Introduction to Soft Matter
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Lecture 28
Three Parameter Model Cont

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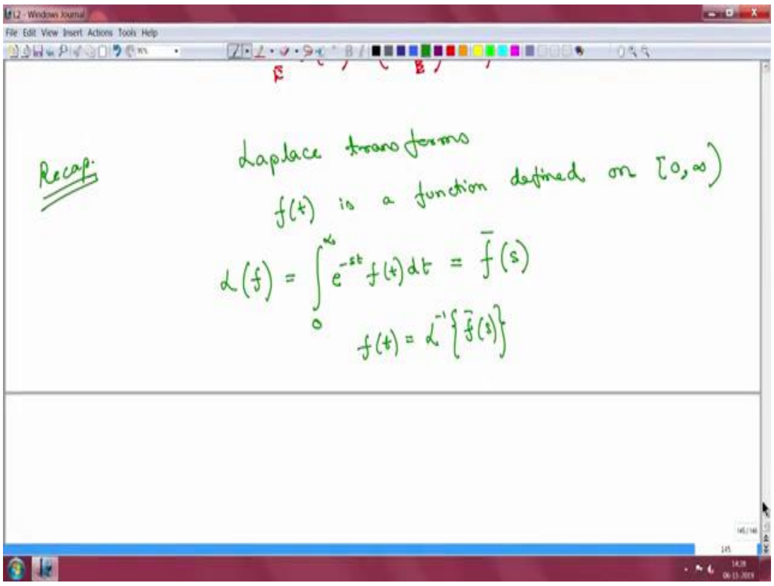

So, last class, we were looking at we just started looking at Laplace transforms with the intention of applying them to solve the ordinary differential equation that we had derived the constitutive relationship. So let us, we have not finished with the Laplace recap of the Laplace transform. So, I will just quickly go ahead and recap for a few other things. So, with this definition, you can apply Laplace to the Laplace operator to lot of different functions and you can show what the Laplace looks like.

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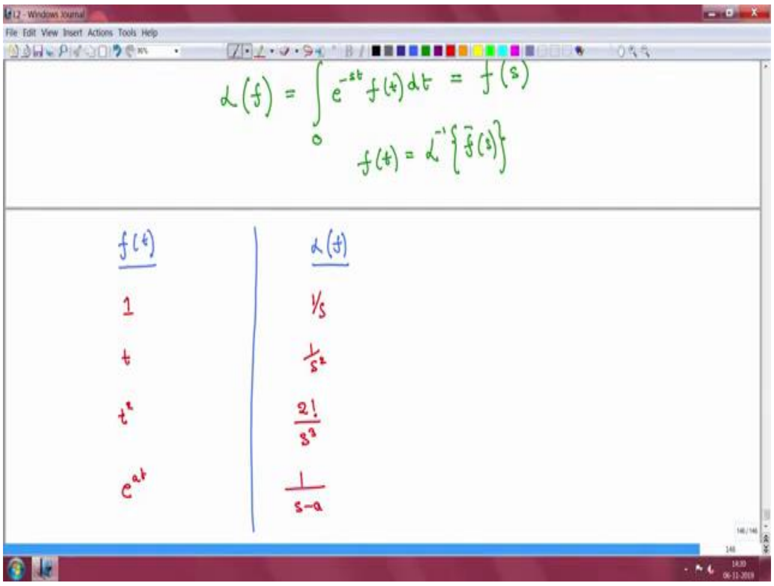

