Introduction to soft matter
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Lecture No 10
Tutorial

Hello, and welcome everybody again to one more lecture on Introduction to Soft Matter. In the last class, we had stopped at an example problem and we had said we had to solve that problem. So, before we solve it, let us just quickly go over that problem one more time.

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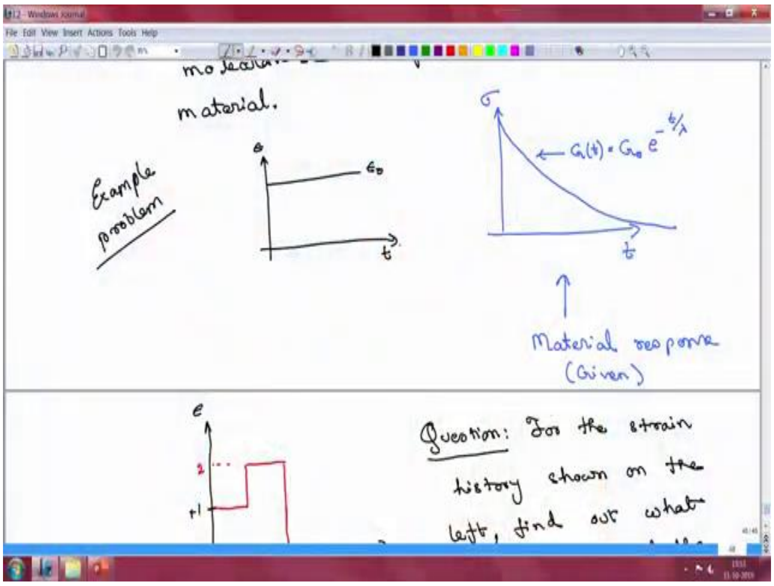

And we had said that there exist some material and that material is subjected to a strain, step strain test. And the material response has been measured and is known and is given to you. So, the material response in the form of the stress relaxation function is given as Gt equal to G naught e to the power minus t by lambda. So, this is the data that you already have.

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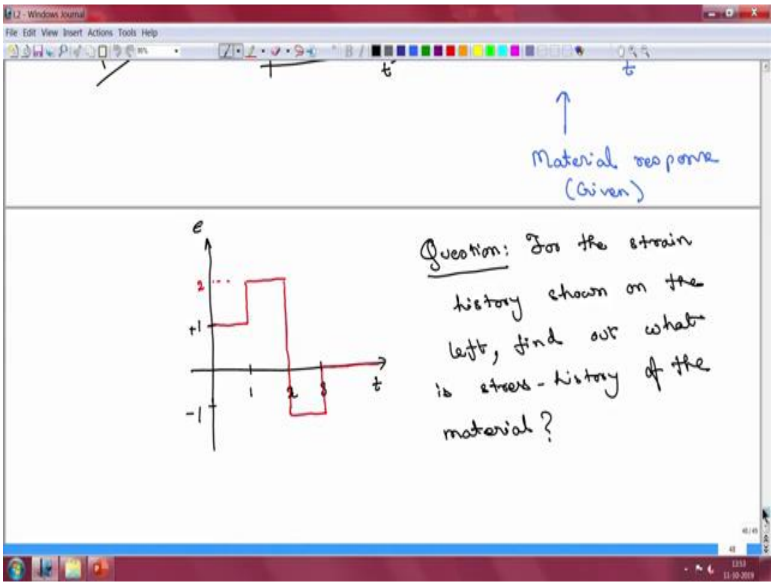

And then the question was asked that if I have a strain history, which looks like what is, what I have drawn on the left hand side, then for this strain history shown on this, on this diagram, what is the sorry, so, for the strain history what is the relevant stress history of the material right. So, we want to solve this particular problem.

