Introduction to Soft Matter
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Lecture 19
Constitutive equations Cont.

So, welcome back to one more lecture on introduction to soft matter. Last time, we were looking at inelastic fluids and we had said that when flow phenomena are dictated only by viscosity and elasticity is absent, then a suitable constituted relationship becomes necessary. And in this kind of constitutive relationship, you have viscosity, which plays an important role but the viscosity need not anymore have a simple linear relationship with shear rate. And that is where the deviation from the simple Newtonian model occurs.

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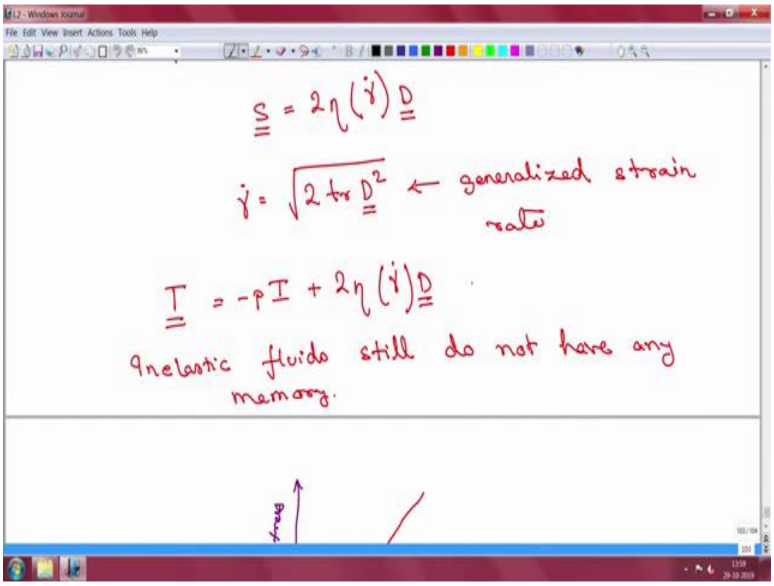