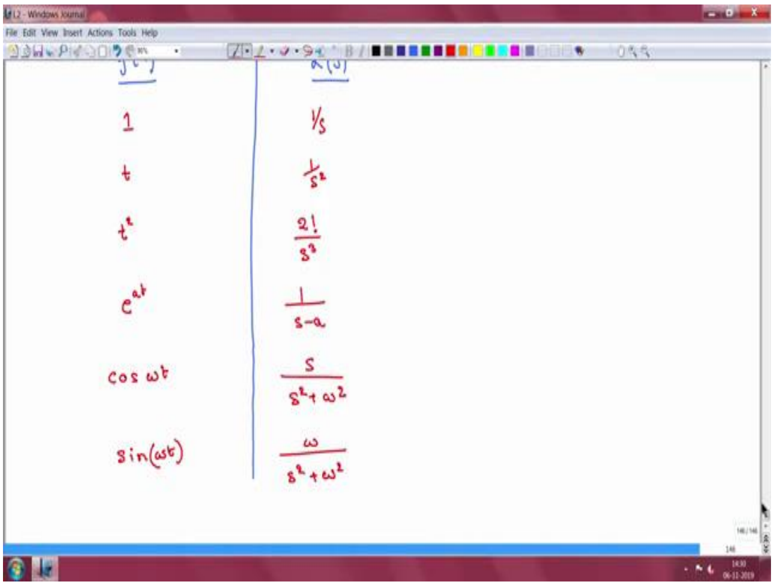

So, some commonly used Laplace transforms are, so if we have a function f t, this is I am just going to list the common Laplacian and if you want you can just go over them and you can derive them for yourself. So, for example, Laplace of 1 comes out as 1 by S. Laplace of T comes out as 1 by S square, Laplace of T square comes out as 2 factorial by S cube.

The Laplace of e to the power a t comes out as 1 minus 1 by s minus a is a fraction. The couple of more. So, for example, Laplace of cos omega t comes out as S by S square plus omega square and the Laplace of sin omega t comes out as omega by s square plus omega square.

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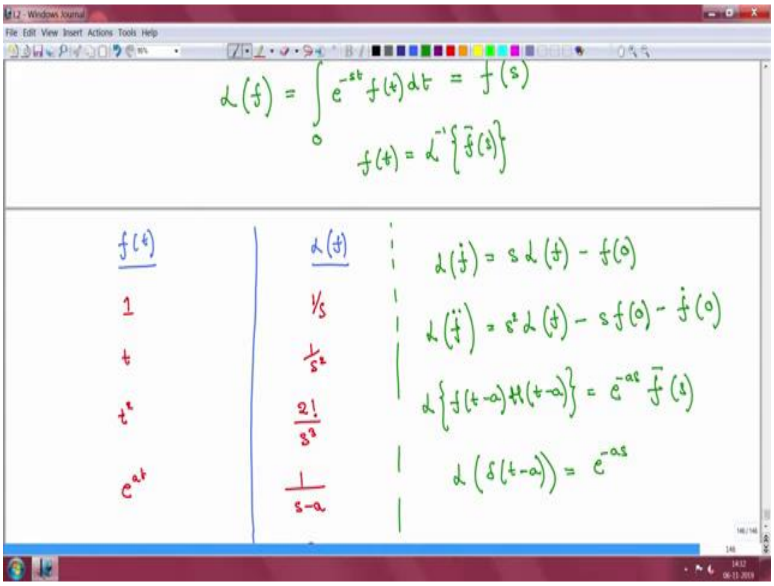

Some other properties that are important, let me just divide this space here. So, Laplace of the derivative of f comes out as S times Laplace of f minus f of at 0, evaluated at 0, the Laplacian of f double dot comes out as S square l f, Laplace of f minus S times f of 0 minus f dot 0. Laplace of f t minus a multiplied by the Heaviside function translocated at t minus a or the Heaviside function which is moved to a, this comes out as e to the power minus a s f bar of s. The Laplace of the Dirac delta function at a is simply e to the power minus a s.

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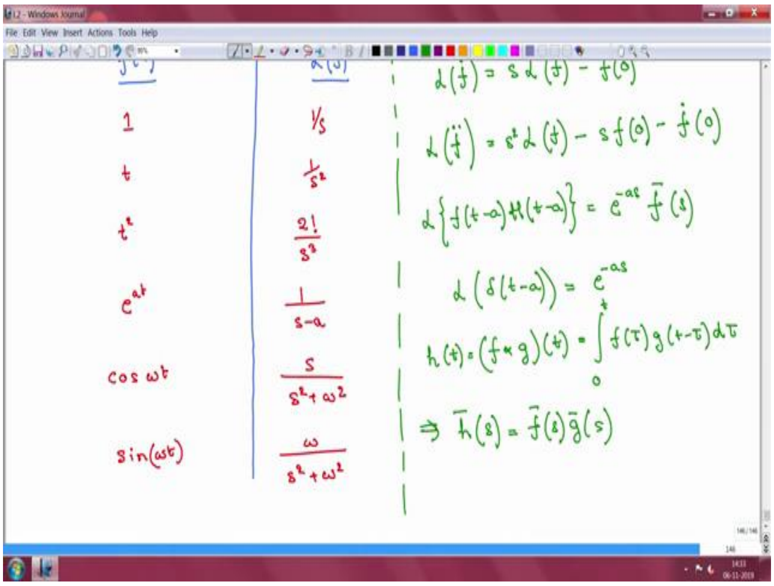

And finally, if you have a function h, which is defined as the convolution of two functions, so let us say you have function h which is defined as convolution of two functions here this star is the convolution operator defined as this particular integral f of sorry, f of tau, g of t minus tau delta, then the Laplace of h is the Laplace of the two functions simply multiplied with each other.