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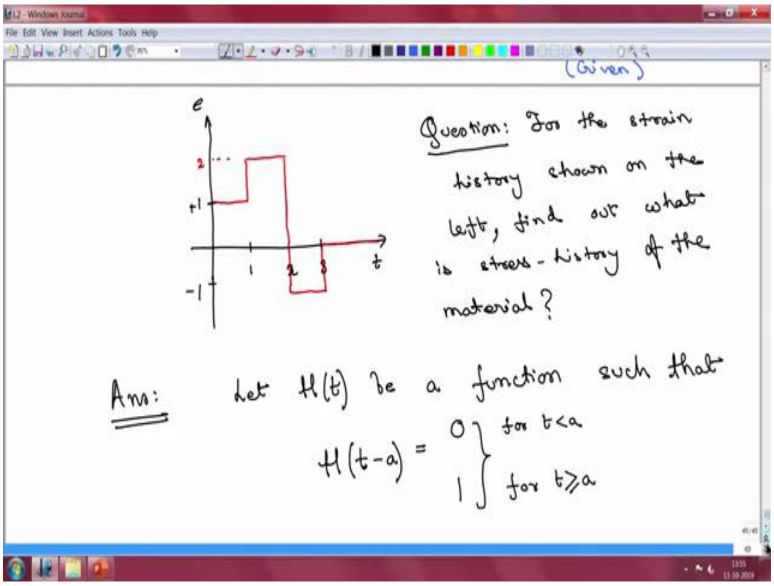

Before we solve this, so, let us say, we are going to say, we are going to look at the answer to this particular problem. Before we solve this problem, I want to introduce a mathematical idea and I want to introduce in particular a function and let us call this function let Ht be a function such that and this is a definition part of Ht that Ht minus a is equal to 0 it takes two values this function takes two values, it takes the value of 0 for t less than a and takes a value of 1 for t greater than equal to A. This is also called the Heaviside function.

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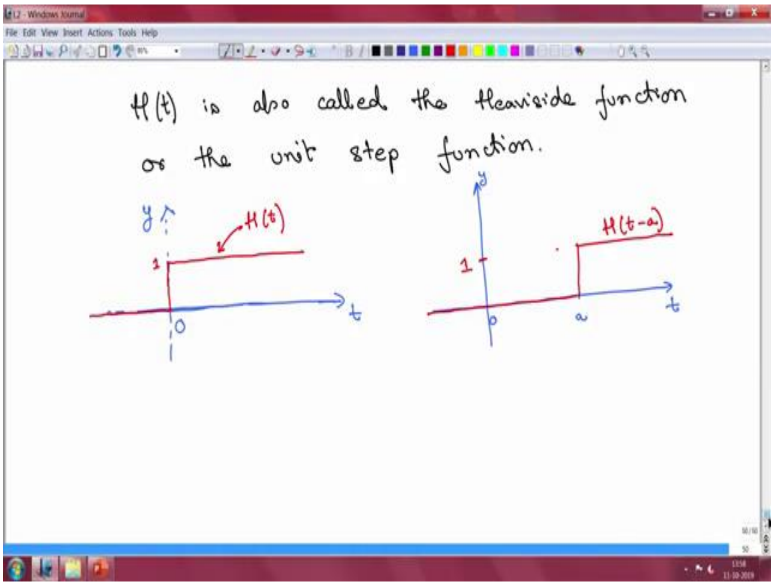

So, Ht is also called the Heaviside function or the unit step function. Why is it called the unit step function? Let us quickly look at this in a graphical form okay. So, let us draw this. So, let us say this is your time axis, this is your 0 and then here on this axis you are going to plot H of t.

Let me just say this simplify our life, we say we have plot y and then so here if a is equal to 0, then this function will be 0 till it reaches a value of 0, t equal to 0 and at t equal to 0 this function will suddenly jump to a value of 1. So, this is 1 and for then for the rest of the time it will stay at a value of 1. So, this is your H of t.

Now, since we are doing this graphical representation, we might as well continue I will just quickly look at what Ht minus a looks like. So, we are still going to plot this is t this is the y axis, this is your 0 and somewhere here is a. So, what happens if when we have H of t minus a, then this function remains at a value of 0 till t takes on a.

And at exactly at this point it will increase. And this on the y axis, this is your 1. So, this is your ht minus a. So, you can see why this is a it is called the unit step function right. And it is also called the Heaviside function in memory of the scientist Heaviside.

