Introduction to Soft Matter
Professor Dr. Aloke Kumar
Department of Mechanical Engineering
Indian Institute of Science, Bengaluru
Lecture 02
Deborah Number

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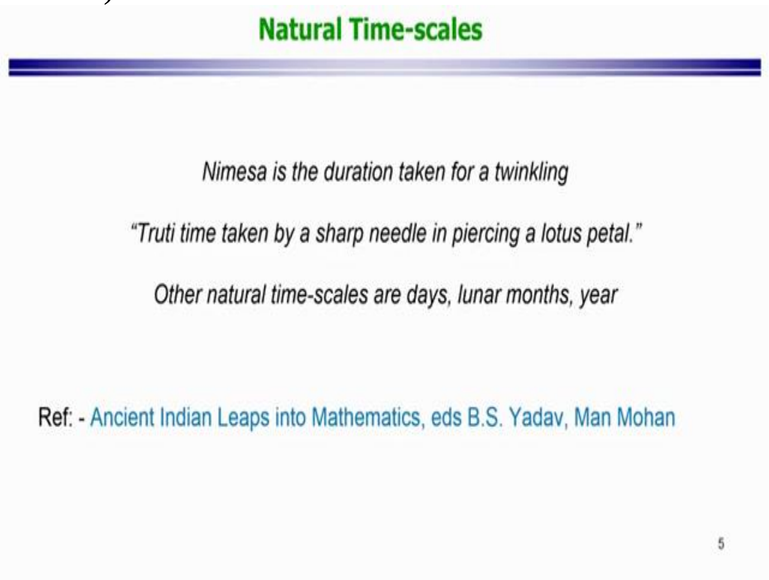

Welcome to the second lecture on Introduction to Soft Matter. We were discussing in last class the various time scales or the natural time scales that were described in some of the indict texts and we have just discussed the for example Nimesa is the duration taken for a twinkling and a twinkling of an eye in this case then, there was a another time scale called the Truti and the Truti is the time taken by a sharp needle in piercing a lotus petal.

Many other time scales were also discussed obviously like days, lunar months, years, etc. and these are some of the Ancient time scales that were discussed in many different texts one of the foremost text being the Suryasiddhanta which was the text on astronomy, you can find many of these reference materials in book called the Ancient Indian Leaps into Mathematics which was edited by Yadav and Mohan.

The edited books has many different chapters contributed by other authors but, the very point that, we have discussing natural time scales was to ensure that we understand that the different phenomena which are associated with their own natural time scales.

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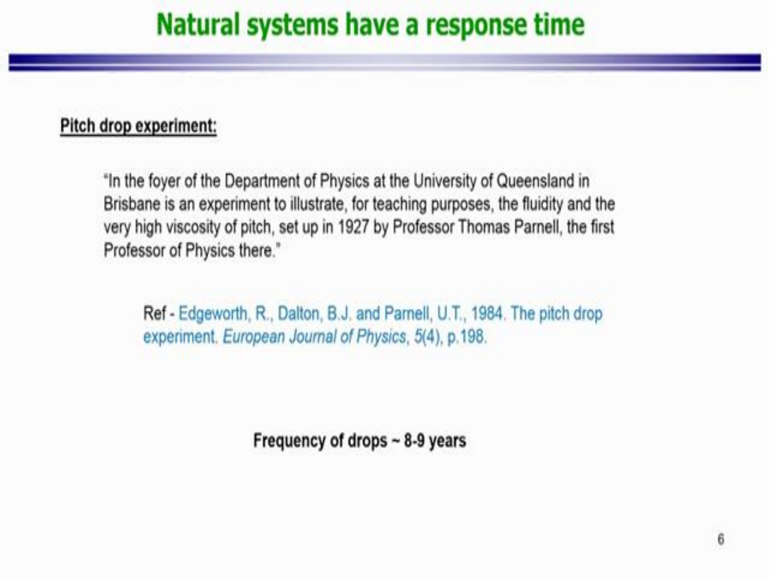

Along with this comes the idea that natural systems have a response time and in order to understand or observe that system we have to naturally be able to first and foremost appreciate what the time scale associated with that system is, a very important experiment in this context is called the very, it is a very famous experiment called the pitch drop experiment.

And in this manuscript which is called the pitch drop experiment was published in the European Journal of Physics, we have authors Edgeworth, Dolton and Parnell, and they describe this, we will go to the pdf in a moment but I just want to describe to you what the pitch drop experiment was.

So, in the foyer of the department of physics at the University of Queensland in Brisbane is an experiment to illustrate, for teaching purposes, the fluidity and the very high viscosity of pitch, set up in 1927 by Professor Thomas Parnell the first professor of physics there.

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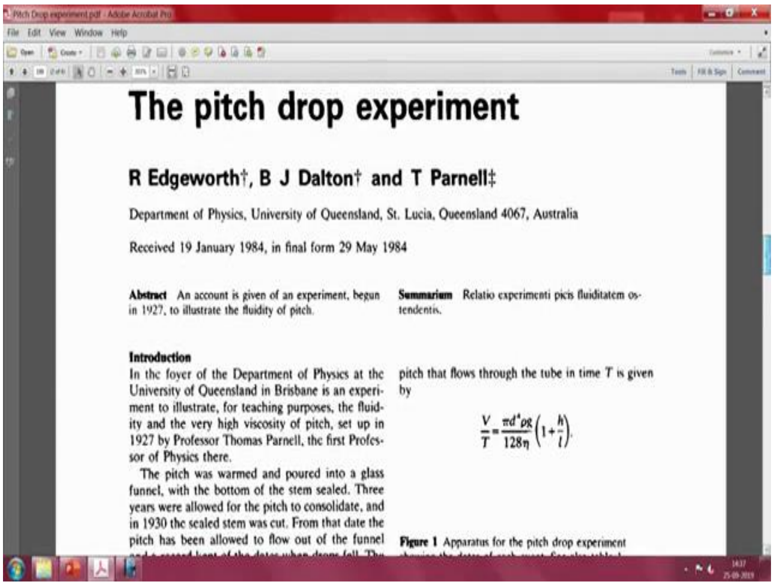

The pitch drop experiment

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Abstract An account is given of an experiment, begun in 1927, to illustrate the fluidity of pitch.

Summarium Relatio experimenti picis fluiditatem ostendentis.

