Introduction to Soft Matter
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Lecture 17
Constitutive equations

So, welcome back to one more lecture on Introduction to Soft matter. So, till last class we were discussing examples of soft matter, and we stopped at we basically discussed polymers, surfactants and pneumatic crystals, etcetera. But today we want to go ahead and we would like to start looking at another important set of issues that affect soft materials and viscoelastic phenomena and that is this concept of constitutive equations.

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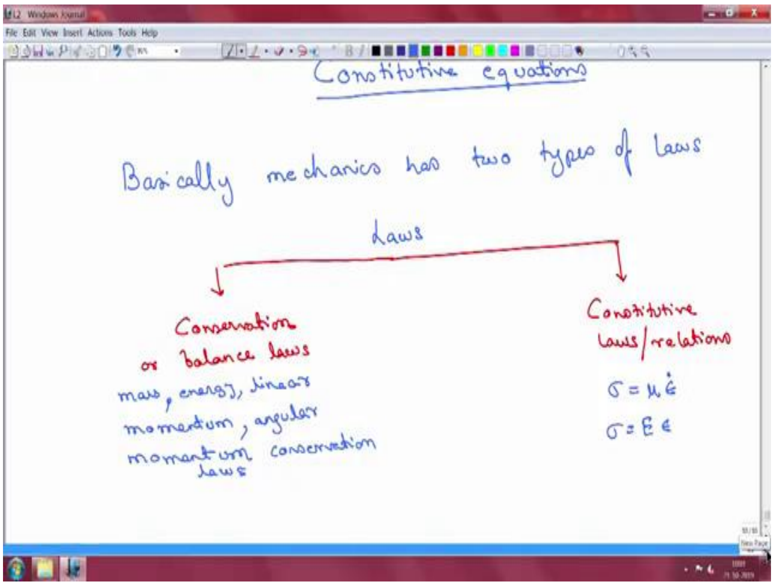

So, we are going to come back to our continuing viewpoint in a sense here. And when we are speaking of constitutive equations, we are implying certain laws that are different from the conservation laws. So in, basically mechanics, so mechanics has two types of laws. So, let us say laws and on one side you have what are called as conservation laws, conservation or balance laws, and on the other side you have constitutive laws or relations.

So, the idea is that when we speak of balance laws, we typically mean the important principles such as the principle of conservation of mass, principle of conservation of energy, principle of conservation of linear momentum, and principle of conservation of angular momentum. So, these are examples of balance laws so here your, so the ones that you will be using in mechanics we use mass, energy, linear momentum, and angular momentum.

I am putting angular momentum separately because it is a separate law so these conservation principles. And on the other side you have constitutive relationships for example, the ones that we saw before. So, sigma equal to Mu times of Epsilon dot or sigma equal to E times of Epsilon that we have already discussed, these are examples of constitutive relationships. Now there is a big difference between the two cases. The conservation laws, when we speak of them, conservation of mass, energy, momentum, etcetera., they are understood to be perfect in a certain sense.

So these are not approximate laws, these are to the best of our understanding to the best of our application we will apply them in the sense that they are exact. And when we applied them, these also while these are laws of mechanics, they also apply to all other branches or all other cases for example, the idea of linear momentum, linear momentum conservation holds beat in the relativistic domain or beat in the quantum mechanics domain.

So, these laws do not change, the idea of conservation of mass holds true irrespective of which domain you are working in. Obviously, this conservation of mass and energy become actually one combined law under different cases. So, in a certain sense these different (relations) balance laws are in a sense exact so, they are truly laws in a sense that they cannot be broken so the conservation of mass and energy to our best of our understanding cannot be broken.

Whereas, constitutive relationships are approximate of phenomenological approximations that encode material properties and they can be approximate, they can be easily broken sometimes. And if they are broken then all we have to do is to change it accordingly. But these, it does not mean that they are universal in the same sense as the balance laws.