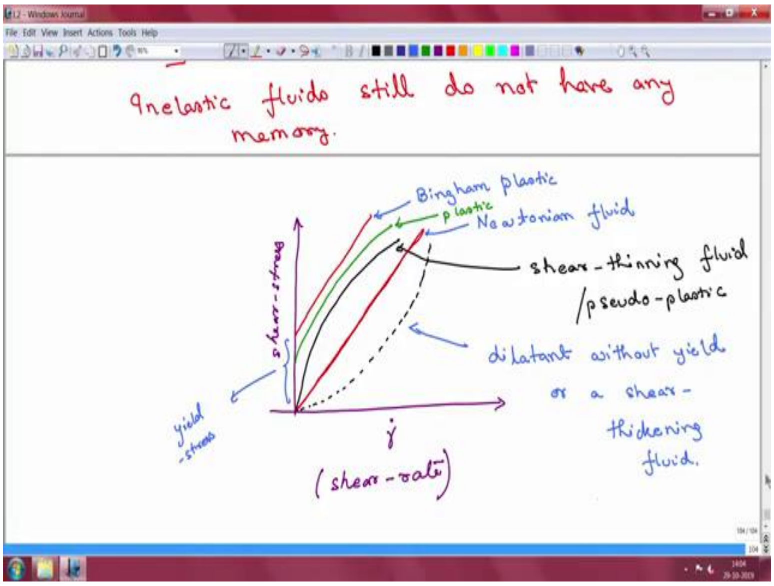

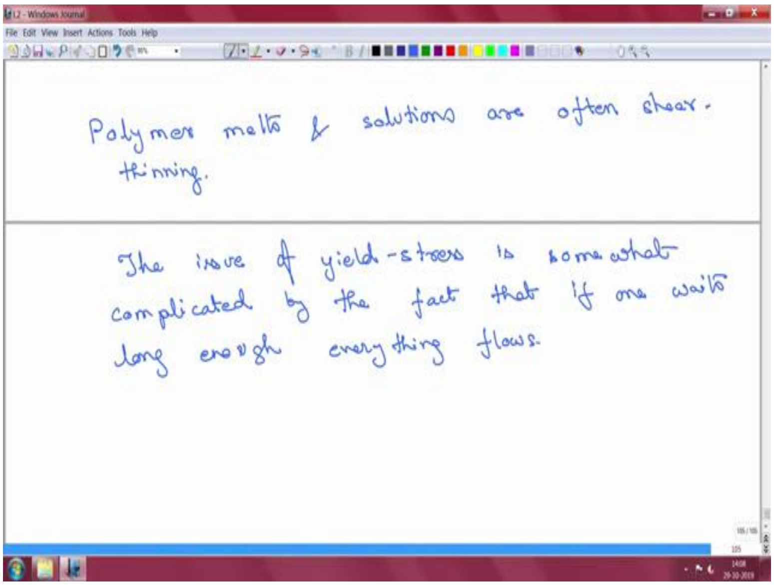

So, let us go ahead and take a look back at that this kind of a phenomena and before that we had also discussed the generalized strain rate and what we had done was we just started plotting the shear stress versus the shear rate and we had drawn one straight line and this straight line is essentially representative of Newtonian fluid.

Now, on this diagram you can have another type of material, which whose stress shear rate graph looks something like this and this you would call as a Bingham plastic, in a Bingham plastic you have a yield stress, that this material can support. So, you have an yield stress here and the material does not freely flow.

So, there is no appreciable shear rate till you exceed a certain yield stress, but after you exceed the shear rate, the material flows almost like a Newtonian fluid. So, you have a relationship which looks exactly like a Newtonian fluid, but once the applied stress exceeds a particular critical value, a slightly different type of material here would look with a yield stress, but where this relationship may no longer be linear, is what is called the plastic (())(2:55), so this is your plastic. Here, the material has a yield stress, so the material starts to flow after a certain critical stress has exceeded, but then the shear stress does not behave linearly with shear rate.

In fact, the shear stress decreases as a shear rate increases or in a sense effectively the viscosity of this material decreases as the shear rate is being increased. So, this is an example of what we also often call the shear thinning phenomena where the effective viscosity decreases as you increase the shear rate. And that brings us to another type of fluid which we where we will use black here and this presents a shear thinning fluid. Also called as a pseudo plastic, you can see that in the shear thinning fluid, there is no yield stress. So, it is actually behaving exactly like a fluid.

But the shear stress is decreasing with a with higher shear rates. So, effectively the viscosity is decreasing there. So, now you can imagine, so this is our control, this is sort of the line of separation, this is our well behaved Newtonian fluid that we have the straight line right in between. And I have drawn all these on the other sides, so you can guess that there will be something on this side as well.

And that is the equivalent of that, maybe I will use a black one again would be a material like this and this is a dilatant without yield or a shear thickening fluid. So, this graph, it has become pretty crowded, but it gives you an idea of the various important inelastic phenomena or inelastic material laws that are there.

And these are important terms that you might see in a textbook, there are other terms as well depending on how the scrub behaves, so we will not go into all of them, we are just going to try and give you a nice pictorial or a plot driven idea of for the various inelastic models can be of these models, one of the most important ones are this pseudo plastic category. And I will just make some space for me to write here. So, pseudo plastic as we said, or shear thinning, polymer melts. So, we had seen earlier that one of the largest classes of soft materials or viscoelastic fluids that we have encountered, maybe the term soft material is more appropriate here are the polymers.

So, the polymer, so most of the polymer melts or solutions of polymers they behave often as shear thinning fluids. So, polymer melts and solutions and I say solutions it means solutions of polymers are often shear thinning. Now, when we say shear thinning, we must understand that in a real situation of polymeric fluid probably has both probably has viscoelasticity, which means it has both characteristics of viscous fluid and the elastic behavior, but when you use the word shear thinning, you must always remember that you are only describing the viscous part of the material and you are implying almost nothing about the elastic part and more often than not that is usually considered that can be considered that the elastic part is maybe totally absent.

In a theoretical model that is possible, because you can create an artificial theoretical model which has only shear thinning, but in a real fluid that might not may or may not be true. So, you can have a polymeric fluid where that can be both viscoelasticity, so both viscous and elasticity, but the viscosity part is shear thinning and the elasticity is somehow is governed by some other equations. So, you have to remember that the word share thinning only describes one part of that real behavior. So, now that make some space for myself.