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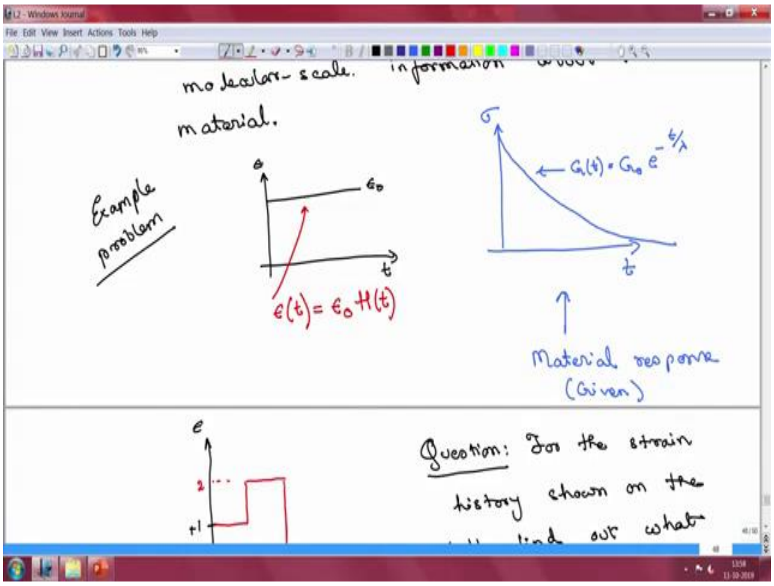

So, your step response test here this can be written as epsilon t is epsilon naught into H of t. If I had to represent this graph in a mathematical sense then I can write this. Now, we will recall the superposition to, superposition of responses and to construct this strain history we will ask, how can we construct this particular strain history that is given on the left in as a sum of different step functions. Okay.

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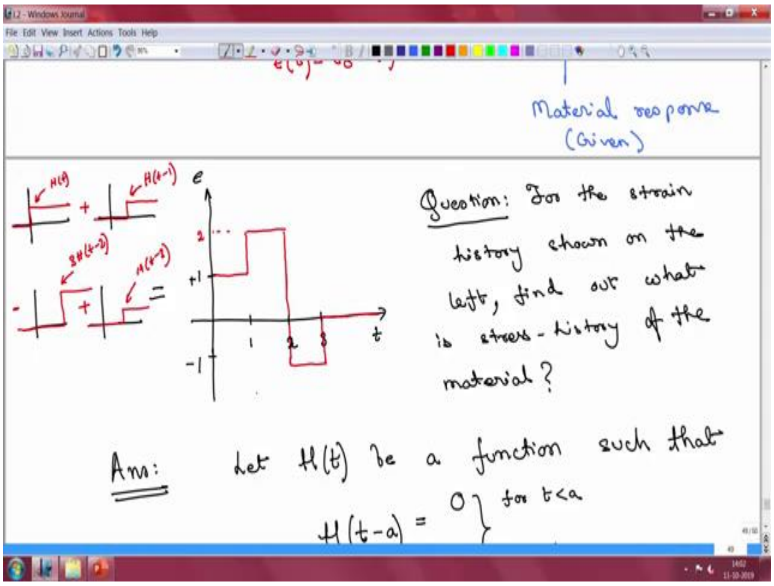

So, do that this curve okay, maybe I will let me see if I have space here. Okay, maybe I just try to fit in, it right here. So, this function can be thought of us, and I will draw this 1, 2, 3, 4. So, this function can be thought of as being equal to this is Ht. This is H of t plus H of t minus 1 plus, actually, in this case, so now what you have, if you add these two, you would have at t equal to 1 you will have a function, which is going to attain a value of 2 and then it is going to stay at 2 forever.

But you at t equal to 2 this function here is going to achieve a value of minus 1. So, here actually instead of a plus, this should be a minus and it should be. So, this is actually 3 times of Ht minus 2. So, if you add these three, if you superpose these three signals, then what you end up getting is at t equal to 2, your function will attain a value of minus 1, but here it will stay at minus 1, but this particular function here it stays till of till value, it stays at a value of minus 1 till t equal to 3 and then achieves becomes 0. So, here sorry okay, this is a small mistake here, let me correct that okay. Okay.

So, here what I wanted to do what I should have done is this goes, so this is your 3 times of Ht minus 2, it should be displaced on the y axis and you add this is lack of space here. So, please accept my poor drawing and you have another Ht minus 3. So, this signal can be thought of as a sum of these 4 signals, right. So, if you add these 4 signals, you end up getting this.

