Introduction to Soft Matter
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Lecture 23
Lagrangian and Eulerian Perspectives

So welcome back to one more lecture on introduction to Soft Matter and last time, so in the lab we had a very nice session where we took a look at actual rheology characterization of some of these fluids and we took a look at how these fluids can be, so how some of the theoretical concepts that we discussed earlier on in the class.

How that looks like in a real experimental situation and some of the things that you probably noticed is that there was noise the curves are not as well behaved obviously as we are drawing it the theoretical setting. At the same time the idea of linearity which seemed most likely would see would have felt to you that must be valid in many cases seemed quite not so applicable in someone of the case at least one of the cases that we saw.

But those experiments you have to do multiple times it has to be repeated which we did not do for the interest of time you can also use different geometries to study to the same rheological characteristics. We did not discuss that because this is an introductory course in the introductory course we are not delving into all the details.

So, if you take a more advanced course that those are some things that you would get to understand, but now that we have taken a look at the relaxation phenomena, we want to ask ourselves as to what are some of the simple models which can give us an elementary relaxation, more behaviour. And before we go so we will look at a model known as Max's model we will also look into the original paper which in which Maxwell offered this model to us.

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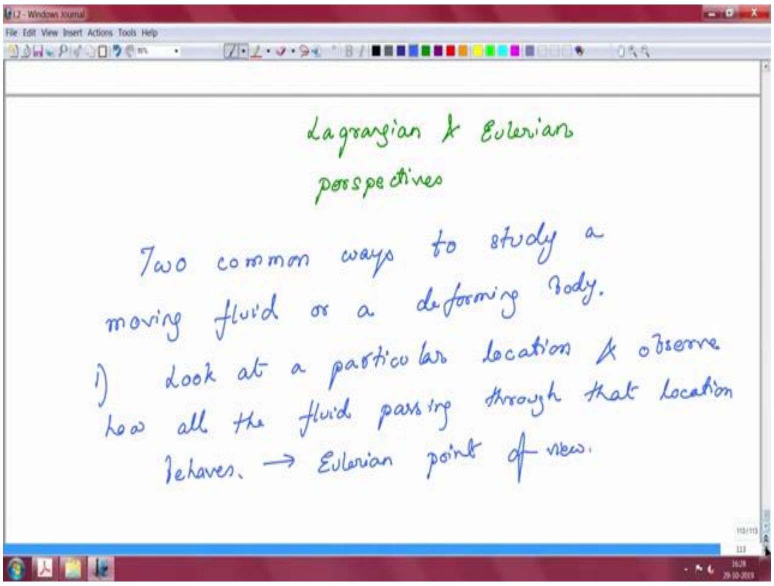

And but before we do that we want to take a quick look at a couple of important topics which are the Lagrangian and Eulerian perspectives. There is an important reason why I am discussing this and it is often not clear in many of the text books. When they derive the Lagrangian Maxwell's model as to what is the basis for application. So, we want to make sure that our case is well understood.

So let us discuss these two important perspectives. Now Lagrangian and Eulerian perspectives are two perspectives by which you can observe or explain deformation of a material or a flow. And these are two model, two perspectives which are based on the person who is observing the flow. And so the two common with these two common ways to study a moving fluid or a deforming body.

So the first is the Eulerian. In the Eulerian one, what we do is we look at a particular location and then we observe how all the particles are all the material points are flowing through that location and how that behaves. So here what we do is we look at a particular location and observe. So this is just like an observe how all the fluid passing through that location behaves. This is the Eulerian point of view.

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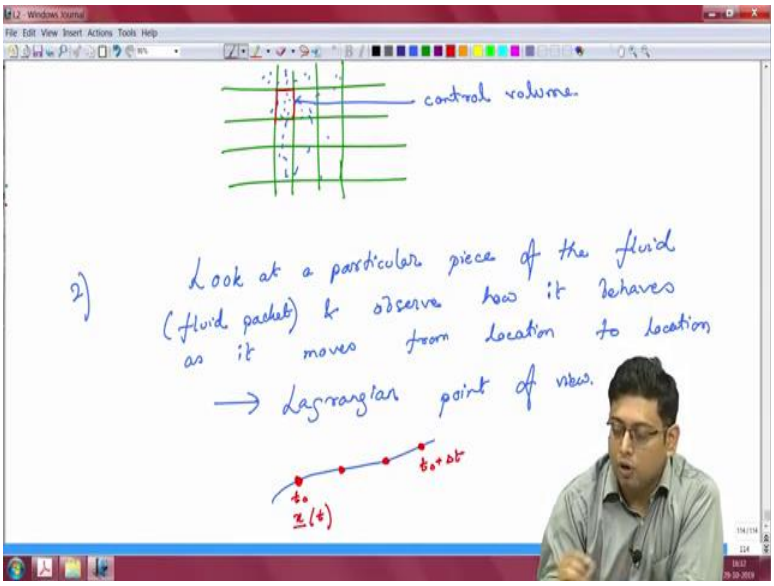