So, this issue of yield stress, so I am just going to write it down for you, this issue of yield stress is somewhat complicated by the fact, so maybe at this time is a good time to recall the original reason why we coined the term Deborah number and we know that if you wait long enough everything flows.

So, in a sense, if you wait long enough, even I mean, this the concept of yield stress can then start becoming a bit vague as to what we mean because if you wait long enough, everything is going to flow. So, what does this term even mean? So this is somewhat complicated by the fact that if one waits long enough everything flows.

We will not address this in too much detail, but I will just simply say that when we say yield stress, it usually means in a sense sort of almost immediately, you are looking at the response and then in sort of instantaneous sense, so I will leave it at that. Now, since we are going to be dealing a lot with polymers, it is and we have one lab session, where we will actually look at polymer solutions. So, see this shear thinning behavior is something that is very common there. So, while we have discussed the general nature of the graph, one has to you have to quantify the shear thinning behavior you have to give up mathematical model for it. So, one of the popular models for shear thinning behavior is the Carreau model.

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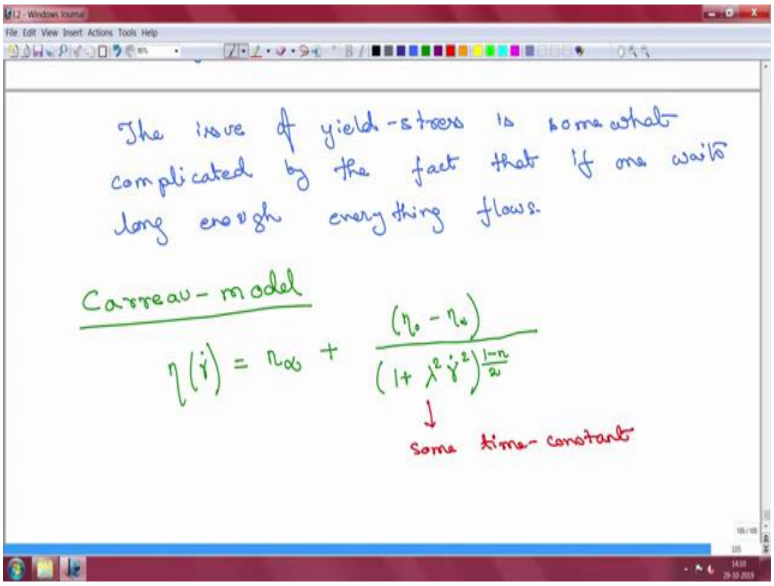