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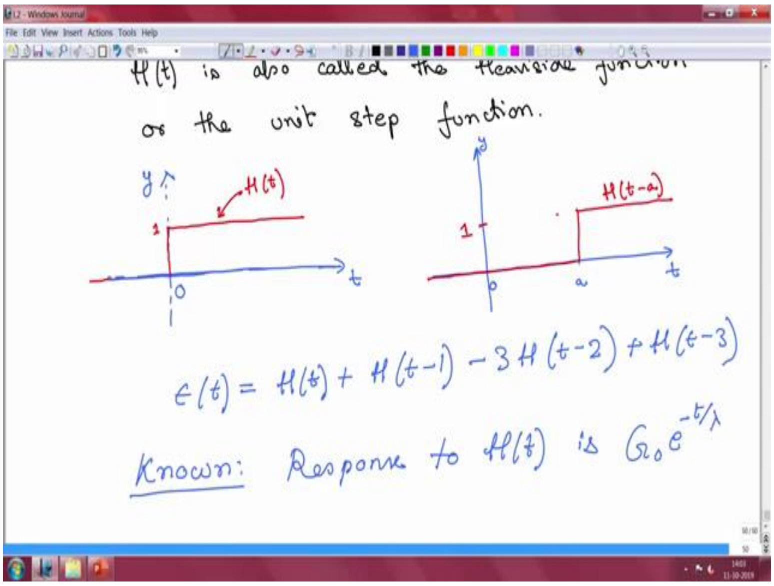

In mathematical form. I can say that the functional form that has been given to me is essentially H of t plus H of t minus 1 minus 3 times of H of t minus 2 plus H of t minus 3. So, you can see that this arc at equal to 3, all these will add up this will become 1, this will become 1, this is minus 3, this is 1. So, it adds up to a value of 0. And what is known is that response to Ht is your G naught e to the power minus t by lambda.

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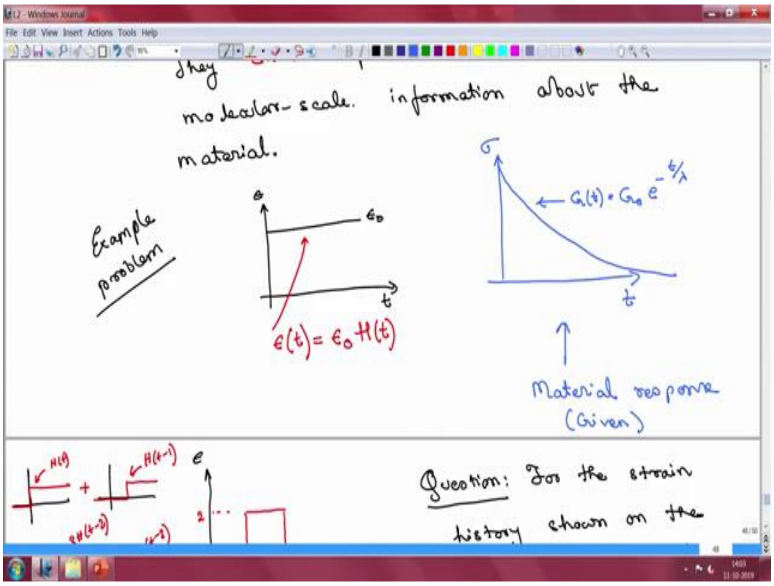

If we just, we go up this functional form was provided to us and this functional form is the stress relaxation curve for this function here.