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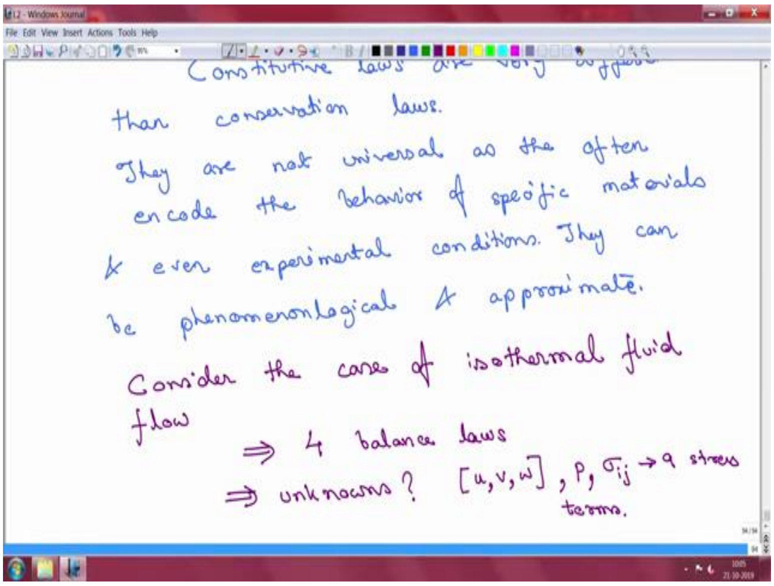

So, we will just write down this important point about constitutive relationships. So, constitutive laws, and again, a lot of people want to call it equations or relations that is a better term but I am just using that here the word laws, constitutive laws are very different than conservation laws. So they are not universal as they often encode behaviour of specific materials, materials and even experimental conditions. So for example, when we speak of the Newtonian fluid, we know that it is already a very special class of fluid.

So, this equation that we just wrote above sigma equal to Mu epsilon dot, we know that this holds only true for a certain class of materials known as the Newtonian fluids and otherwise it would not hold true. So, it encodes behaviour specific materials and there can be many other materials for which this law is easily broken or the deviation is very, very strong from this.

So, they are also very, so they can be phenomenological and approximate. And many times they can even be oversimplified expressions of the true behaviour. So, for example, the true behaviour might have many different order terms, but you realize that the different order terms do not contribute so much to the numerical value so you can drop some of them and say that okay I will only use some part of this.

In any case constitutive laws are very important for the solution of various problems of mechanics and that comes because so for example, why are constitutive laws very important, just consider the case of, let us consider the case of isothermal and even incompressible fluid flow. So, let us say isothermal fluid flow. So when you have this, obviously you can apply the four different balance laws, mass, energy and linear momentum and angular momentum.

So you have the 4 balance laws. But how many unknowns? So, what are the unknowns? And you have 4 unknowns? No because you have at least 3 unknowns from velocity, see the u, v, w in a 3 dimensional system you have u, v, w is the components of velocity there are unknowns. Pressure is an unknown, it is isothermal so the temperature we will just say that it is known. And then you have sigma i j which are the stress terms, and these are 9 stress terms. They are also unknowns, so you have a set of 9 or 10 and 13 equations here, which are going to make life very, very difficult for us.