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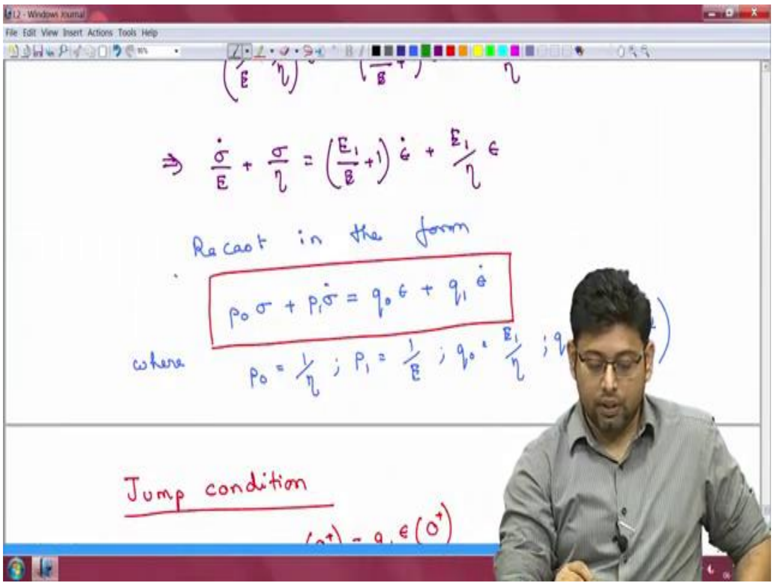

So, now with this quick recap we are ready to solve the particular ODE that we wanted to, what is that ODE in question? It was this, we have P naught sigma plus P 1 sigma dot into q naught epsilon plus q 1 epsilon dot, we want to solve this and at the beginning we will not care what P naught, P 1, et cetera. are. Once we solve this, then we will insert these values to get our result.

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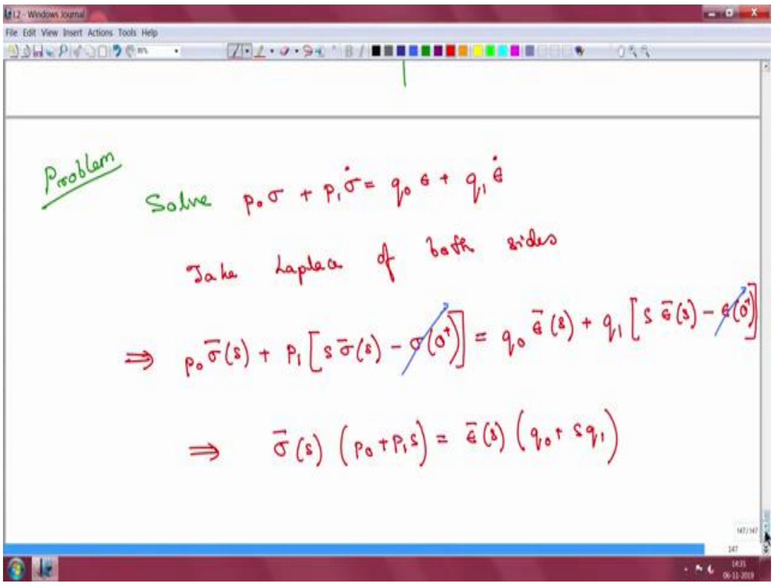

So, if you take the Laplace of this, you will. So maybe I will rewrite that equation we have. So, our problem is problem is solve this equation, P 0 q plus P 1 q dot equal to epsilon dot. So, let us take Laplace of both sides and if you do that, you will end up with P 0t q bar s plus P 1, please note that this is now a derivative, so you are going to apply that particular form that we had just see, equal to q naught epsilon bar s plus q 1 s times epsilon bar s minus epsilon 0.

Now, I had already said that this is from the jump condition which we did not derive and that was left as a homework for you. We can take this quantity out. So, we have left with a simpler situation, we have q bar s into P 0 plus P 1 s equal to epsilon bar s multiplied by q naught plus s of q 1. So, now, we have to take, so now we have to decide what we are solving for. So, this is very general Laplace one, the Laplace equivalent for that above equation. So, let us say we are solving the stress for the stress relaxation response.