So what do we mean here? We mean that in this condition we will create, let us say you have you will create a mesh and this makes meshes are static and you have all the different particles, say these are fluid particles. And these fluid particles all have a velocity and they are moving in different directions. Now instead of observing the individual fluid particles you say that I will confine my attention to this particular window.

So let us just say this particular window I am just taking an example and then you will say that, I will look at all the different fluid particles behaving how they and how they behave as they pass through this, but once they pass out I of this control window of this section I will not bother. Then it is to be studied in the next control window perhaps.

So I only confined my attention to this small section and you can probably realize this is your control volume. And if you take this same control volume and then you say that the in the limit of the size as it shrinks down to 0, that will also become your differential control volume.

So this is Eulerian perspective where you do not track individual fluid particles but rather say but my the way I am going to look at it I am not going to bother about all the different fluid particles, rather I will keep my view fixated on certain meshes or grids. And these grids are your control volume and I will keep I will quantify the behaviour in these grids.

So this was 1, so the second one is the Lagrangian perspective where you look at a particular piece of the fluid packet. So bracketing and put a fluid packet and observe how it behaves as it moves from location to location. This is your called your Lagrangian point of view. So what do you do here? Let us say you have a fluid particle. And let us say it is possible for you to identify that particular. So, how you identify that I am not discussing that.

But let us say somehow you are able to identify this particular particle and this particle particular particle is now going to move through these different locations at different times and this is your path of that. So, this is let us say at some time T naught so this is some T naught plus delta T. So, this is the initial position so your position now the position vector is a function of your time.

So, this is your Lagrangian point of view where you are looking at individual fluid particles and you are following them. So, why are we discussing these two by way? Why is this important? Well because when you apply Newton's laws they are applicable for a fluid particle when you are looking at it from a Lagrangian perspective.

So Newton's laws are applicable to this mass as it moves through but it may not be it is not applicable in the same way in the in its original form to this control volume. So, for application to the control volume we have to do something different

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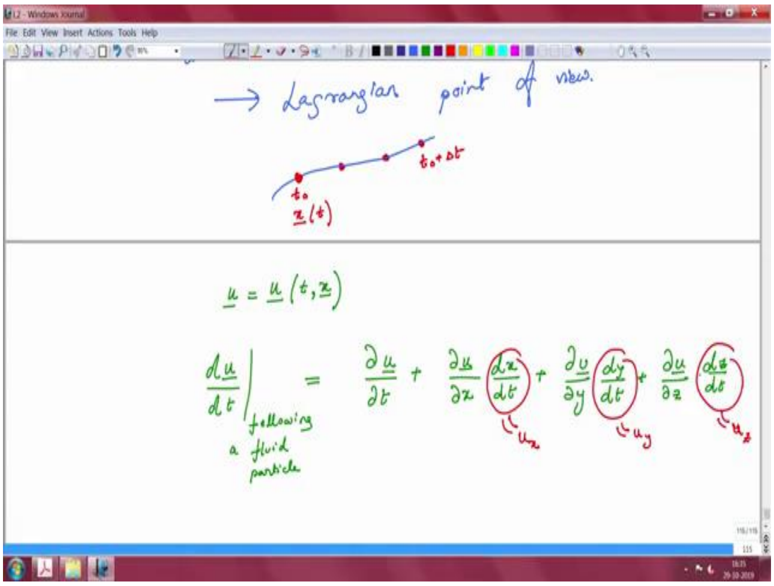

So the difference is in the control in the Lagrangian point of view is that, for example if you have the position vector that is just simply a function of time here. The position vector value itself is changing just as a function of time. Whereas, if you have if you are looking from the Eulerian point of view, then and let us say you are tracking the velocity field in a differential control volume, then your velocity vector is so you are u is now a function of your time as well as X bar which where X bar is now the control volume location.