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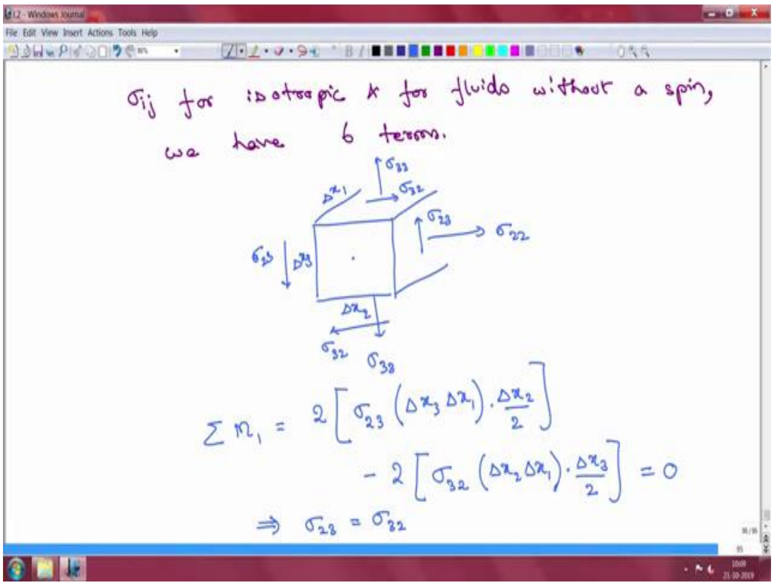

But interestingly at least the Sigma i j can be simplified. So, the in case of Sigma i j, so Sigma i j for isotropic fluid which just not have a spin of its own, and for fluids without a spin an intrinsic spin we have 6 terms. So why do we have 6 terms? Because this is something that you probably have seen in your undergraduate class, but I am just still just quickly going to work this out. So you have delta x 1 here, let us say you have delta x 2, delta x 3 we have stressed terms, sigma 2 2, sigma 1 1 is into the planes, I am not drawing that.

Then you will have sigma 3 2, you will have sigma 2 3 here and then you have sigma 3 2 in the other direction and then you have sigma. So you have 9 stressed terms and fluid which does not have an interest spin, your moment should go to 0. So, you take the moment about let us say centre and what you find is, so, if you take it such that you take the moment term is given by delta x 2, then you have two forced terms, so I will just multiply it with 2 initially. So, you have sigma 3 2 or rather 2 3 first, let me write down 2 3 first.

So, you have sigma 2 3, that is going to be multiplied by delta the area of the phase which is x 3 into delta x 1 and multiplied by the moment term, which in this case is delta x 2 by 2. And this moment is being balanced by the other permutation, which is sigma 3 2, so sigma 3 2 then you may again have to multiply with the area and that is now this is, so to get the force and then the moment term in this case becomes this. So from the conservation of angular momentum, we want that this should be equal to 0 and this will give you sigma 2 3 equal to sigma 3 2.

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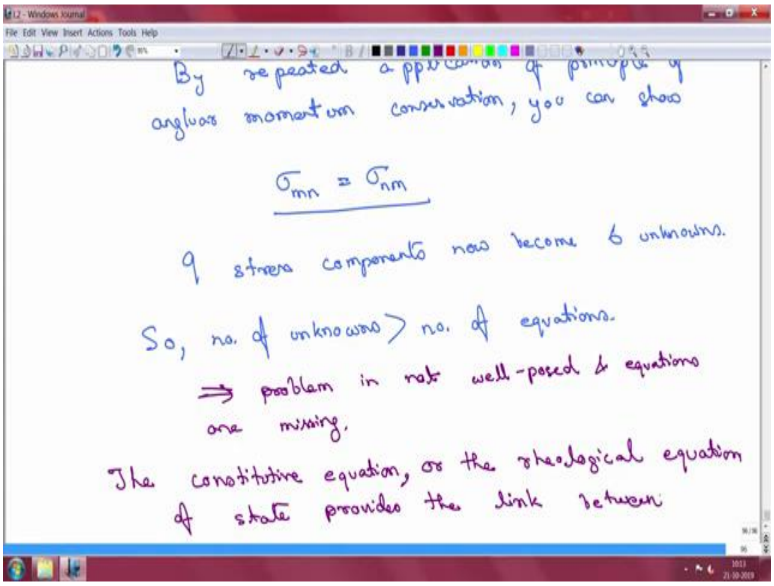