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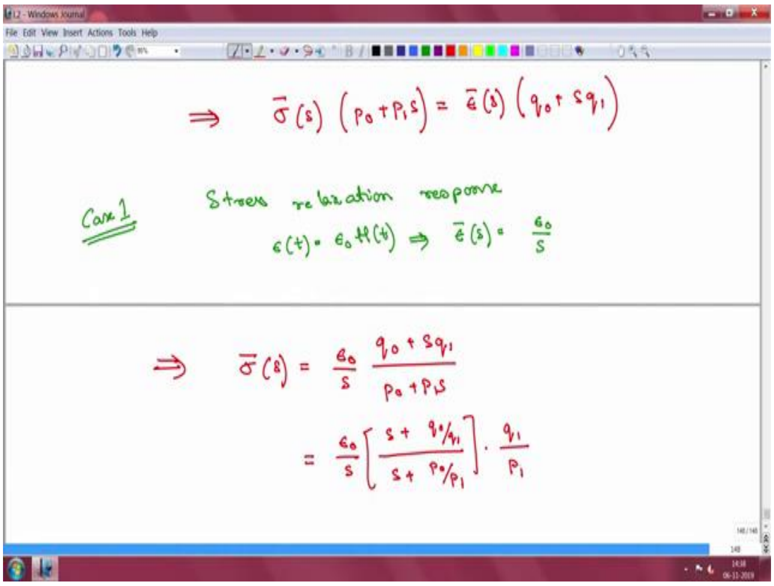

So, case one stress relaxation response. So, in this case, we know that epsilon t is equal to some epsilon naught into the Heaviside function. So, if you take the Laplace of this, we know that epsilon bar s is equal to epsilon naught by s. Now, we can replace, so here we did not know what epsilon bar s was and now we have a functional form for that. So, what we will do? Is just we will replace it, second.

So, let us go ahead and replace, so you have q bar s equal to epsilon naught s, you have q naught by s, q 1, P 0 plus P 1 s. So here we have, now, if you take the Laplace inverse, you will be able to compute what the stress is, but this is a sort of rational fraction format, if we have to simplify this a little bit before we can easily take the Laplace. So, let us go ahead and try to simplify your right hand side.

So, left hand side you do not have to do anything anymore, the left hand side you have to, the right hand side has to be simplified. So, we will try to write it as s plus q naught by q 1 and. So, now I am going to try and get to a simple rational function format for this and I just divided here by q 1, so I multiplied by q 1, here we did that divided by P 1.

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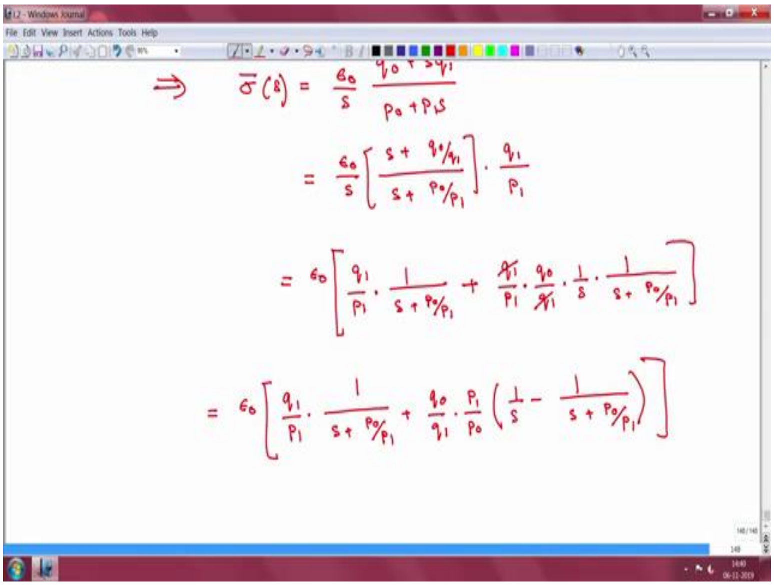

So, now this is equivalent to epsilon 0, this s I am going to take it inside and I am going to open the bracket here and I will see what I get. We have q 1, P 1, 1 by s plus P naught by P 1, this s and this s it gets cancelled for this first case. In the second one we have q 1 on P 1 multiplied by q naught, q 1 into 1 by s, s plus, so this simplifies. So, this part I have already got into a form which I can easily take the Laplace inverse of.