So when you apply the derivative if you want to apply that derivative you want to apply it such that the derivative with respect to time is following a fluid particle. And if you are looking at it from the Eulerian perspective, so if u is an Eulerian variable you cannot compute this time derivative easily. So for that what you have to do is now it is a multi there are two variables here. So, you have to individually take derivatives and by the chain rule you will know that you know the derivative will look something like this.

So I am just applying the chain rule here, because here to evaluate this was not to evaluate this I must apply the chain rule and look at it from that perspective. Now you will notice that in the case of the Eulerian system, these are nothing but your Eulerian velocities

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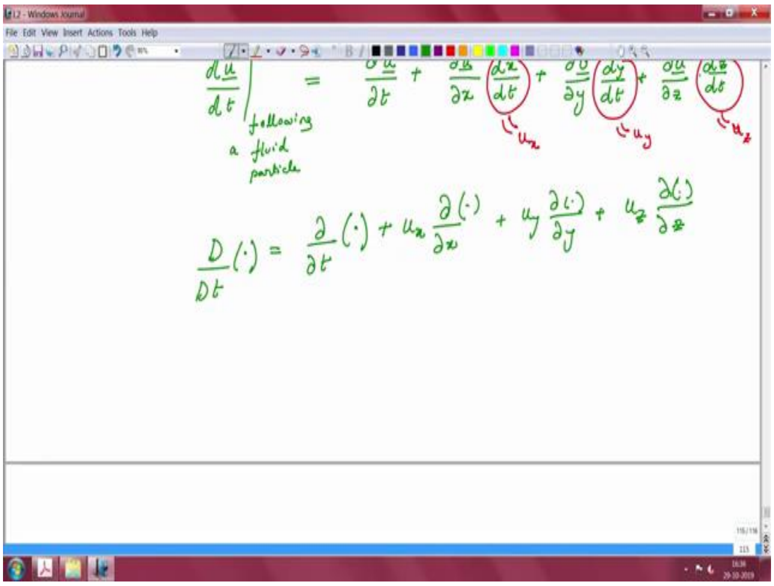

So that is why you have the concept of when you take the Eulerian derivative, in the Eulerian sense you have to take material derivative and that is usually a derivative of this form. So, I am just so I am saying when you have it if this is an Eulerian variable, then when you take the derivative following a particle you have to take it in this particular form. And this can be any quantity, it can be a scalar it can be a vector and. So with that small introduction we are now ready to discuss constitutive relationships.

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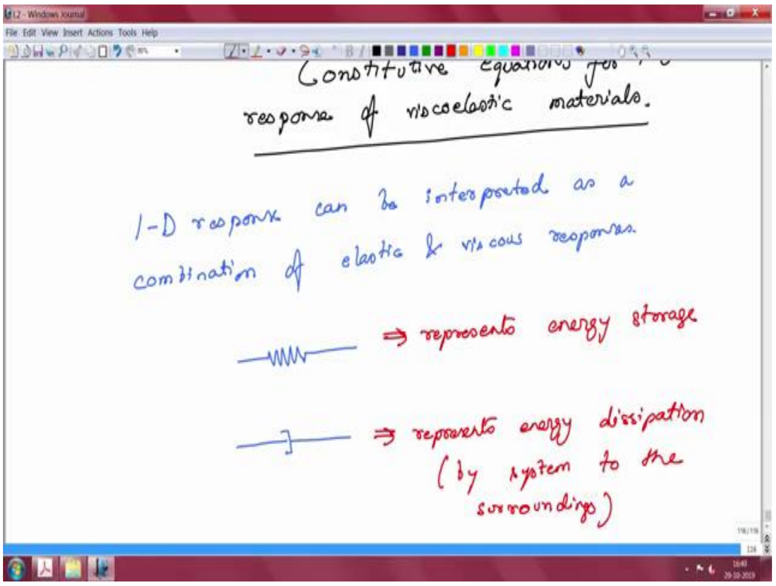

So what we want to do now is we want to look at constitutive equations for 1D response of his riocolastic materials. And to do that, now we had discussed if some time ago just scrolling through the notes to find where we had discussed that. So, we had discussed some time ago that 1D response can be interpreted as a combination of elastic and viscous responses.