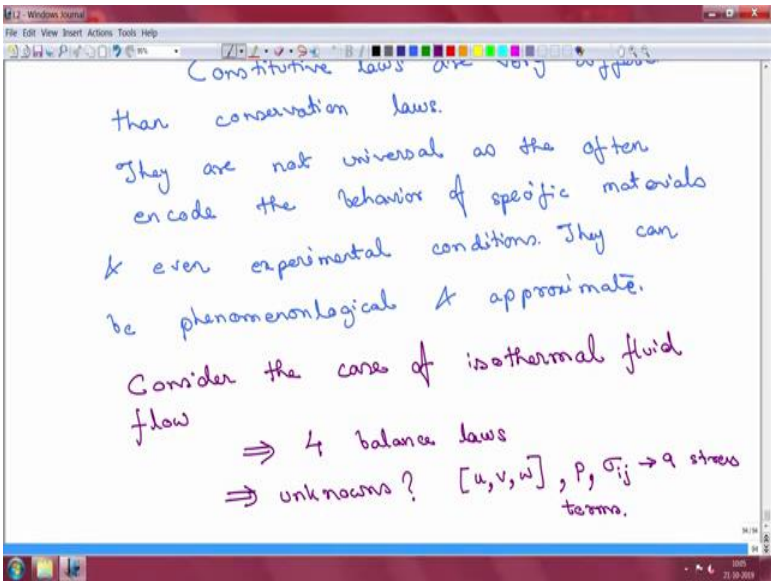

By repeated application of Principle of angular momentum conservation, you can show sigma m n in general is equal to sigma n m. So that reduces your stress so the 9 stress components now become 6, 6 unknown components. So even after that, even if you have 6 if you just go back to this problem, you already have 3 unknowns here then this is the 4th one, you only had 4 balance laws. So, even if you reduce the sigma i j from 9 stress components to 6 components, you still have more number of unknowns than the number of knowns. So, despite this simplification that you have been able to do, so your still number of unknowns is greater than number of equations.

So this problem is not well posed which implies that this problem is not well posed and equations are missing, and this is where the constitutive relationship comes in. The constitutive relationship here what we often call for soft materials, so the constitutive relationship constitutive equation. So as I said, the problem is not well posed and equations are missing so, somebody has to provide you that equation. And that equation, which relates the stress and the strain and provides you with extra equations to solve the problem here that is called the constitutive equations. So the constitute equation are often called the rheological equation of state provides this linkage between stress and kinematics.

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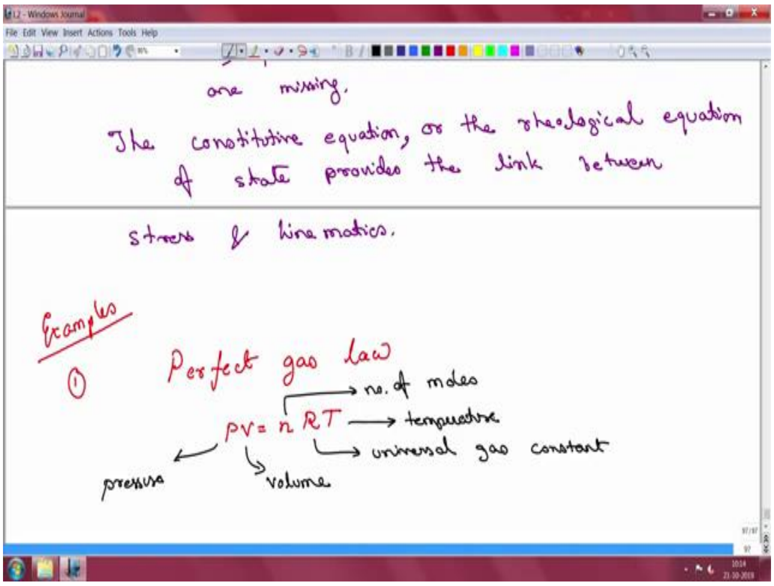