If you look at the tables of the Laplace function the general Laplace functions that you know that we can easily take the Laplace inverse of this. So, we have, so this first part, we will leave it as it as it is, we still have to simplify the last part a little bit before we can take the Laplace inverse. So, what we will do is? This part, we are going to write it as two other functions q naught by q 1, sorry I have to this will be P 1 by P 0.

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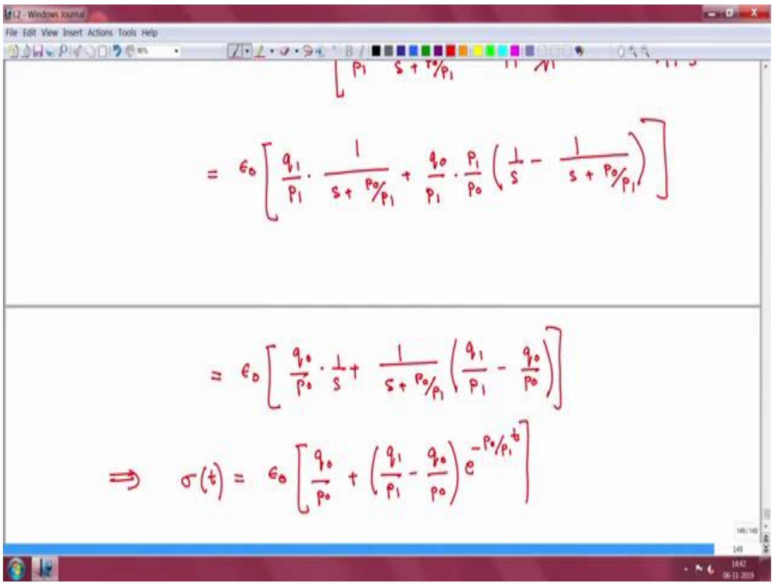

So, now if you take the two, so now you have these two are common. So you can combine these two, and then you have rational function of the form 1 by s. So, you have basically two separate rational functions, one is q naught by P naught, 1 by s, this, so this is not q 1 this is P 1. So, then you have the other one part which is s naught plus P naught by P 1 and that is q 1 by P 1 minus q naught by P naught.

So, this implies if you now take the Laplace inverse of both sides, you will get on the left hand side you will get the stress sorry on the right hand side you on the left hand side you will get the stress on the right hand side you will recover these terms, so q 0. So, we know that the Laplace of 1 was 1 by s. So, this is just the fraction and on here you have q 1 by P 1 minus q 0 by P naught, e to the power minus P 0 by P 1 t.

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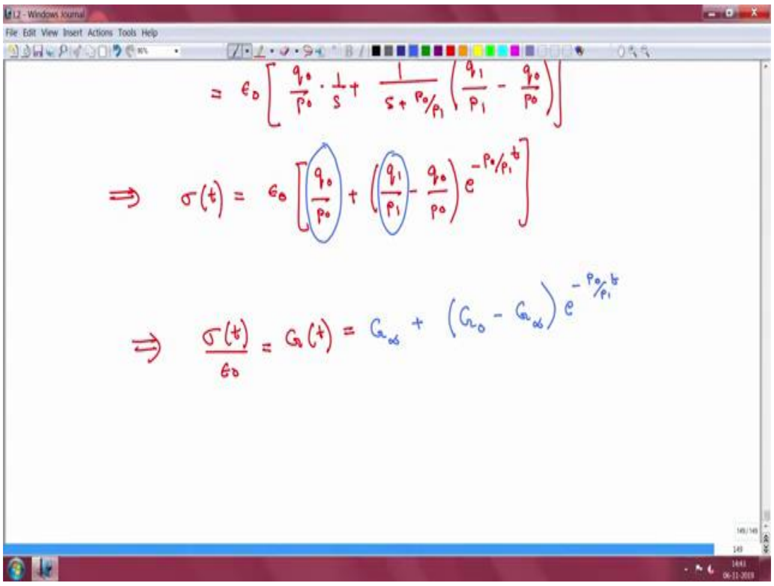

So, if I just divide these two, you have sigma t by epsilon 0, which is a stress relaxation function, right. So, we are seeking that anyway that we get and now that will have two parts, so this part which is a constant and then you have this part minus again the same fraction here. So, if I decide to call this G infinity, then you have the function can be written as some G naught minus G infinity e to the power minus P 0 by P 1 t.