This is something that we had said long ago, when we had discussed what a viscoelastic phenomenon is. So we are saying that in the case of 1D response we can obviously go back to that idea where the viscoelasticity is represented as a combination of elastic response and viscous response.

So, let us say take the case of elastic body. We had just said that there was 1 analog that we can use and that analog allows us to interpret our model elastic response and that analog was your spring. So, your spring represents energy storage and your dashpot this represents energy dissipation my system to the surroundings.

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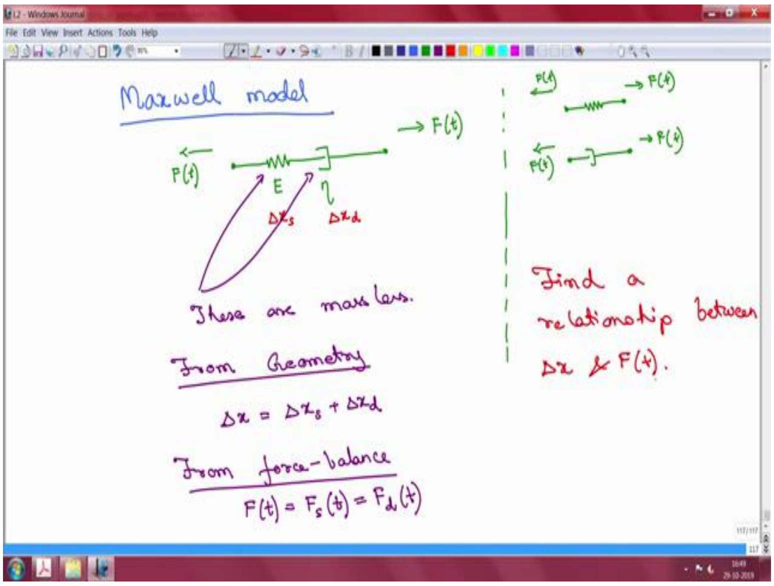

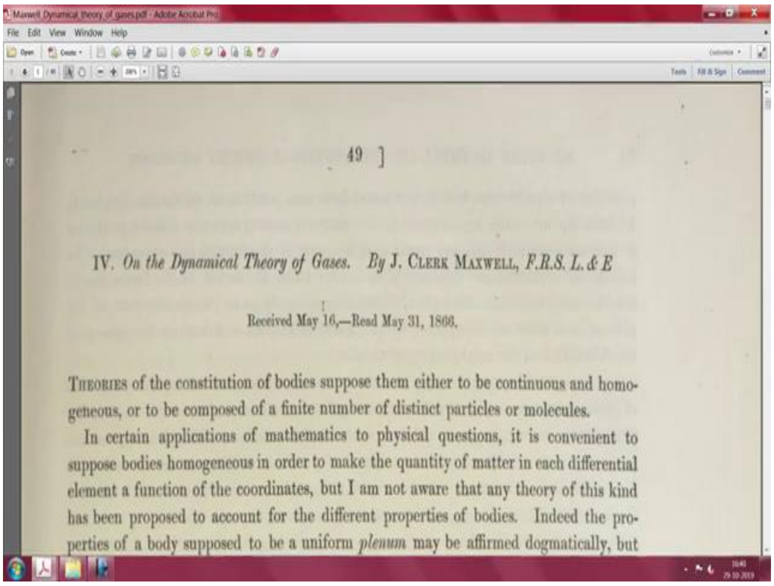

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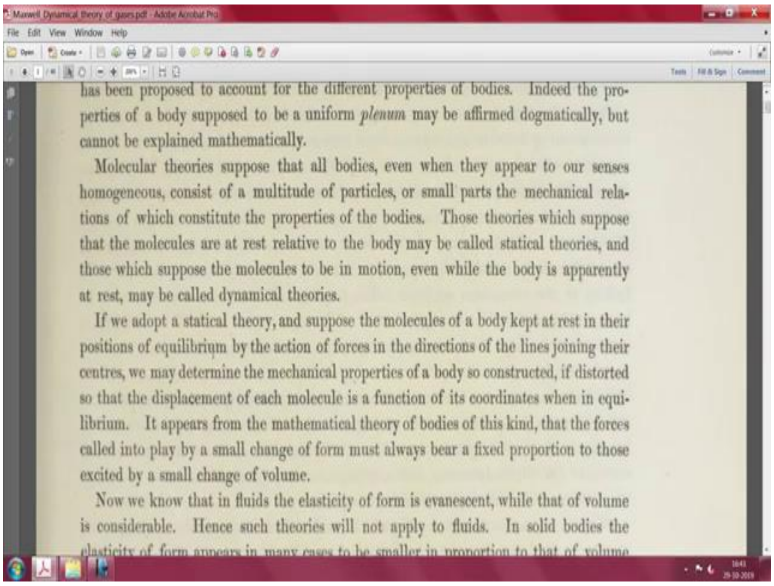
IV. *On the Dynamical Theory of Gases.* By J. CLERK MAXWELL, *F.R.S. L. & E.*

Received May 16,—Read May 31, 1866.

