this model is appropriate for viscoelastic fluid and hence said it is also known as Jeffrey Street. So, we have the Kelvin Voigt model once again. So, we are just going to borrow from last time what we had written, we had said here, we are just going to reuse this one more time. So, eta D plus E epsilon, so for F eta D plus E times the force is equal to, sorry, this displacement and this is equal to force.

So, I am just reusing the expression from there. For the dashpot we are just using the same methods. So, you understand that when I say delta XKV, then that means that the displacement in this the Kelvin Voight body,. So, none of that has changed. So, when I, when

I am using that you can understand that this force again is the force in this particular subsystem. So, for the dashpot, you again now have FD equal to eta 1 delta X dot D. And now we have to figure out what are the relationships between the different quantities.

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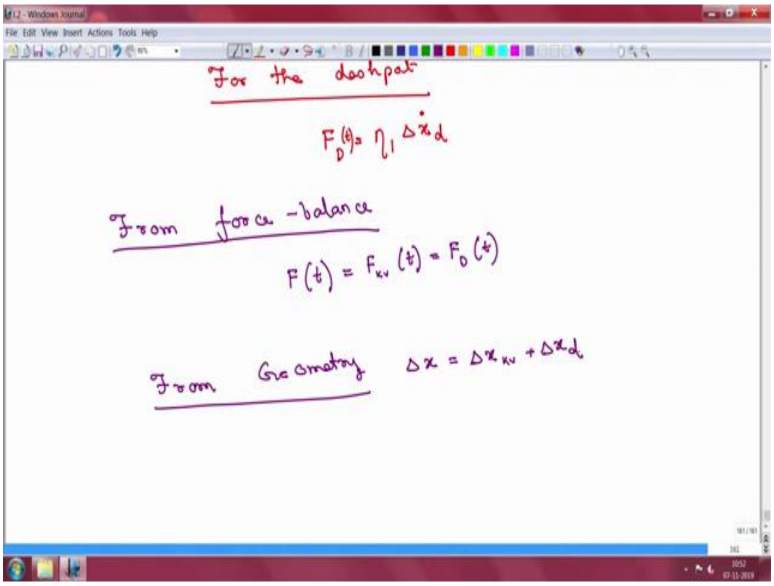

So, from force balance, now this system is actually exactly equivalent to the previous case. So, what we had written the expressions that we had discussed for the other three parameter model are actually going to be valid even in this case. So, the for the force parameter, for just for the force balance, we can just use the previous particular case and we can just say that the force in the system is going to be equal to the two individual forces.

And from geometry, we are once again going to have the same expression which was that the net displacement is equal to the delta XKV plus delta XD. So, now, what is where do we want to get to? We want a relationship between the force and this displacement here, the left-hand sides of the two, the force balance of the, from the geometry the two relationships that we have.

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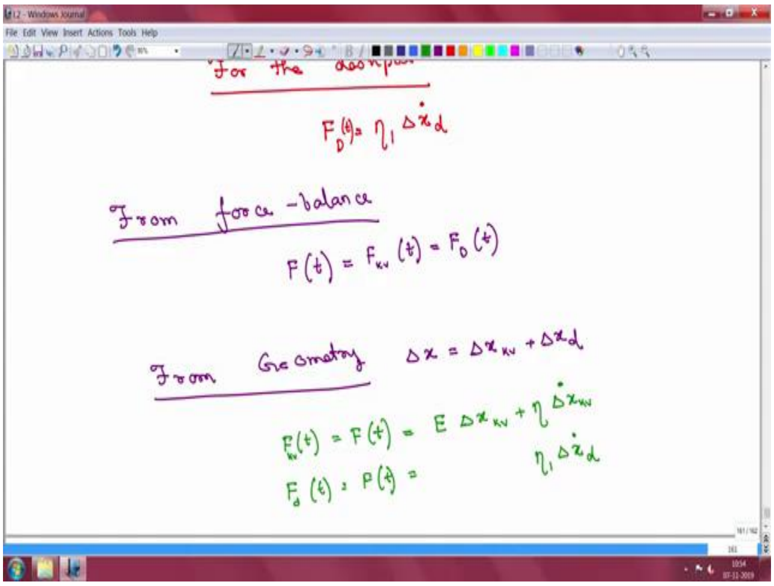

So, let us write down the expressions. So now we have FT, which KV which is the same as the net force is equal to it E times delta XKV. And in this case, I am just going to use this style. So, this is the expression for the first case. And for the second case, we have force in the dashpot again; equivalent to the system force because we have seen that and that is equal to now eta 1, delta X dot.

And now we want to add these two expressions. So that is what we want to do. And we cannot add these two up easily. Because you have different, she can. So at this point, what we have to do is that we have to simplify or we have to do the addition in such a way that we can add all these up and we can get the net force on one side and we could get the net displacement of the other side. So, this is something that I had leave you to do for the next class.

And then I am going to do, I am going to solve it for you in the next class for you. So, we will stop here today. And what we are done is we have looked at another three-parameter model. And we discovered that the solution for that is interesting the same as the solution for the previous case, except now the coefficients are something that are just going to be determined by the particular model. And we are going to see that the same is going to be true for the Jeffery’s case. And we are in the process of solving for that. So, next class will complete the solution for the Jeffrey’s fluid.