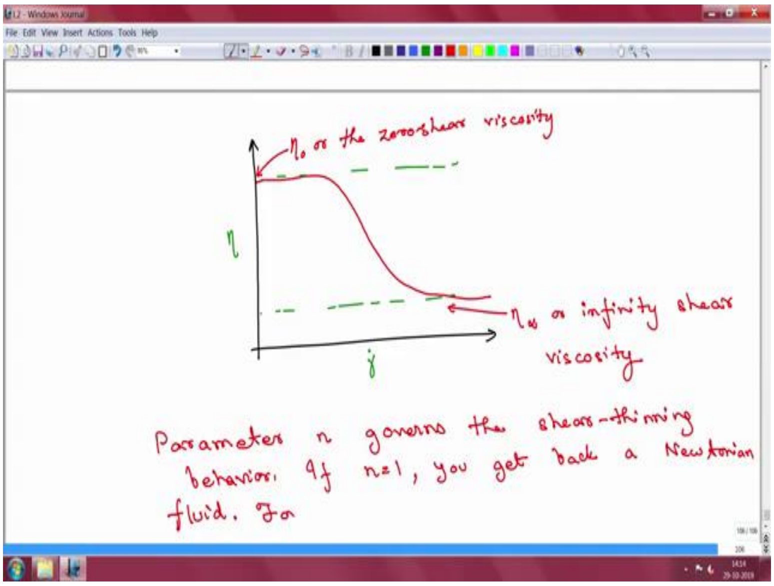

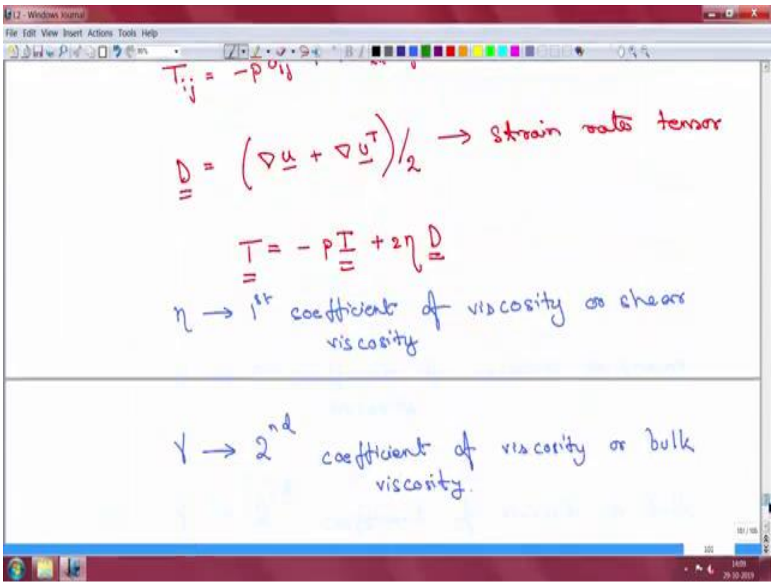

So, and in a Carreau model, the equation that governs this is given as, I am just quickly check what was the term I was using for, so we have been using eta for viscosity. So, maybe we will just stick with that. So, eta gamma dot, so the viscosity when we wrote this in it in this way, it implies that your viscosity is a function of the shear stress is usually it denotes one constant, which is eta to the subscript infinity.

And then you have a fraction which is given by eta naught minus eta infinity and so 1 plus lambda square gamma dot square 1 minus n by 2. And here, I have to specify that this lambda is some time constant and it can or need not be related to your viscoelastic timescales, so lambda is just a parameter in this equation.

And this equation relates a few quantities and maybe it is best to draw graph here and then explain, I will just draw it here, so here, this value this intercept is also called eta naught or the 0 shear viscosity. So, in this model you have an initial time, initial part of the graph which based almost a Newtonian fashion and then the shear rate decreases very quickly with time sorry, the viscosity decreases with shear rate very quickly.

So, in this graph what you have is eta naught is the zero shear viscosity and the viscosity is more or less constant with respect to shear rate in the beginning, then you have another section where the shear, the viscosity decreases sharply with increasing shear rates and then it finally stabilizes to another value. And this other intercept and this intercept of this asymptotic value this is your eta infinity or the infinity shear viscosity.

So, this explains the two few terms here. So, we have eta infinity here, this is the infinity shear viscosity you have eta naught, which is the 0 shear viscosity again you have eta naught minus eta infinity in the numerator and this is again the infinity shear viscosity and then in the denominator you have lambda squared lambda b some time constant associated with this phenomena, gamma dot is a shear rate is and then you have a parameter n.

So, this parameter n and governs the shear thinning phenomena. If n is equal to 1, then you can see easily, you get a Newtonian fluid, you get back a Newton fluid. And usually, your n for shear thinning fluids, this model is meant only for shear thinning fluids. So, for shear thinning fluids your n is lies between 0 and 1. So, we can see even from the graph that there are three basic regimes of this and you can figure out what the regimes are by from this equation.

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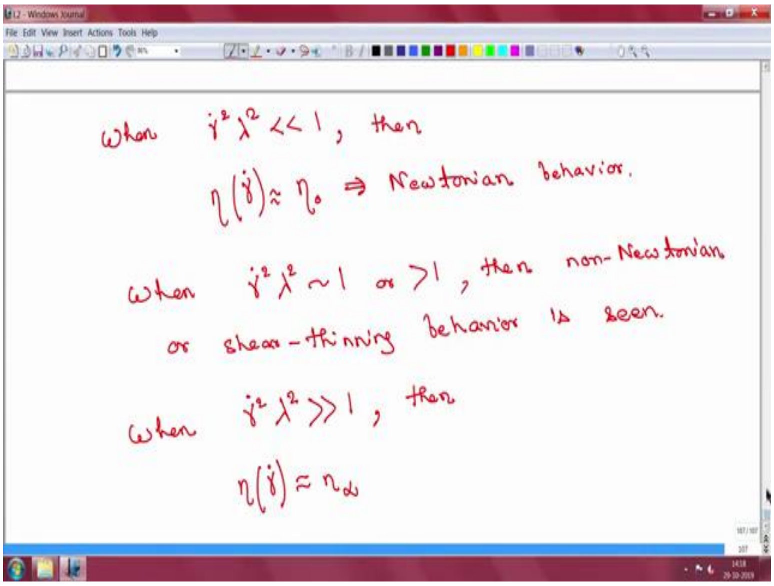