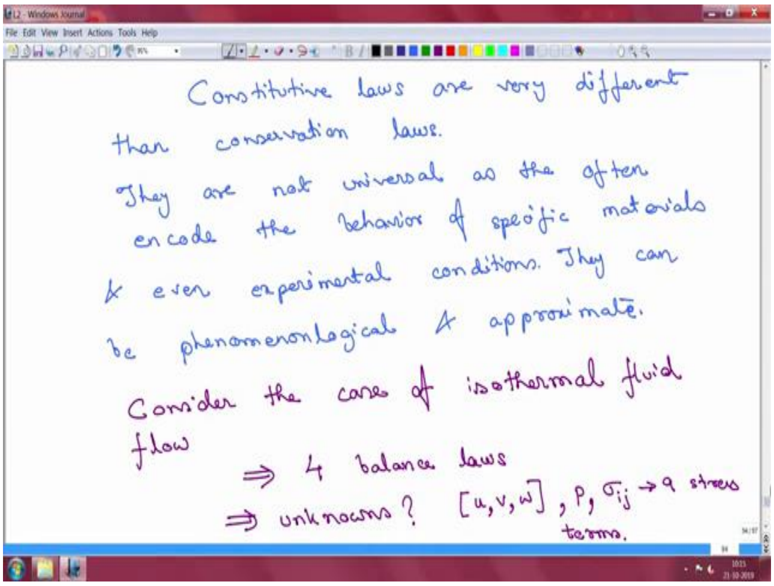

So you have to get these extra equations and then your problem, you are going to ensure that the number of equations and the number of unknowns are the same and then you are going to solve the problem so that is why the constitution comes in. So, let us take a look at some of the examples and we will just take a look at some broad examples of constitutive equations. What are the broad examples?

So, probably one of the simplest (conservation) constitutive equations that almost everybody is familiar from school days is the perfect gas law. So, here we state that P V is equal to n R T, where P is the pressure of an ideal gas, V is the volume, n is number of moles, R is the universal gas constant, and T is the temperature. So this equation P V is equal to n R T, this is not a conservation law.

So this is not a very fundamental law of nature, this is a law that has been determined from many different experiments. And it has found to be generally true for a class of materials, which we would call the perfect gases. And in the case of perfect gases, it has not found that this relationship does hold true. So, if we go back to our description of other constituted relationships, where we said that the input of the behaviour of specific materials so, this and even experimental conditions.

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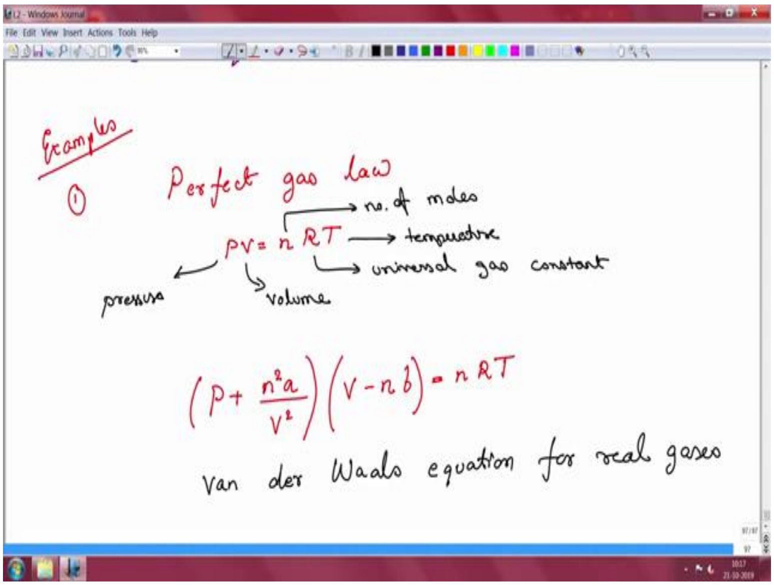

So this particular gas law, the perfect law, this is important with respect to a particular type of fluid. And if you have an adiabetic expansion, then you have polytrophic process, then this equation changes, you can apply maybe P V to the power with Gamma equal to constant that would also be applicable. So, we see that this does satisfy the important characteristic of constitutive equation that we said and then it also satisfies the other characteristic which we said that a constitutive equation need not be universal or fully correct so, they are just approximate relationships.