THEORIES of the constitution of bodies suppose them either to be continuous and homogeneous, or to be composed of a finite number of distinct particles or molecules.

In certain applications of mathematics to physical questions, it is convenient to suppose bodies homogeneous in order to make the quantity of matter in each differential element a function of the coordinates, but I am not aware that any theory of this kind has been proposed to account for the different properties of bodies. Indeed the properties of a body supposed to be a uniform *plenum* may be affirmed dogmatically, but

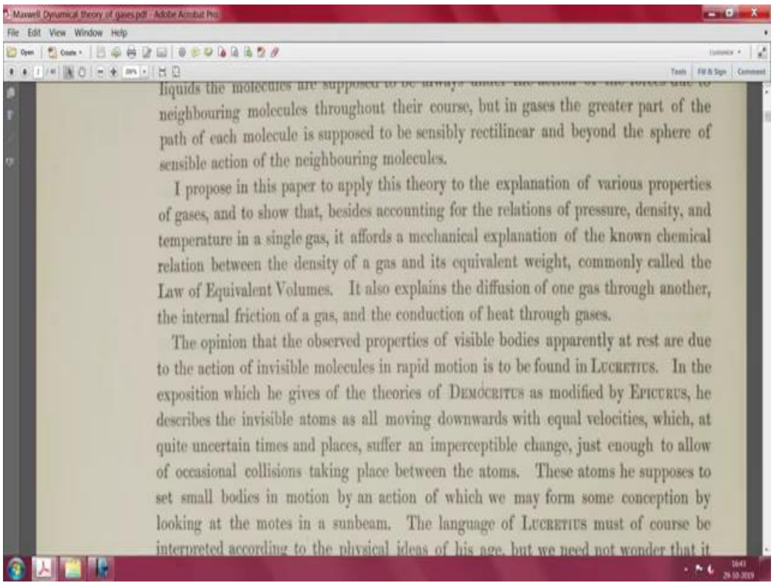


has been proposed to account for the different properties of bodies. Indeed the properties of a body supposed to be a uniform *plenum* may be affirmed dogmatically, but cannot be explained mathematically.

Molecular theories suppose that all bodies, even when they appear to our senses homogeneous, consist of a multitude of particles, or small parts the mechanical relations of which constitute the properties of the bodies. Those theories which suppose that the molecules are at rest relative to the body may be called statical theories, and those which suppose the molecules to be in motion, even while the body is apparently at rest, may be called dynamical theories.

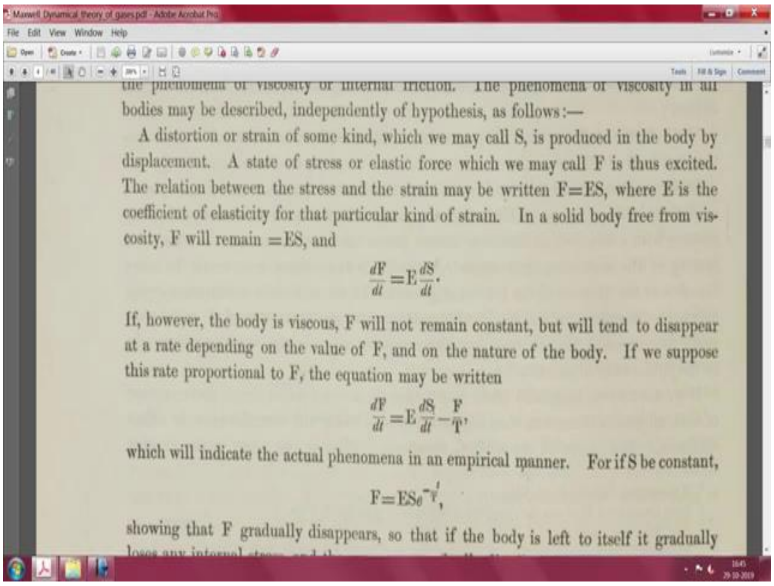
If we adopt a statical theory, and suppose the molecules of a body kept at rest in their positions of equilibrium by the action of forces in the directions of the lines joining their centres, we may determine the mechanical properties of a body so constructed, if distorted so that the displacement of each molecule is a function of its coordinates when in equilibrium. It appears from the mathematical theory of bodies of this kind, that the forces called into play by a small change of form must always bear a fixed proportion to those excited by a small change of volume.