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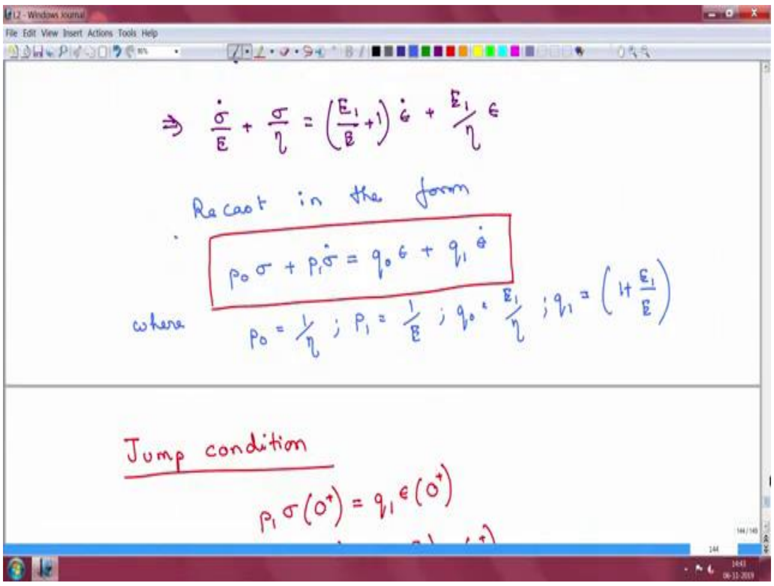

What is P 0 by p 1? Let us quickly go back and check. So, P 0 was 1 by eta and P 1 is 1 by E. So, this is back to our lambda, eta by E.

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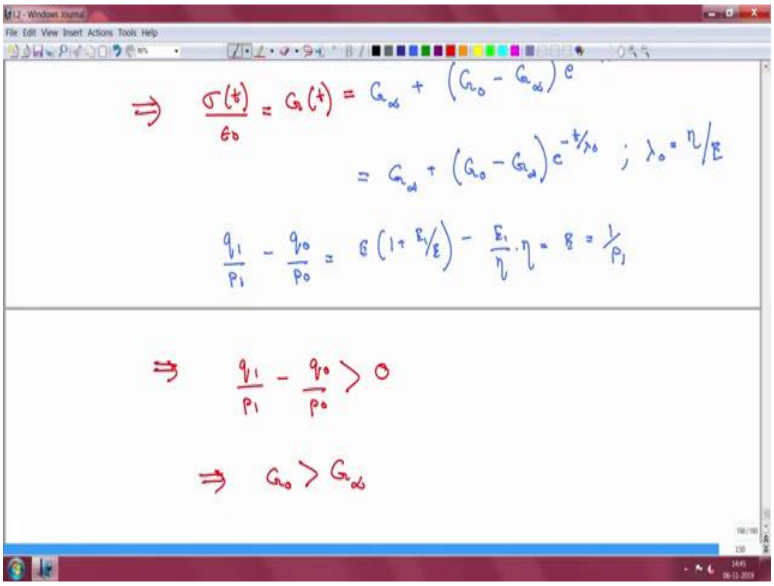

So, this quantity I can replace and I can or I can just say that is equal to this G infinity plus G 0 minus G infinity e to the power minus t by lambda. Actually, what we will do is? We will call this lambda 0 and we will see why that has to be there, where lambda 0 is now eta. Now, convince yourself that this quantity is in fact positive.

So, what is q 1 by P 1 minus q 0 by P 0? Can we quickly calculate this? So, if you calculate this, this should come out to your E times of 1 plus E 1 by E minus E 1, eta into eta equal to E equal to 1 by P 1. So, this fraction is greater than 0. So, is actually a positive number, which implies at your G 0 is greater than G infinity.

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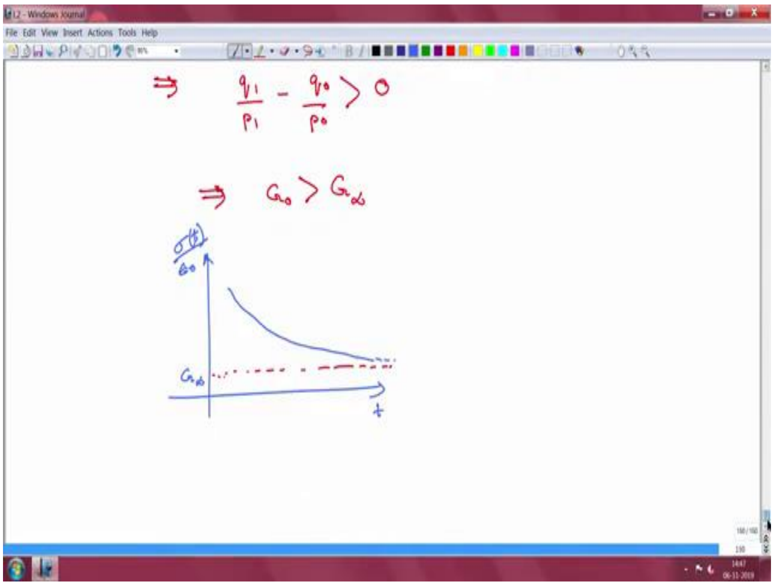