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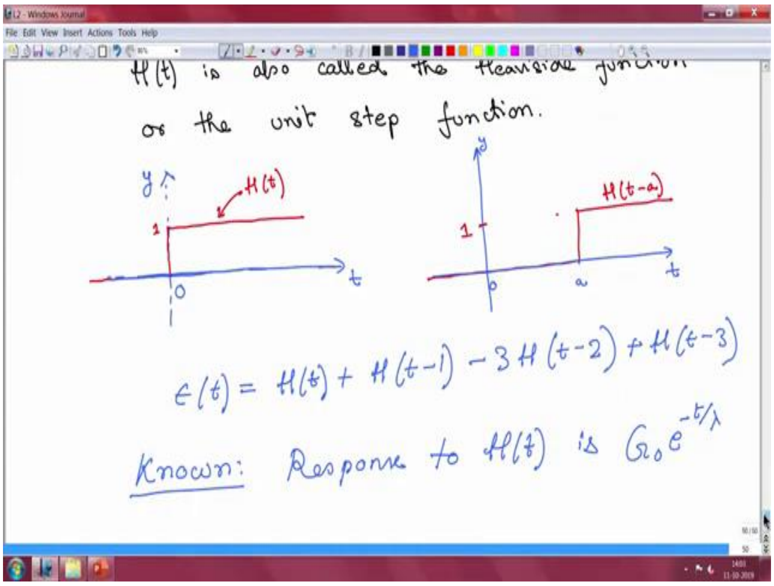

Actually this is to be mathematically fully correct this is epsilon naught and epsilon naught value is 1 in this case. Okay.

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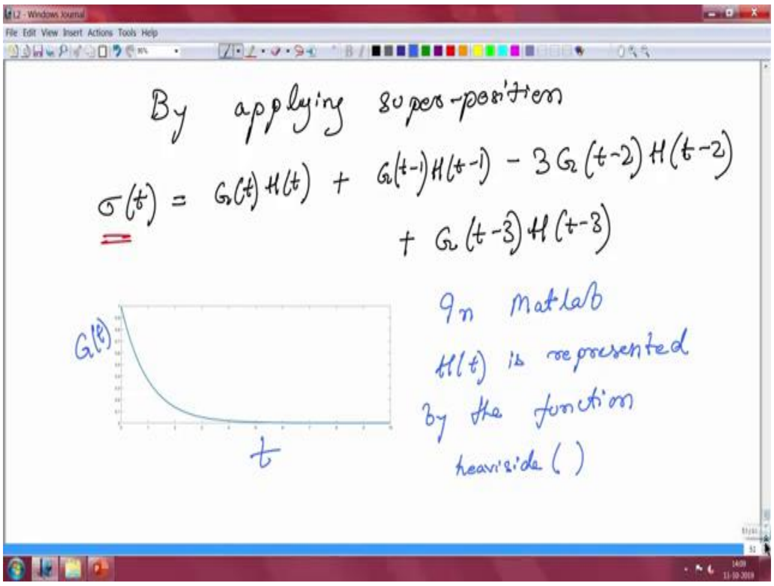

So, now, if you, if superposition is valid, so by applying superposition we can say that the stress history that we are after is equal to Gt multiplied by Ht. You can drop Ht also here in the first case because it does not. So, the function Gt if you remember when we are first discussing it. We had already said that 40 less than 0, this function GT is also 0. So, we have defined it like that.

So, this function would automatically become 0, but if you multiplied it to Ht, it does not make a difference. I am going to multiply to the Ht so that I can write a consistently nice equation. So now, so, this is the response to the first one, what is the response? So, the next one is your Ht minus 1. So, we have to multiply some Gt with this Ht minus 1, but should we multiply Gt or it should be Gt minus 1.

See, here, when you are giving your input at t minus 1, this function also has to be translated to t minus 1. Okay, so this cannot just be G of t, so this has to be G of t minus 1 to be accurate. So, similarly, you have 3 G minus 2, H of t minus 2 plus, let us just be consistent. We are first writing G. So, this t minus 3 and this is Ht minus 3. So, actually, this is the answer. This is the answer that you are after. This is what had been asked of you.

So, what I have done for you is, so this is a mathematically correct or the proper mathematical formulation for this. But let us say somebody asks you to go ahead and numerically compute it. So, what I did is, and please, you know, don't just take whatever home I am discussing here for face value. Check these yourself whenever you get time. So, what I have done here is I have plotted this is your Gt.

I have taken a value of G naught equal to 1, and I believe lambda was also 1. So, this is a particular case of this problem. And this is plotted in MATLAB. This function and this is an exponentially decaying function as you can see, I have not plotted the minus 0 part because this is trivially going to be equal to 0. So, given if somebody says they give you this, then what do you do?