Introduction

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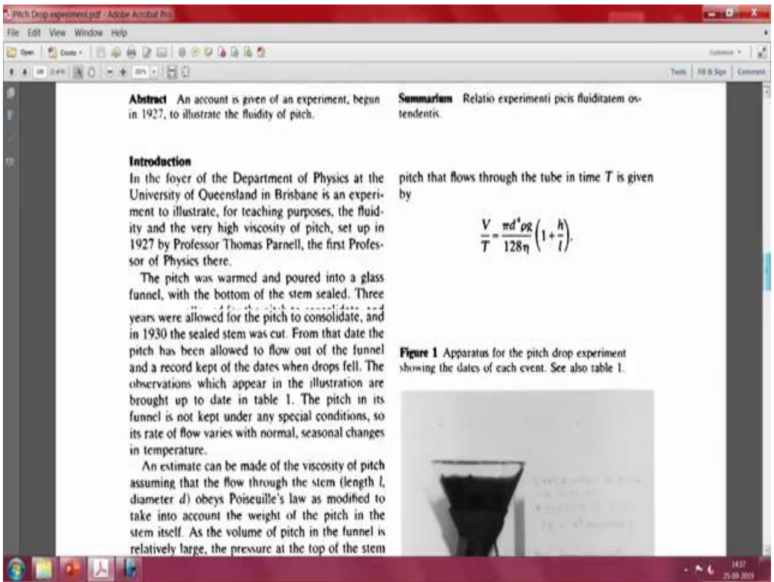
The pitch was warmed and poured into a glass funnel, with the bottom of the stem sealed. Three years were allowed for the pitch to consolidate, and in 1930 the sealed stem was cut. From that date the pitch has been allowed to flow out of the funnel

pitch that flows through the tube in time T is given by

$$\frac{V}{T}=\frac{\pi d^4 \rho g}{128\eta}\left(1+\frac{h}{l}\right).$$

Figure 1 Apparatus for the pitch drop experiment



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Introduction

In the foyer of the Department of Physics at the University of Queensland in Brisbane is an experiment to illustrate, for teaching purposes, the fluidity and the very high viscosity of pitch, set up in 1927 by Professor Thomas Parnell, the first Professor of Physics there.

The pitch was warmed and poured into a glass funnel, with the bottom of the stem sealed. Three years were allowed for the pitch to consolidate, and in 1930 the sealed stem was cut. From that date the pitch has been allowed to flow out of the funnel and a record kept of the dates when drops fell. The observations which appear in the illustration are brought up to date in table 1. The pitch in its funnel is not kept under any special conditions, so its rate of flow varies with normal, seasonal changes in temperature.

An estimate can be made of the viscosity of pitch assuming that the flow through the stem (length l, diameter d) obeys Poiseuille's law as modified to take into account the weight of the pitch in the stem itself. As the volume of pitch in the funnel is relatively large, the pressure at the top of the stem

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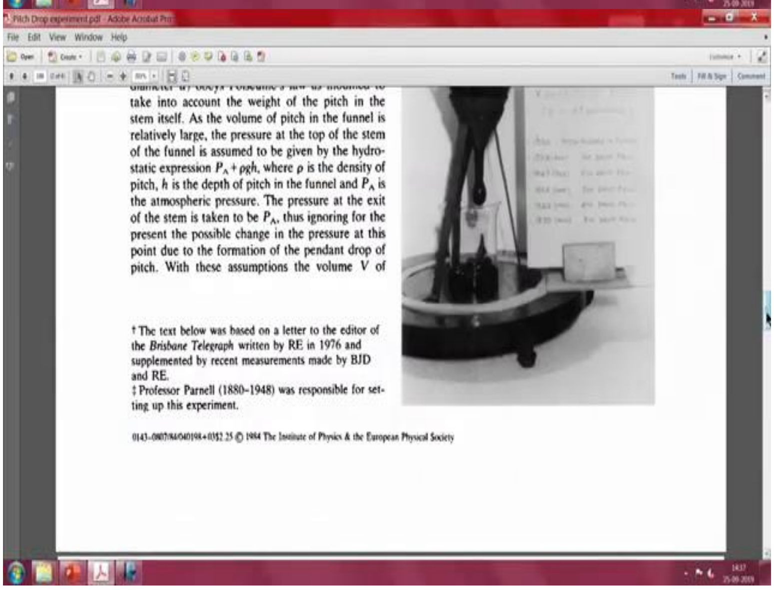
Figure 1 Apparatus for the pitch drop experiment showing the dates of each event. See also table 1.



take into account the weight of the pitch in the stem itself. As the volume of pitch in the funnel is relatively large, the pressure at the top of the stem of the funnel is assumed to be given by the hydrostatic expression $P_A+\rho gh$, where ρ is the density of pitch, h is the depth of pitch in the funnel and P_A is the atmospheric pressure. The pressure at the exit of the stem is taken to be P_A, thus ignoring for the present the possible change in the pressure at this point due to the formation of the pendant drop of pitch. With these assumptions the volume V of

† The text below was based on a letter to the editor of the *Brisbane Telegraph* written by RE in 1976 and supplemented by recent measurements made by BJD and RE.
‡ Professor Parnell (1880–1948) was responsible for setting up this experiment.

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Now, when we talk of something of pitch which is usually. A buy product, a natural product, we think of it in terms of a as a solid with this very famous experiment challenged our view point and is often looked upon as a one of the pinioning experiments in trying to understand viscoelasticity.

So, here is, in the beginning I already read it out to you that we have at the department of physics a very high viscosity of pitch, which was professor Parnell wanted to illustrate and what he did is, he setup a very simple experiment you have a funnel shaped here, let just a minute to show that so, this is the apparatus and then you have pitch that is content within it.

And what happens is much like water or much like a fluid you have the pitch starting to form droplets and then this falls down into this beaker now, obviously pitch has a very very high viscosity and because, it has a very high viscosity this is not going to be fast in human terms.

So, this experiment was set up and it was one of the very long running experiment and this experiment, they found that the drop, actually, a single drop usually comes out at around 8 to 9 years of time, so they went around and you are welcome to more read this paper at your leisure, which is again the pitch drop experiment by Edgeworth Dolton and Parnell, it was published in the European Journal of physics in 1984.

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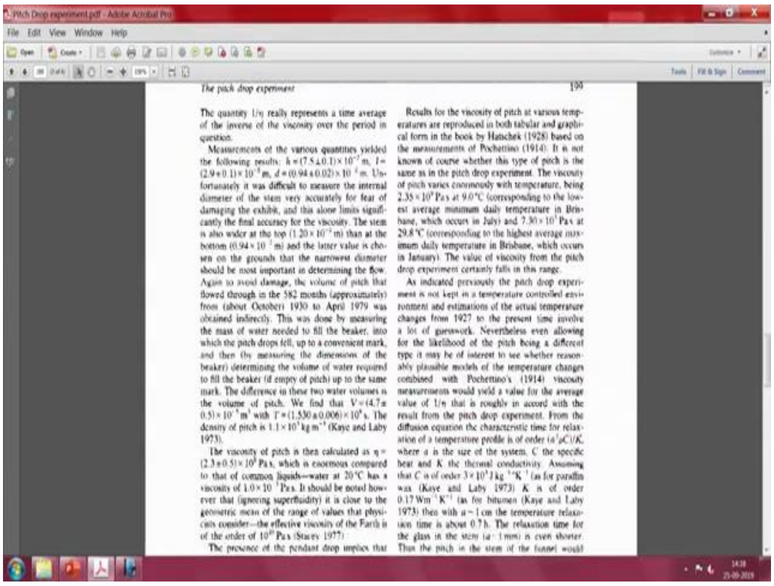

The pitch drop experiment 199

The quantity $1/\eta$ really represents a time average of the inverse of the viscosity over the period in question.