So, if you wanted to plot this curve, what it will look like? It will be some exponential function, some constant, plus an exponential decay will there, so the exponential decay will be will not go down to 0, because you always have this constant G infinity that will be left. So, this is your sigma t by epsilon 0, then this will be G t, maybe it is not legible, let me rewrite this sorry.

So, this was your plotting that this is your G infinity. So, G infinity is a nonzero value, so, your stress relaxation function does not decay down to 0, which is a viscoelastic solid type of response. And that is what we had said in the very-very beginning itself that this by intuition are simple physics based arguments.

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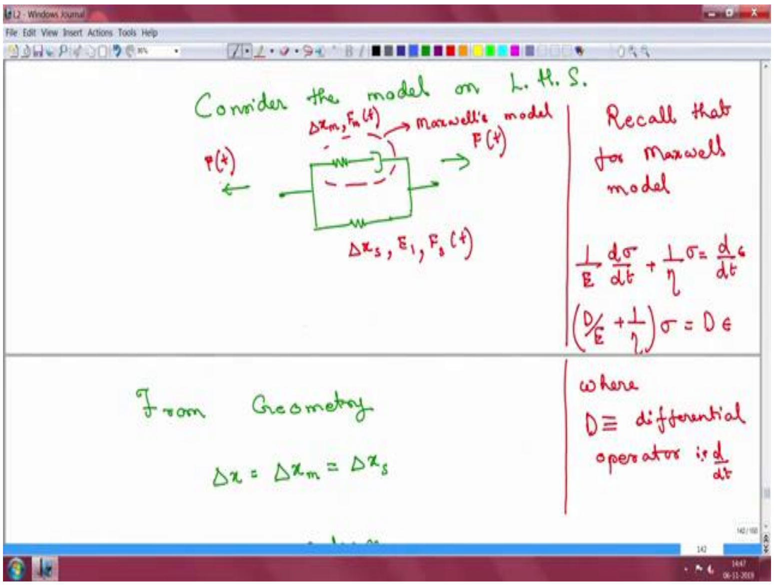

We had said that this model has to behave like a solid body, because the displacement and the system, the springs displacement is arresting the total displacement of the system. So, this has to be a solid like situation and that is what exactly what we have found out that this behaviour does satisfy that of a simple solid or a viscoelastic that this response is in line with what we would expect from viscoelastic solid.

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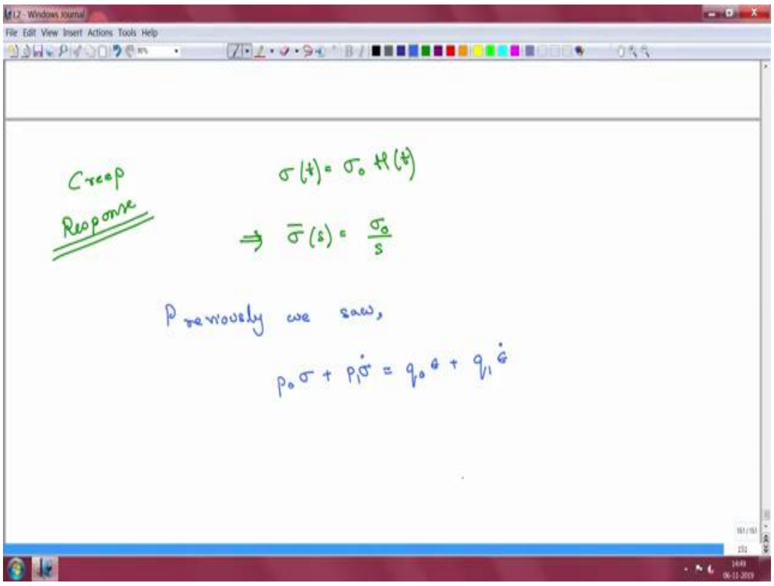

So, similarly, to what we have done, we can now also look at Creep response. So, in Creep response you have your sigma t is equal to some sigma naught, into our Heaviside function and this implies again that if you take the Laplace of both sides end up with sigma naught by s. So, previously we saw and we will just rewrite that equation because otherwise it becomes difficult to solve it without looking at it.