Now we know that in fluids the elasticity of form is evanescent, while that of volume is considerable. Hence such theories will not apply to fluids. In solid bodies the


liquids the molecules are supposed to be always under the action of the forces due to neighbouring molecules throughout their course, but in gases the greater part of the path of each molecule is supposed to be sensibly rectilinear and beyond the sphere of sensible action of the neighbouring molecules.

I propose in this paper to apply this theory to the explanation of various properties of gases, and to show that, besides accounting for the relations of pressure, density, and temperature in a single gas, it affords a mechanical explanation of the known chemical relation between the density of a gas and its equivalent weight, commonly called the Law of Equivalent Volumes. It also explains the diffusion of one gas through another, the internal friction of a gas, and the conduction of heat through gases.

The opinion that the observed properties of visible bodies apparently at rest are due to the action of invisible molecules in rapid motion is to be found in LUCRETIUS. In the exposition which he gives of the theories of DEMOCRITUS as modified by EPICURUS, he describes the invisible atoms as all moving downwards with equal velocities, which, at quite uncertain times and places, suffer an imperceptible change, just enough to allow of occasional collisions taking place between the atoms. These atoms he supposes to set small bodies in motion by an action of which we may form some conception by looking at the motes in a sunbeam. The language of LUCRETIUS must of course be interpreted according to the physical ideas of his age, but we need not wonder that it


the phenomena of viscosity or internal friction. The phenomena of viscosity in all bodies may be described, independently of hypothesis, as follows:—

A distortion or strain of some kind, which we may call S, is produced in the body by displacement. A state of stress or elastic force which we may call F is thus excited. The relation between the stress and the strain may be written F=ES, where E is the coefficient of elasticity for that particular kind of strain. In a solid body free from viscosity, F will remain =ES, and

$$\frac{dF}{dt}=E\frac{dS}{dt}.$$

If, however, the body is viscous, F will not remain constant, but will tend to disappear at a rate depending on the value of F, and on the nature of the body. If we suppose this rate proportional to F, the equation may be written

$$\frac{dF}{dt}=E\frac{dS}{dt}-\frac{F}{T},$$

which will indicate the actual phenomena in an empirical manner. For if S be constant,

$$F=ESe^{-\frac{t}{T}},$$

showing that F gradually disappears, so that if the body is left to itself it gradually

So that will help us create this first in the simplest of the viscoelastic models. Where what we will do is we will put them in series, but before we go here, as we have been doing frequently in this course whenever we introduce an interesting and important topic we try to see if we there is if we can go back and look at the original works where this was represented.

So this is the manuscript by James Clerk Maxwell which is titled on the dynamical theory of gasses. It was published in 1866 and this is the manuscript where he proposes the Maxwell's model. The history of why he was trying to look into the dynamical behaviour of gases is an interesting something you can look up later on by itself but we are not going to concern ourselves with that.

So in the beginning he talks about the theories of the Constitutions of bodies and if you know at that time they were not they had still not seen an atom. This is eighteen hundred and sixtysix this is still before we have been able to image an atom and we have been conclusively able to prove that materials exist as atoms and molecules.

So this is predating their time but there are different philosophical or there are people who had sort of guessed that material is going to be made a lot of small particles. So, in the beginning he talks about that and that is quite interesting to read because he says that the theories of the constitution of bodies suppose them either to be continuous and homogeneous or to be composed of a finite number of distinct particles or molecules. So, he is talking about the two ways in which we believe that the material exists.

The first is the continuum model where the continuum case where the material is just believed to be homogeneous at any scale even if you go down smaller and smaller and in the other case it is always supposed to be composed of a finite number of particles of molecules. And then he goes over the different ideas that are prevalent at that time says that if we adopt a statical theory and suppose the molecules of a body kept at rest in their positions of equilibrium actions of forces in the directions of the lines joining their centres, we may determine the mechanical properties of a body so constructed if distorted so that the displacement of each molecule is a function of its coordinates when in equilibrium.