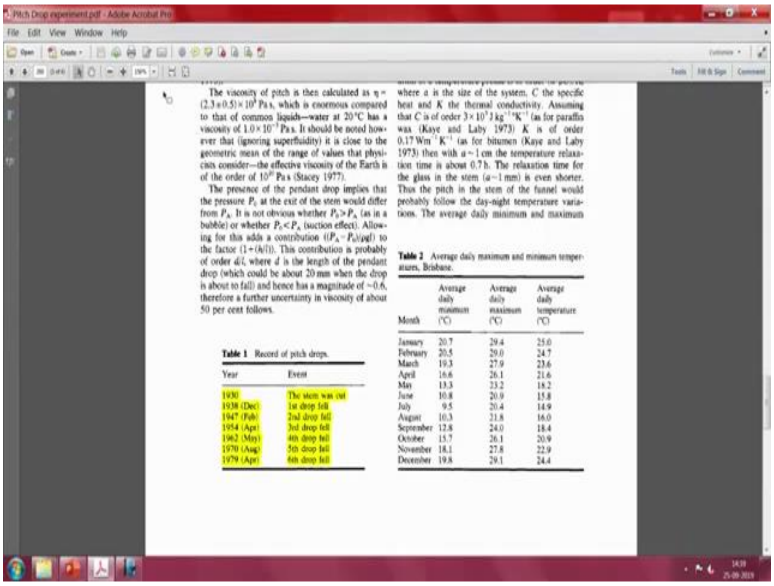
Measurements of the various quantities yielded the following results: $h=(7.5\pm0.1)\times10^{-2}$ m, $l=(2.9\pm0.1)\times10^{-2}$ m, $d=(0.94\pm0.02)\times10^{-2}$ m. Unfortunately it was difficult to measure the internal diameter of the stem very accurately for fear of damaging the exhibit, and this alone limits significantly the final accuracy for the viscosity. The stem is also wider at the top (1.20×10^{-2} m) than at the bottom (0.94×10^{-2} m) and the latter value is chosen on the grounds that the narrowest diameter should be most important in determining the flow. Again to avoid damage, the volume of pitch that flowed through in the 582 months (approximately) from (about October) 1930 to April 1979 was obtained indirectly. This was done by measuring the mass of water needed to fill the beaker, into which the pitch drops fell, up to a convenient mark, and then (by measuring the dimensions of the beaker) determining the volume of water required to fill the beaker (if empty of pitch) up to the same mark. The difference in these two water volumes is the volume of pitch. We find that $V=(4.7\pm0.5)\times10^{-5}\,m^3$ with $T=(1.530\pm0.006)\times10^9$ s. The density of pitch is 1.1×10^3 kg m^{-3} (Kaye and Laby 1973).

The viscosity of pitch is then calculated as $\eta=(2.3\pm0.5)\times10^8$ Pa s, which is enormous compared to that of common liquids—water at 20 °C has a viscosity of 1.0×10^{-3} Pa s. It should be noted however that (ignoring superfluidity) it is close to the geometric mean of the range of values that physicists consider—the effective viscosity of the Earth is of the order of 10^{20} Pa s (Stacey 1977).

The presence of the pendant drop implies that

Results for the viscosity of pitch at various temperatures are reproduced in both tabular and graphical form in the book by Hatschek (1928) based on the measurements of Pochettino (1914). It is not known of course whether this type of pitch is the same as in the pitch drop experiment. The viscosity of pitch varies enormously with temperature, being 2.35×10^9 Pa s at 9.0 °C (corresponding to the lowest average minimum daily temperature in Brisbane, which occurs in July) and 7.30×10^7 Pa s at 29.8 °C (corresponding to the highest average maximum daily temperature in Brisbane, which occurs in January). The value of viscosity from the pitch drop experiment certainly falls in this range.

As indicated previously the pitch drop experiment is not kept in a temperature controlled environment and estimations of the actual temperature changes from 1927 to the present time involve a lot of guesswork. Nevertheless even allowing for the likelihood of the pitch being a different type it may be of interest to see whether reasonably plausible models of the temperature changes combined with Pochettino's (1914) viscosity measurements would yield a value for the average value of $1/\eta$ that is roughly in accord with the result from the pitch drop experiment. From the diffusion equation the characteristic time for relaxation of a temperature profile is of order $(a^2\rho C)/K$, where a is the size of the system, C the specific heat and K the thermal conductivity. Assuming that C is of order 3×10^3 J $kg^{-1}\,K^{-1}$ (as for paraffin wax (Kaye and Laby 1973)) K is of order 0.17 W $m^{-1}\,K^{-1}$ (as for bitumen (Kaye and Laby 1973)) then with $a\sim1$ cm the temperature relaxation time is about 0.7 h. The relaxation time for the glass in the stem ($a\sim1$ mm) is even shorter. Thus the pitch in the stem of the funnel would



The viscosity of pitch is then calculated as $\eta = (2.3 \pm 0.5) \times 10^8$ Pa s, which is enormous compared to that of common liquids—water at 20°C has a viscosity of 1.0×10^{-3} Pa s. It should be noted however that (ignoring superfluidity) it is close to the geometric mean of the range of values that physicists consider—the effective viscosity of the Earth is of the order of 10^{21} Pa s (Stacey 1977).

The presence of the pendant drop implies that the pressure P_0 at the exit of the stem would differ from P_A. It is not obvious whether $P_0 > P_A$ (as in a bubble) or whether $P_0 < P_A$ (suction effect). Allowing for this adds a contribution $((P_A - P_0)/\rho g l)$ to the factor $(1 + (h/l))$. This contribution is probably of order d/l, where d is the length of the pendant drop (which could be about 20 mm when the drop is about to fall) and hence has a magnitude of ~0.6, therefore a further uncertainty in viscosity of about 50 per cent follows.

where a is the size of the system, C the specific heat and K the thermal conductivity. Assuming that C is of order 3×10^3 J kg^{-1} K^{-1} (as for paraffin wax (Kaye and Laby 1973)) K is of order 0.17 W m^{-1} K^{-1} (as for bitumen (Kaye and Laby 1973)) then with $a \sim 1$ cm the temperature relaxation time is about 0.7 h. The relaxation time for the glass in the stem ($a \sim 1$ mm) is even shorter. Thus the pitch in the stem of the funnel would probably follow the day-night temperature variations. The average daily minimum and maximum

Table 1 Record of pitch drops.

Year	Event
1930	The stem was cut
1938 (Dec)	1st drop fell
1947 (Feb)	2nd drop fell
1954 (Apr)	3rd drop fell
1962 (May)	4th drop fell
1970 (Aug)	5th drop fell
1979 (Apr)	6th drop fell

Table 2 Average daily maximum and minimum temperatures, Brisbane.

Month	Average daily minimum (°C)	Average daily maximum (°C)	Average daily temperature (°C)
January	20.7	29.4	25.0
February	20.5	29.0	24.7
March	19.3	27.9	23.6
April	16.6	26.1	21.6
May	13.3	23.2	18.2
June	10.8	20.9	15.8
July	9.5	20.4	14.9
August	10.3	21.8	16.0
September	12.8	24.0	18.4
October	15.7	26.1	20.9
November	18.1	27.8	22.9
December	19.8	29.1	24.4

And the other point thing I wanted to quickly point out is the record of the pitch drops or the so 1930 when the first it was started so 1938 the first drop fell, 1947 is when the second drop fell so almost a gap of a 9 years there then the third drop falls in 1954 almost a gap of 7 years here, then 54 to 62 so that is about 8 years I am not counting the months but, approximately that is the time scale. So, imagine you are a creature which would only view something at a interval of 6 years or 7 years.