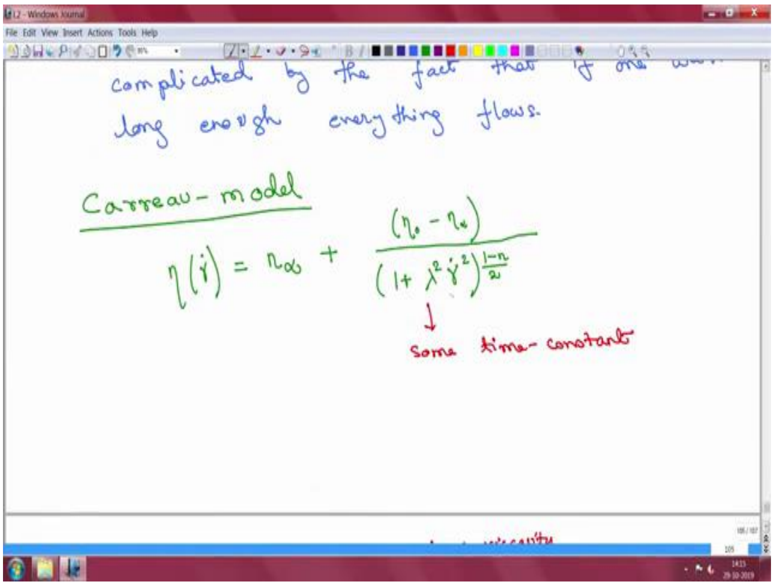

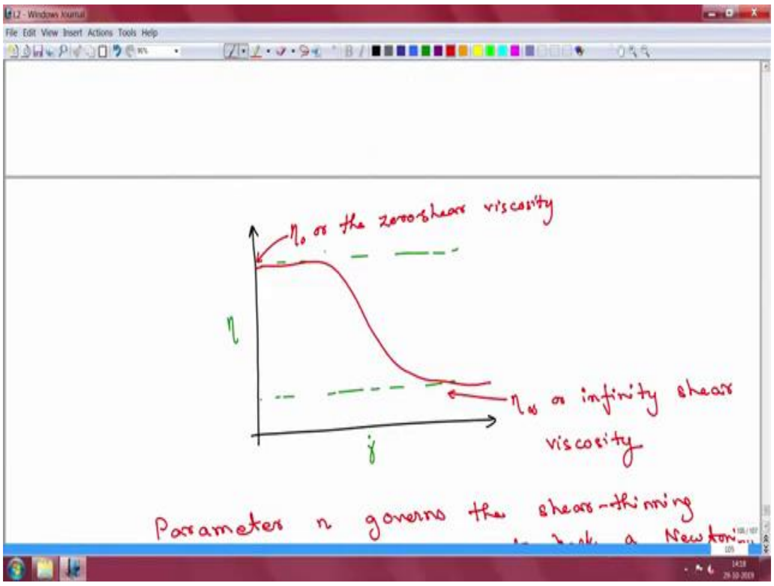

So, maybe we will discuss that a little bit of detail. So, for example when, so you have this term in the numerator. So, let us go back to this. So, in this particular term lambda square into gamma dot square, if this term is much much smaller than 1, then in the denominator, you will have 1 to the power of something. So, you can see that when this term is very very small, the behavior should be approximately Newtonian. So, from the mathematical model, we can say see that when is much much less than 1, then it is more or less eta naught or you get a Newtonian behavior in the beginning.

For small enough, shear rates maybe I will just put an approximately sign here. It is not exactly what is it but it is approximately equal to eta naught. When you have a region where this term will start to become comparable to 1 and that is where you have you start to have appreciable shear thinning behavior and that is when you will see.