So he is saying that if you knew the internal forces, then you would be able to figure out how the body is going to react to that. So, this this is something that you can read at your leisure it is downloadable it is also available on archive I believe one of the archive platforms has all of Maxwell's papers.

Even a section very goes into the idea is where so is the sum of this I had referred to in one of the earlier classes that the opinion of the observed properties of visible bodies apparently at rest are due to the action of invisible molecules in rapid motion is to be found in Lucretius. So he is referring to the Roman and the Greek literature on the idea that matter is composed of small indivisible parts.

And we know that there was a similar school of belief in India at that time, which is called the vaisesika system of which Kanade is probably the most well-known propounded which also believed that the materials are composed of small atoms and molecules. So he goes through all that and then he introduces this particular equation where thus there is a solid body.

So there is a solid body that has so dF by dT equal to e times of dS by dT minus F times of T. Where the there is a body which has both elasticity and viscosity at the same time. So this T is a relaxation time scale, so we will go down, we will what we will do is we will try to derive this from our first principles and we will see whether we end up using this we end up with this equation or not.

So, let us go back to the Maxwell's model. So here we said what we are going to do is, we are going to put into series, let us say this is represented by E and the modulus here and this represents a viscosity of eta. So the viscosity and the e are constants and you are applying a force and as a result both the spring and the dashpot will experience some displacement.

So, let us say that the displacement observed here is delta XS, the subscript s standing for spring and the displacement by the dashpot is delta XD, D being a subscript for dashpot. And these themselves are massless, so we will also make a note that these are massless. So now the total displacement in the system is, so let us say that this spring dashpot combination is representing a Lagrangian material point.

So now if that is the case then what we want to do is, we want to apply force balance and we want to see where that leads us to, but before we do that then let us quickly write that from geometry of the current system, from geometry, the total displacement in the this body is equal to delta XS, plus delta XT. We want to apply force balance so consider these separate entities.

So I am just going to create a separate plane here and draw that. So if the force applied is F, then my force balance on the system requires me and since these are massless that the forces on both sides will be F(t) for the spring and, similarly for the dashpot. So if I were to write the so from force balance we have F(t) of the force F(t) is equal to FS, where FS is the force being experienced by the spring and FD is the force that is being expressed by the dashpot.

And our question that we want to answer is we have to find a relationship. So the question is find a relationship between delta X and F(t). So this is where we are trying to get. So now what do we know about FS and F T so FS we know is the force that is in the spring.

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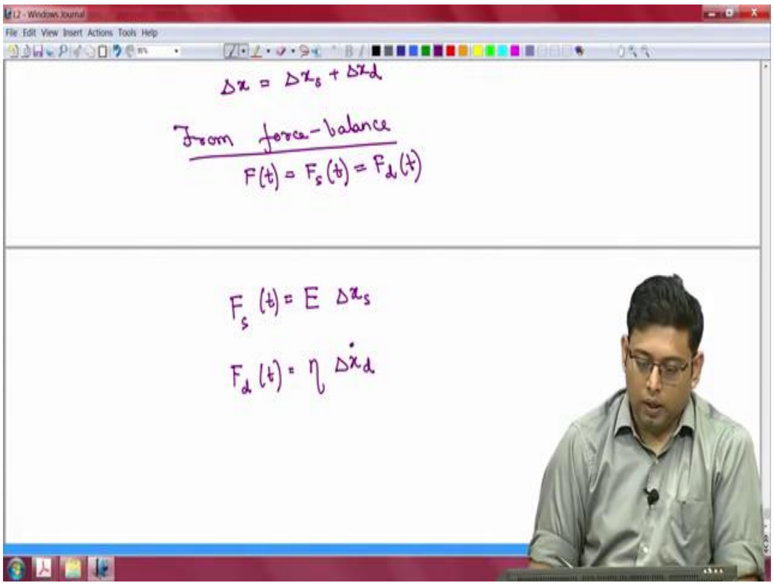

So if that is the force in the spring then from our analogy the relationship between the force and the displacement in the spring should be this because this is a linear spring and the force in the dashpot should be delta XD. So what we will do is we will stop here today and the next class we will take out from here I will try to derive the formula.