For you with every, if you are twinkling of your alien creature so, let us imagine for a second that there is an alien creature where he is twinkling of the eye, or they are twinkling of the eye, is takes 6 to 7 years of human time for them the moment he blinks eye the next moment that drop has fallen, has fallen. In a sense for him the material would not seem like a solid it would seem more like a liquid quit which is, you have droplets coming out.

So, but for humans our twinkling or the blinking of an eye is much faster so for us it (alw) seems as a rather long time to wait for a drop to form and it is appropriate in the min time if you are using pitch or a material like that in your laboratory for an experiment that is only lasting 10 minutes, it is appropriate to discuss that as a solid body. What this experiment or what the framework, what we are trying to understand is the idea that. Just a second I will open up the other document that I want to now discuss.

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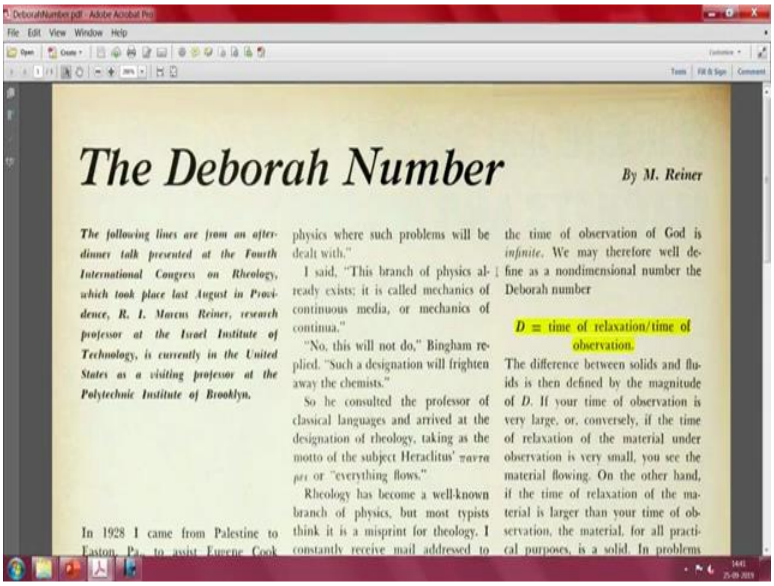


The Deborah Number

By M. Reiner

The following lines are from an after-dinner talk presented at the Fourth International Congress on Rheology, which took place last August in Providence, R. I. Marcus Reiner, research professor at the Israel Institute of Technology, is currently in the United States as a visiting professor at the Polytechnic Institute of Brooklyn.

In 1928 I came from Palestine to

physics where such problems will be dealt with."

I said, "This branch of physics already exists; it is called mechanics of continuous media, or mechanics of continua."

"No, this will not do," Bingham replied. "Such a designation will frighten away the chemists."

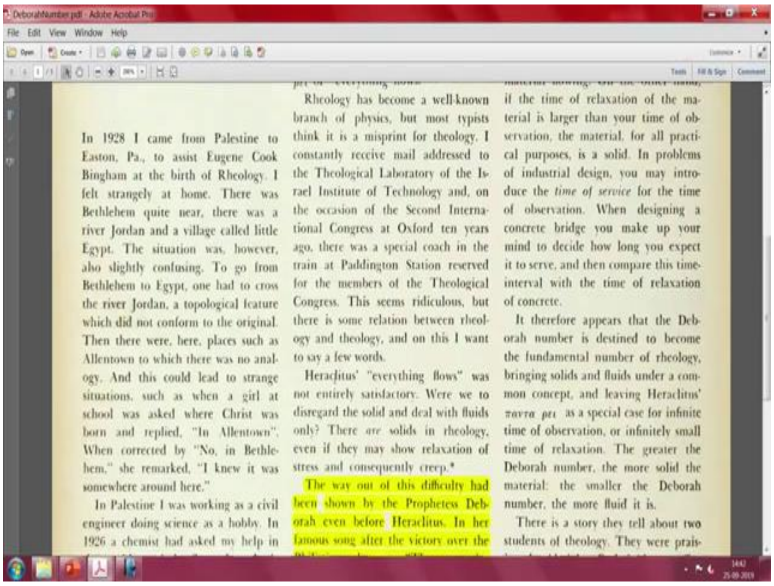
So he consulted the professor of classical languages and arrived at the designation of rheology, taking as the motto of the subject Heraclitus' *παντα ρει* or "everything flows."

Rheology has become a well-known branch of physics, but most typists think it is a misprint for theology. I

the time of observation of God is *infinite*. We may therefore well define as a nondimensional number the Deborah number

D = time of relaxation/time of observation.

The difference between solids and fluids is then defined by the magnitude of *D*. If your time of observation is very large, or, conversely, if the time of relaxation of the material under observation is very small, you see the material flowing. On the other hand, if the time of relaxation of the material is larger than your time of observation, the material, for all practical purposes, is a solid.



Rheology has become a well-known branch of physics, but most typists think it is a misprint for theology. I constantly receive mail addressed to the Theological Laboratory of the Israel Institute of Technology and, on the occasion of the Second International Congress at Oxford ten years ago, there was a special coach in the train at Paddington Station reserved for the members of the Theological Congress. This seems ridiculous, but there is some relation between rheology and theology, and on this I want to say a few words.

In 1928 I came from Palestine to Easton, Pa., to assist Eugene Cook Bingham at the birth of Rheology. I felt strangely at home. There was Bethlehem quite near, there was a river Jordan and a village called little Egypt. The situation was, however, also slightly confusing. To go from Bethlehem to Egypt, one had to cross the river Jordan, a topological feature which did not conform to the original. Then there were, here, places such as Allentown to which there was no analogy. And this could lead to strange situations, such as when a girl at school was asked where Christ was born and replied, "In Allentown". When corrected by "No, in Bethlehem," she remarked, "I knew it was somewhere around here."

In Palestine I was working as a civil engineer doing science as a hobby. In 1926 a chemist had asked my help in

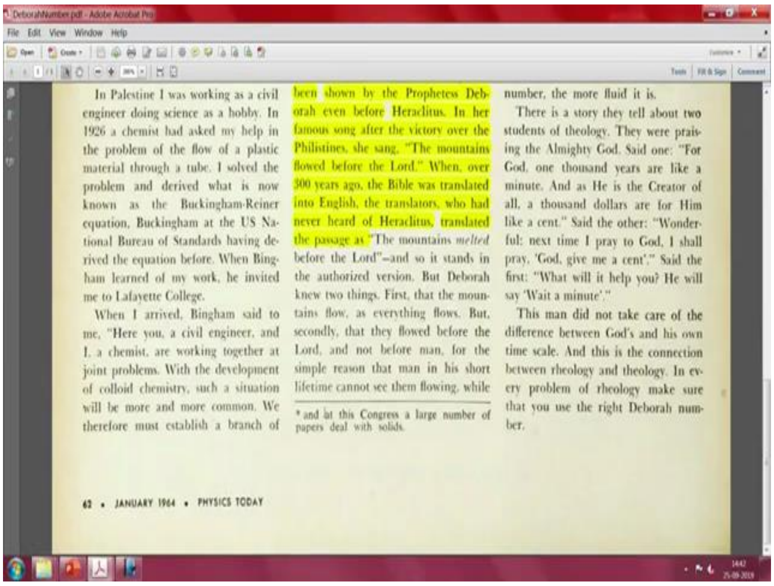
Heraclitus' "everything flows" was not entirely satisfactory. Were we to disregard the solid and deal with fluids only? There *are* solids in theology, even if they may show relaxation of stress and consequently creep.*

The way out of this difficulty had been shown by the Prophetess Deborah even before Heraclitus. In her famous song after the victory over the

if the time of relaxation of the material is larger than your time of observation, the material, for all practical purposes, is a solid. In problems of industrial design, you may introduce the *time of service* for the time of observation. When designing a concrete bridge you make up your mind to decide how long you expect it to serve, and then compare this time-interval with the time of relaxation of concrete.