So, this relationship we know that for real gases it actually breaks down, this does not hold true and for real gases you will probably need, different people have provided different constitutive relationships, but probably the most famous one that you might almost all be familiar with is the Vander Waals equation of state.

So, in the Vander Waals equation of state you have correction terms and this equation goes like, I will use the same colour, so, you have pressure, n square which is still the number of moles, then you have a constant a which is a Vandal as a constant of this particular equation, and then you have V minus n, again n being the number of moles, V being the volume and then another constant appears, it is called B and now this is equal to n R T. So, this is your famous Vander Waals equation for real gases.

And these correction terms as we remember they take into account that the molecules in the gas they do have a volume, so they are not point particles. So, despite this correction, this is still applicable to gases. So, this is not for different condensed matter forms so, this is why this equation still satisfies the constitutive relationship idea that we had said. So, we will take a look at another example.

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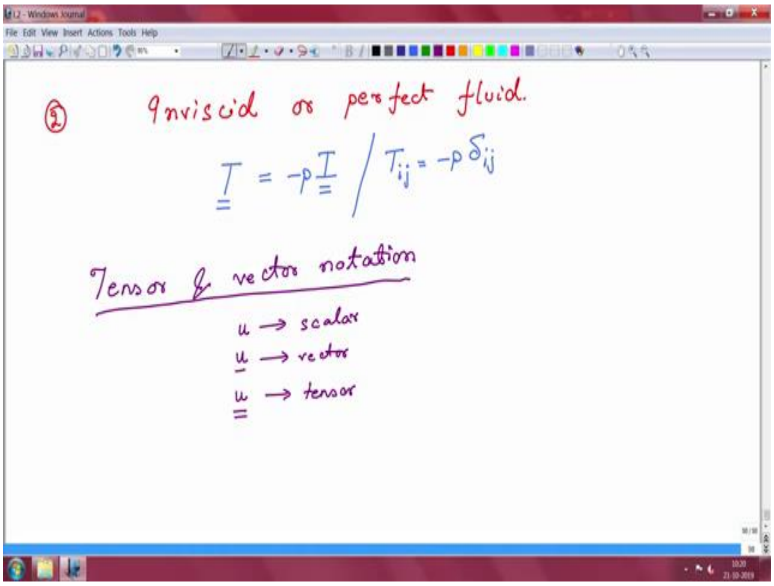

And this is the Inviscid of perfect fluid, and in the Inviscid fluid, there is no viscosity that is why it is called Inviscid, the viscosity is totally missing. So, what kind of forces can Inviscid fluid exert? An Inviscid fluid if it cannot exert any force due to shear viscosity, then the only other force left is your pressure. So, your stress tensor and I will explain the so till now I have not used tensors or I have not had a lot of need to introduce tensors so much, but let us just say T is the stress tensor here, this is equal to minus p, I will explain the symbols that I am going to use in a slight bit.

So, the stress tensor is equal to minus p which is just a number into I or the identity matrix or you can even write this as Tau i j equals minus P del i j. So before I explain this, maybe I should introduce my tensor notation, and what we will do is, so let us say if u is a scalar, we will just write it as u from now on. If u is a vector then we will put a bar underneath, so this denotes a vector.

And if u is a tensor, then we will put two bars and that would denote that is a tensor. So, this class we will just stop here, and in the next class will go on and discuss more about these various, we will discuss about the tensor notation a little bit more and then we will go on and discuss a few more constitutive that are relevant to us. Okay, so we will stop here today.