So, when this is the order of 1 or greater than 1, then non-Newtonian or shear thinning behavior is seen. And finally, if you go back to this equation, obviously when this start term starts to dominate, becomes very very large then the, so then this, so when this term becomes very very large this term this entire denominator starts to go towards infinity.

So, in that case your much much greater than 1 starts becoming, so this particular Carreau model if you take the situation where we are looking only at the part where the shear thinning behavior is very very dominant. So, if you are already in a range that the shear thinning behavior can be seen very predominantly. Then you end up with what is called as a power law.

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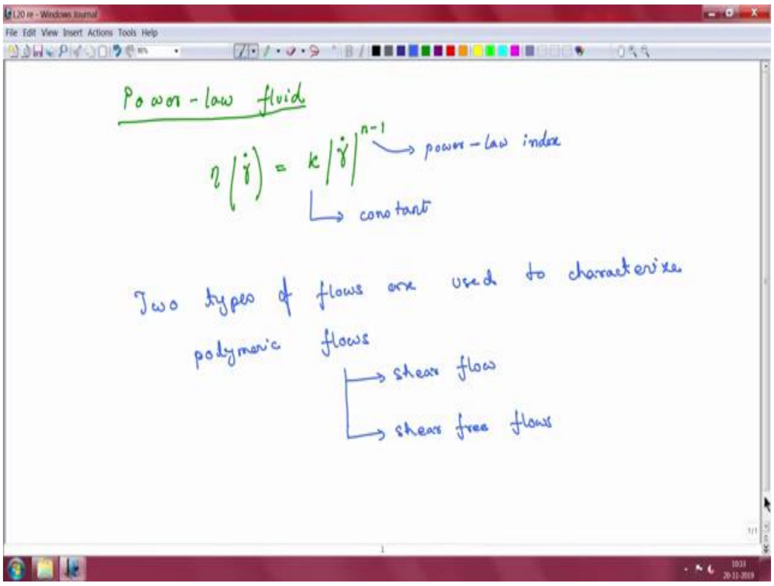

You can devise another constitutive relationship, just call the power law fluid. Here your viscosity as a function of gamma dot is given by k times the absolute value of gamma dot raise to power of n minus 1, where this k is a constant and n is the power law index, if you assume a value for n eta you assume a value of 0. For high shear rates, you can show that the Carreau model can also can be reduced to this particular form. But usually the power law fluid formulation is meant for calculating the viscosities in a given shear rate regime.

So now, this brings us to another important idea that is often used in polymeric fluids and to characterize polymeric fluids we what we do is we often use two separate type of flows. So, there are two flows, two types of flows are used characterize volumetric flows and one of them is called a shear flow, a simple shear flow and the other one is called the shear free flow. So, today we will only look at in detail at the sheer flow or simple sheer flows.

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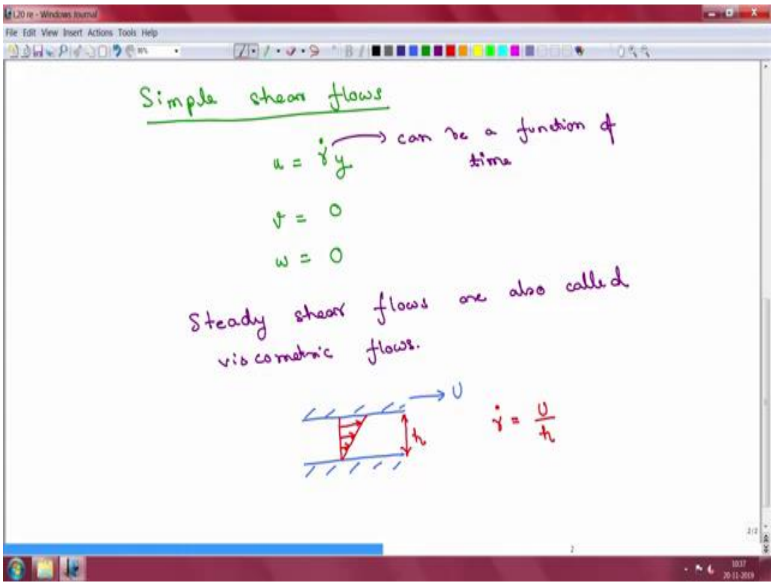