It therefore appears that the Deborah number is destined to become the fundamental number of rheology, bringing solids and fluids under a common concept, and leaving Heraclitus' *παντα ρει* as a special case for infinite time of observation, or infinitely small time of relaxation. The greater the Deborah number, the more solid the material: the smaller the Deborah number, the more fluid it is.

There is a story they tell about two students of theology. They were prais-



In Palestine I was working as a civil engineer doing science as a hobby. In 1926 a chemist had asked my help in the problem of the flow of a plastic material through a tube. I solved the problem and derived what is now known as the Buckingham-Reiner equation, Buckingham at the US National Bureau of Standards having derived the equation before. When Bingham learned of my work, he invited me to Lafayette College.

When I arrived, Bingham said to me, "Here you, a civil engineer, and I, a chemist, are working together at joint problems. With the development of colloid chemistry, such a situation will be more and more common. We therefore must establish a branch of

been shown by the Prophetess Deborah even before Heraclitus. In her famous song after the victory over the Philistines, she sang, "The mountains flowed before the Lord." When, over 300 years ago, the Bible was translated into English, the translators, who had never heard of Heraclitus, translated the passage as "The mountains *melted* before the Lord"—and so it stands in the authorized version. But Deborah knew two things. First, that the mountains flow, as everything flows. But, secondly, that they flowed before the Lord, and not before man, for the simple reason that man in his short lifetime cannot see them flowing, while

* and at this Congress a large number of papers deal with solids.

number, the more fluid it is.

There is a story they tell about two students of theology. They were praising the Almighty God. Said one: "For God, one thousand years are like a minute. And as He is the Creator of all, a thousand dollars are for Him like a cent." Said the other: "Wonderful: next time I pray to God, I shall pray, 'God, give me a cent'." Said the first: "What will it help you? He will say 'Wait a minute'."

This man did not take care of the difference between God's and his own time scale. And this is the connection between rheology and theology. In every problem of rheology make sure that you use the right Deborah number.

So, I hope you sort of understand that, when you are discussing the material properties or the material response there are two things that are important and that is why I discussed the two issues, one was the time scale associated with that phenomena and then the other is the time of your observation, and these two become very very important in understanding the phenomena itself.

Now, this idea was proposed that this idea can lead to a non-dimensional important number and this was proposed by Reiner in a paper that was published in 1964 January in physics today. And in the beginning he is so, this is after dinner talk a so, there is a prelude to this manuscript.

And in the introduction he starts describing situation which sort of lead to this entire, to this discussion but, the important thing that here, he wants to say is that he was inspired by the story of Prophetess Deborah and in a famous song after the victory over the Philistines she sang. The mountains flowed before the Lord.

When over 300 years ago the bible was translated in to English the translators who had never heard of Heraclitus translated the passage as the mountains melted before the Lords, before the lord. But, Deborah knew of two things first, that the mountains flow and as everything flows. But, secondly that they flowed before the Lord and not before the man for the simple reason that man in his short lifetime cannot see them flowing. Whereas, the time observation of God is infinite.

So, using so from what I understand these lines are from the old testament of bible and this is the story from there, but he is using it is as an inspiration to go ahead and define a very very important number for us which is going to become the Deborah number and this Deborah number he defines as the time of relaxation divided by the time of observation.

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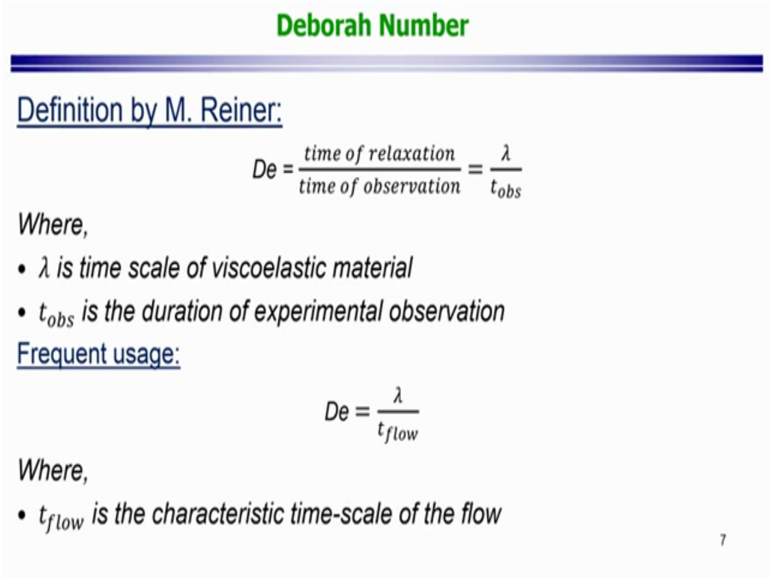

So, you are more than welcome to read this paper also your leisure but for us the important thing is, the definition of the Deborah number, which is defined right now, as time of relaxation divided by the time of observation. What is this time of relaxation? So, I will come to that but, for the time being I will suffice to say that we are going to use lambda as a symbol for time of relaxation and for the time of observation we are going to use t subscript observation.

There is another small change that we are going to do is that we are going to use the letters De to denote Deborah number rather than just D as the original definition so, let us go back to our writing pad and I will come back top this.

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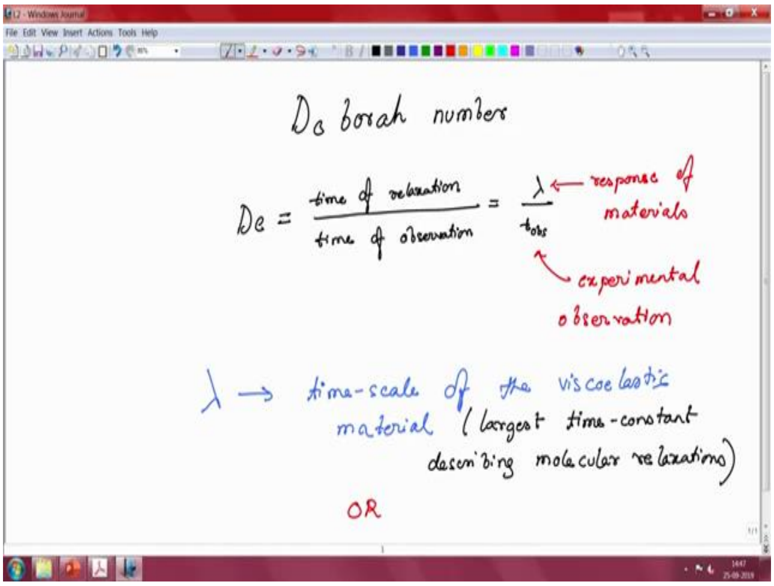

So, we are discussing a number called the Deborah number, and we just said that we will write it as De this non-dimensional number and this is the ratio of two time scales and this is the time of relaxation and this is divided by the time of observation, so this should account for, it accounts for two things, one is response of materials and the whole issue of human observation or experimental observation.