So, what we saw was, P naught sigma plus P 1 sigma dot was equal to some q dot epsilon plus q 1 epsilon dot. There is one more thing I want to point out that there are four coefficients here in the ordinary differential equation, but your original model only has three constants E 1, E and eta. So, the 4 coefficients here are not independent, there are actually in reality only 3 coefficients and the fourth one is a dependent one.

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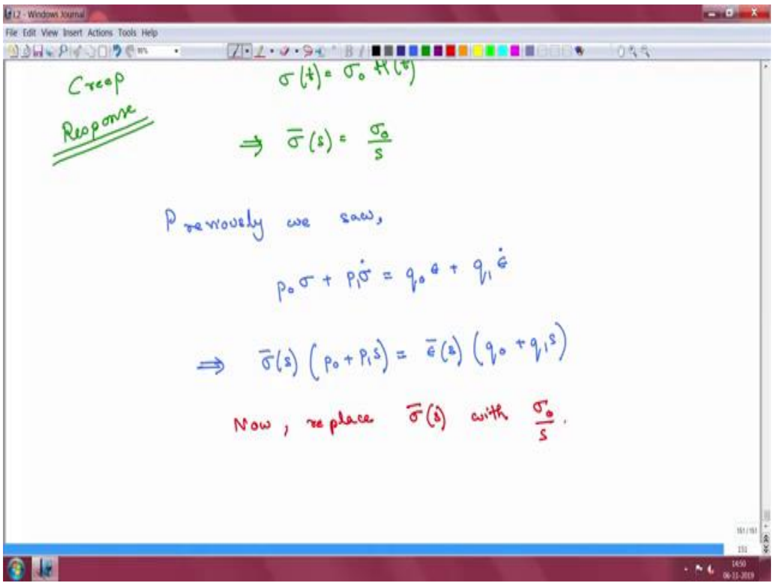

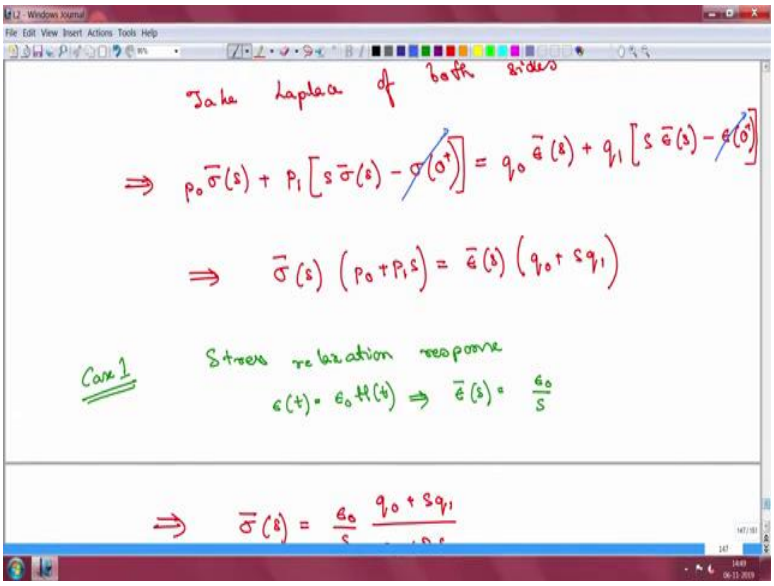

So, now, we again we are this part is something that we can still reuse. So, we will just write that, sigma s, so, from the constitutive equation you have, this is generally true, for this ordinary differential equation, we have not applied anything till now. So, now apply now, replace sigma bar s with sigma naught by s, and what I want you to do for the next class? Is I would like you to give it a try, where you replace this and you try to solve the equation all over again.

And now, you are instead of solving for the stress relaxation function, you are going to solve for the Creep function. So, I would like you to do that and I would like you to convince yourself that the system response is still in line with the idea of a viscoelastic solid, if it is not, then we have a problem. So for today, what we will do is we will stop here, and I would like you to do this and next class I will work it out by myself. So we will stop here today, thanks.