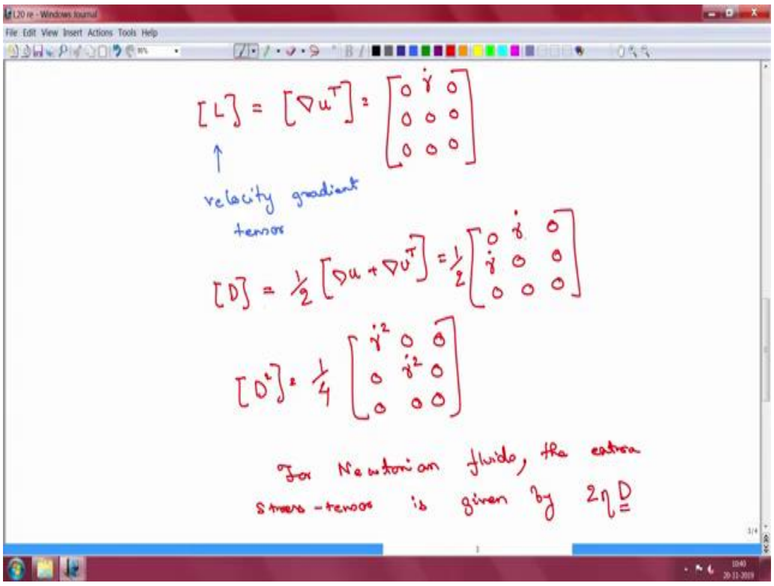

So, let us look at simple shear flow is such flow, where the velocity field is given by, so you will have three components of velocity for any given general flow. So, for a simple shear flow, we say that the velocity should be of the form you should be have some gamma dot times of y, we will clarify why we are writing it as gamma dot is basically some constant times y.

The other velocity u, v and w which are the other two components of the velocity this should be 0. So, it should be a one dimensional flow, such that the flow is a function of the x direction flow is a function of y. And in general, this gamma dot can be a function of time. Now, steady sheer flows, so when you when this is not a function of time, is this can be a function of time in general.

But when this is not a function of time and this is a steady here situation steady shear flows are also called viscometric flows. Now, how are you going to cause such a flow? It is a very simple way of creating this kind of a flow structure and that is by taking two parallel plates which are separated by a small distance and I will tell you why is small distance.

The bottom plate is stationary and the top plate is being moved ahead by some velocity capital U. So, this top plate is being dragged at a constant velocity. In that case, the flow here will look like this in a laminar flow case, the flow and if this is a height h, then gamma dot in this particular case is given by for a steady flow system.

Now, this kind of a flow is very important because it helps us characterize some important quantities which are, we will go see that there are important material functions for polymeric flows or in viscoelastic flows in general. Now, when you have this kind of a simple shear flow, your velocity gradient tensor, if you calculate that this is simply my gradient of this quantity will have all zeros at all locations except for once case which is the second entry here, rest of all these entries will be 0. You can probably see that with rest of all the velocities are 0. So, all the derivatives are also zero, this is the velocity gradient tensor.

Now, in this particular case, if you calculate the strain rate tensor then that is going to be that is half times transpose. Now, this will have two entries, you will have the half here and this gamma dot here will also appear here, the rest of the terms again going to be 0 and gamma dot is the share rate for this entire system. And you can actually quickly double check also if you want, you can calculate for example, you can calculate the square and you can see that that will actually be equal to gamma dot square 0, 0, 0 gamma dot square. And from this you can see, if you calculate the generalize shear rate then you find that resist gamma dot in this case.

But the important point I want to is we saw that for the Newtonian case, your extra stress tensor for Newtonian fluids, the extra stress tensor is given by 2 eta times D. So, here you can look at the form of this matrix and you see that all these terms are zero, right? So, the normal stress terms for a simple shear flow for a Newtonian fluid is such that the normal stresses in all the cases actually the total, when you calculate the total stress that will have the normal stresses will only be minus p minus p minus p. So, all the normal stresses are actually equal in that case.

So, this is an important point with respect to Newtonian fluids, there in a simple shear flow, the normal stresses are such that they are always equal, but we will see that that is not the case for non-Newtonian or viscoelastic fluids. So, we will do that in the next class. So, we will leave it here for today. Thank you.