Now this, so the question is what is the first question that should come to you is, what is lambda, so lambda is a relaxation time scale and it is usually taken as the time scale of the viscoelastic material. I have not yet discussed what viscoelasticity is, so in some sense I am having to use that but, for the time we will come back to this obviously at a later course, at a later stage so this is usually taken as when we say the time scale of viscoelastic materials it usually implies the largest time constant describing molecular relaxations, or.

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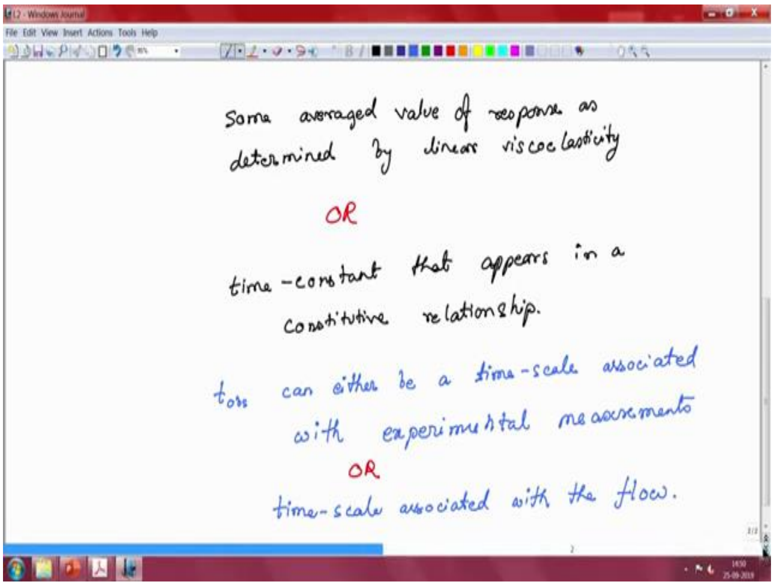

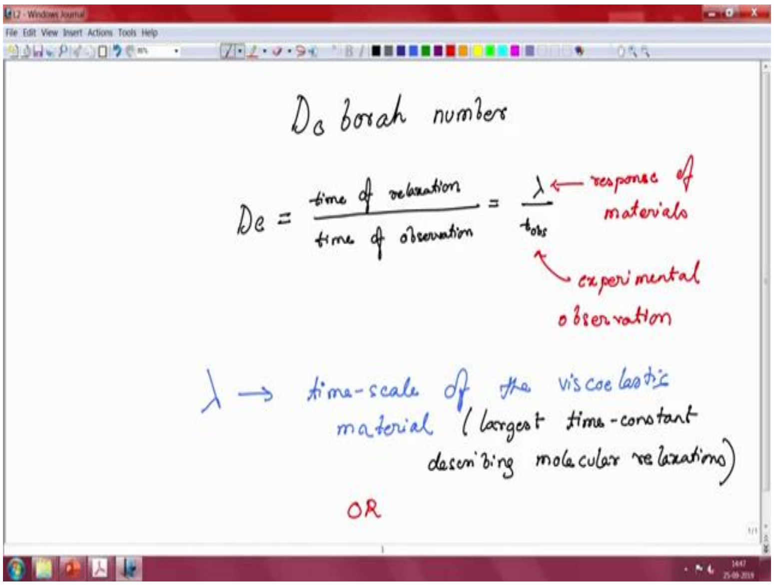

Or, it could be some average time scale, some averaged value of response time, of response as determined by linear viscoelasticity or, it could also be taken as some time scale, sometime constant that appears in a constitutive relationship. So, I must point out here that there are some terms that I am using that we have not yet come across.

So, lambda is going to be something that we are just familiarizing ourselves with or we are going discuss it in more detail in the course so I have not discuss what linear viscoelasticity is I have not discuss what a constitutive relationship is, or what a molecular relaxations is okay we will come back to that.

But more importantly we had another term just quickly going back to this there is t observation so this term is also there, this term is lightly easier to understand and t observation can either be a time scale associated with experimental measurements, or it can also be the time scale associated with the flow.

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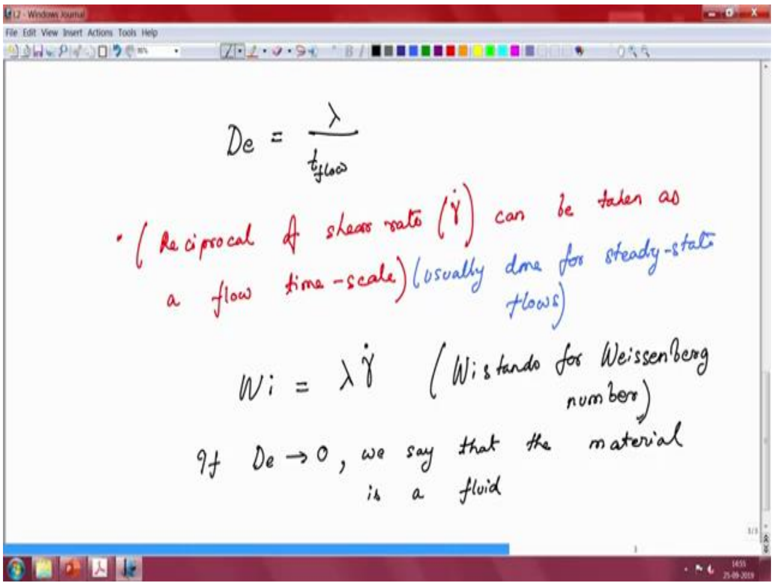

If you take it as a time scale associated with the flow often people tend to write the Deborah number, in that case they have tend to write it as t by t of flow. Now, the question is what is the time scale of flow? The beat way to understand that is through some examples but we will like to note one thing here that is the reciprocal of the average shear rate can also be taken as a time scale of the flow. So, we note here that reciprocal of the shear rate which we will write here as gamma dot can be taken as a flow time scale.

And this is usually done for steady state flows okay, so you are make another note here which means a usually done for steady state flows. But, if you do that then the equation for Deborah number this becomes lambda into gamma dot that is usually a another number that we use often in a viscoelasticity or soft matter which is although Weissenberg number the Wi which is usually defined as always as lambda into gamma dot, so this is called Wi stands for Weissenberg number.

So, if you start using the shear rate or the reciprocal of the shear rate as the appropriate time scale then the number the Deborah number of that, in that case becomes identical with the Weissenberg number so, this is an important point. Now, so I will just quickly summarize this, or before we summarize there is another couple of things I would like to mention. So, now see, in the very very beginning, I will just go back to this definition.

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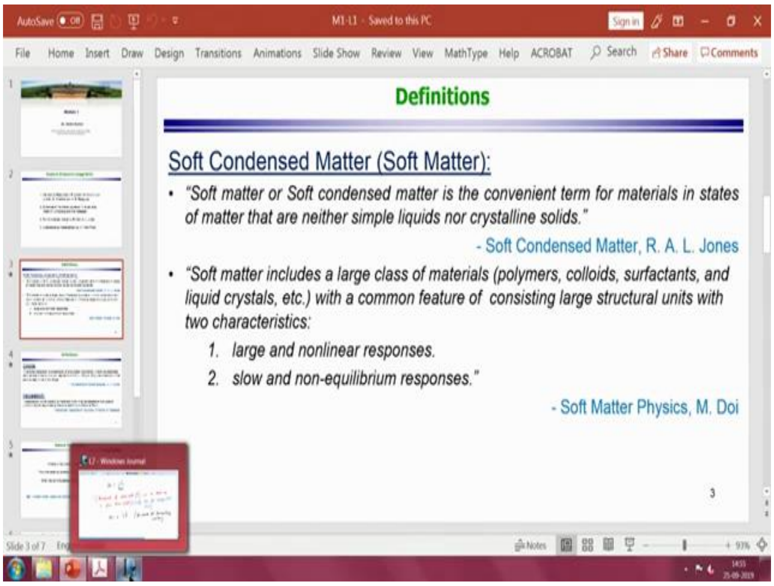

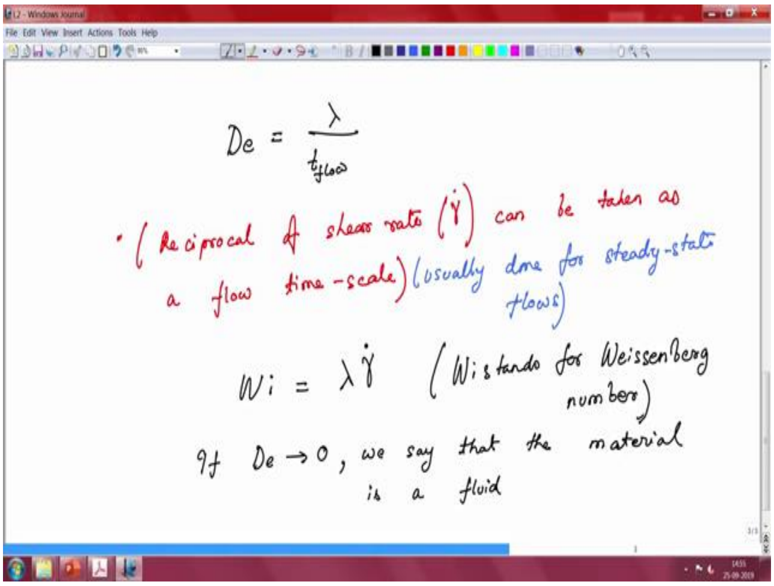

In the very beginning, we had said that soft matter or soft condensed matter is convenient term for materials in states of matter that are neither simple liquids nor crystalline solids. So, what happens to relaxation time scale in case of simple liquids? Now, we know, if you recall some of your undergraduate courses, we used to say that a liquid is something that cannot stand shear at all. Which means that it starts to response instantaneously.

So, if it response instantaneously, then this flow time scale or lambda becomes 0 in that case so, it becomes infinitely small which means that the material is responding almost immediately to a force and that is why it starting to flow. So, it is useful to understand that if, Deborah number tends to 0, we say that the material is fluid or like very much like a fluid.

On the other hand, despite the application of force from our common understanding we see that the solid never flows so the time scale associated with flow of a solid body is almost infinitely large compared to that of a liquid. So, the other extreme that we can consider here is the crystalline solid.

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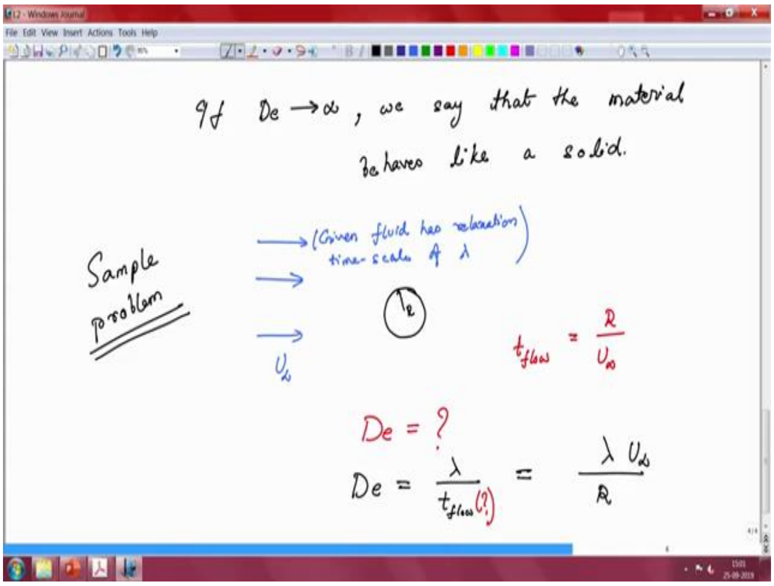

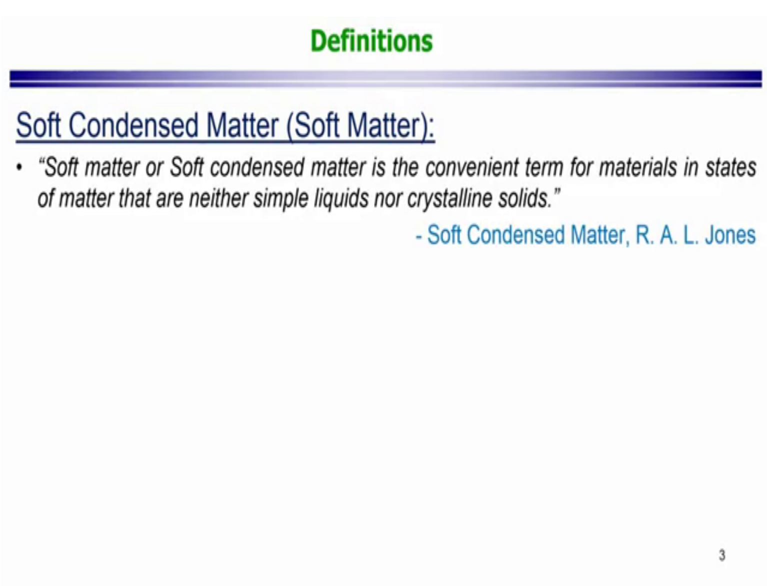

So, if Deborah number tends to infinity we say that the material behaves like a solid, behaves like a solid. So, the advantage of considering these two extreme cases is, we are now ready to in a sense, began discussing what a viscoelastic or material or soft matter would be and you can probably guess that when we say viscoelastic materials or soft materials the Deborah number associated with them will be a finite quantity. So, it will neither be somewhere between 0 and infinity and that where it lies between 0 and infinity gives you an idea of whether it is behaving more like a liquid or more like a solid.

So, some of the classes of materials we had discusses and we will see later are examples of so like polymers will have a finite Deborah number and the Deborah number becomes important similarly, for other types of soft materials. So, now we see that although the original definition here appealed to intuition in some senses.

So, although the this here it is not really stated what a simple liquid or crystalline solid means with the use of Deborah number we can start quantifying our understanding of soft materials in a sense.

Now, what I had like to do is I would like to consider a couple of problems okay, so we go ahead and have a sample problem, so let say you have a sphere of radius r, and some fluid is coming with the velocity of u or u infinity the freestream velocity of u infinity, then in this system, and it is given to you that this fluid has a relaxation time of lambda so this is given okay, so I will just write it down here, given fluid has relaxation time scale of lambda.

So, the question that we want to ask is, what is Deborah number in this case? So, this is just go back to our formal definition we write Deborah number as lambda by t of flow, lambda is already given to you and this is the more appropriate definition to use in this case, because, we are not stating the this is an experiment or anything, so there should be a time scale associated with the flow. So, then your question simply becomes, what is this time scale of flow?

So, the time scale of flow in this case should be R by U infinity this is a time scale that is associated is appropriate to describe this flow. R is the radius of sphere and U infinity is the freestream velocity which means that we can write our Deborah number as lambda by R into U infinity.

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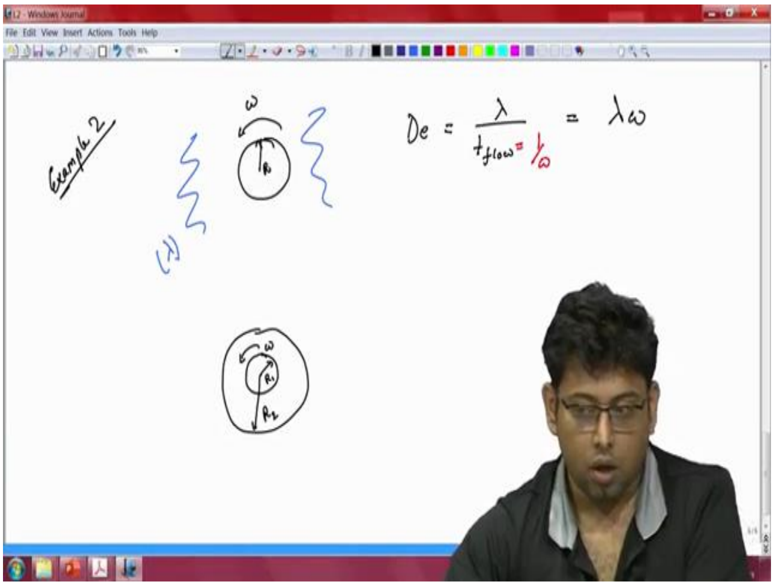

Okay, we will do one quick other example which is you have a sphere of radius R and that is rotating in a fluid with an angular velocity of omega and again the fluid here has relaxation time scale given by lambda so this number is given to you. So, in this case again you have Deborah number as lambda by t of flow, and the natural time scale here, is actually given by omega itself, because omega is angular velocity.

So, t of flow, in this case is simply equal to 1 by omega. So, your Deborah number now becomes lambda times omega. So, I will give you one problem that you might want to think over and we will discuss that in the next class okay, and you have let say two spheres, one has R1 radius, the other one has R2 radius. So, these are concentric cylinders think of this as problem of two concentric cylinders. Sorry, I had said spheres in the beginning sorry. So, let us think of this as two concentric spheres and then one of them the inside one is rotating at an angular velocity of omega.

So, what I would like you to is to, given today’s lecture material tries to think as to what would be an appropriate Deborah number in this case.

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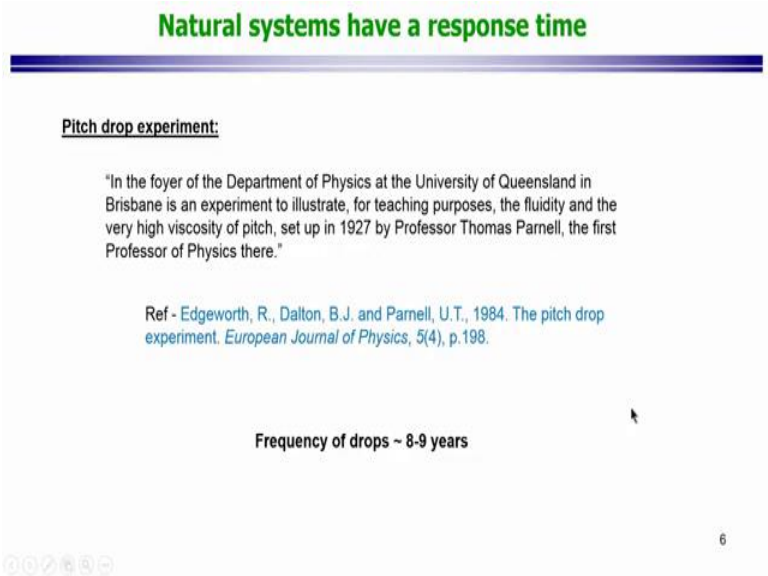

So, what we did today is that we discussed how natural systems have their own time scale and how your observation of that also has an effect of your understanding of the material itself. And, the, we discussed the famous pitch drop experiment and how a pitch drops. So, if you are an alien being you would not necessarily understand it in terms of a solid material.

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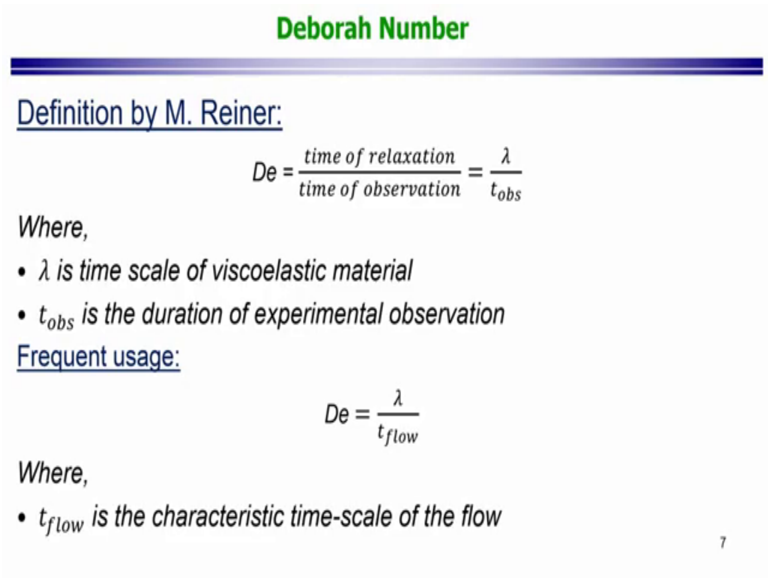

So, that brought us to the very important concept of the Deborah number and we saw that Deborah number in the original definition as given by Reiner and his paper in the 1964 paper if I am correct, he gave it us time of relaxation divided by the time of observation. Where lambda is the time scale of the viscoelastic material and t observation is the duration of the experimental observation.

In frequent usage, the definition of or the format of the Deborah number that is typically used is we replace this t observation by the time scale of the characteristic time scale of the flow. We saw some of the examples and there is a problem for you to also think about before the next class. So, we will stop here for today's lecture. Thanks.