Then what you can do is in math, you can compute, so in MATLAB so you are free to use any computing language. But most of the problems if whenever I work out I will try to work out in MATLAB. So, in MATLAB Ht is represented by the function Heaviside. And Heaviside takes a scalar input. And this will compute for you a given function.

So, if you feed in a matrix of values, let us say you create a t which is a vector, which spans from 0 in this case you have a vector which spans from 0 to 10 and there are, you provide a spacing and then you create a vector t, which is all the values of that t is going to take place. And then when you feed this to Heaviside you need not just put in a scale and it can also take vector here. So, if you feed in that then it will the output will also be a vector which will be the vector of all values for that particular time period.

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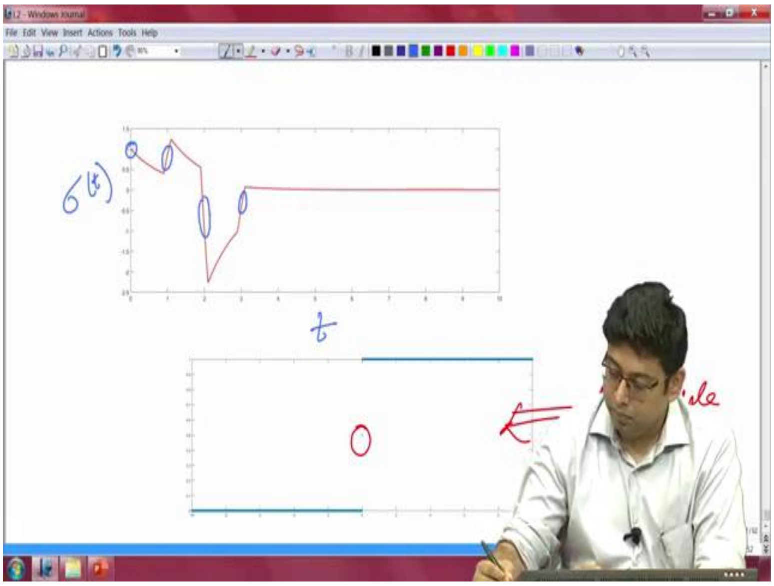

So, if you do this, then what you will end up with and if you write those equations you end up with a curve like this. So, this curve takes the value of 1 and then it decreases and then at t equal to 1 there is a sudden increase then it goes down. And then finally it is decaying to 0 overtime. Because if your material itself is such that the Gt is going to decrease to 0, then your final solutions should also decrease to 0 overtime. So, that is a quick simple check that you can apply to your solution.

So, at t equal to 10, which is a very long time scale, if you, this is an exponentially decaying function at a long time, this will also decay to 0. But when you are doing something in numerical software, you are again as I said you are free to use any software you want, you have to be a little bit, a little bit careful, okay. Because there will be some issues and I will tell you one particular issue that I found out and it so happens that if you compute the Heaviside function in MATLAB.

So, this is your Heaviside function H of t, where t is 0 in this particular case. So, I just did the numerical evaluation of Heaviside function and then I plotted it and here time goes from minus 10 to plus 10. So, you can see that from minus 10 to 0, it should remain at a value of 0 which is what you will find and for values of t greater than 1 or greater than 0 it takes on a value of plus 1.

But, but there is a value at 0.5. So, the place where the next step where it is going to make a jump, it does, does not jump straight away to 1 where it takes this 1. So, if the vector is 1000 size long, there will be one vector in between where this jump is taking place and there the value actually goes to 0.5. But that is not as per the definition that we discussed. So, this is a numerical issue.

And when you apply this Heaviside function, what ends up happening is exactly at the locations of the jumps, you end up getting some problems. So, for example, here, you will not get this value here there is a small issue. So, these values you have to be mindful of that when you are doing a numerical evaluation it does not lead, it might or might not always lead to the correct result, which is why whenever you are doing some solution using a numerical scheme, it is important that you check and you apply simple sanity checks to your data.

Now, MATLAB also allows Symbolic Logic calculations and I am sure this entire thing can also be done in a Symbolic Logic form. Although I did not do it, but you are more than welcome to do that, there are other softwares also which allow for Symbolic Logic calculation and you can also use those although for this particular class I have not prepared such a solution okay. So, this completes our discussion on this particular problem.

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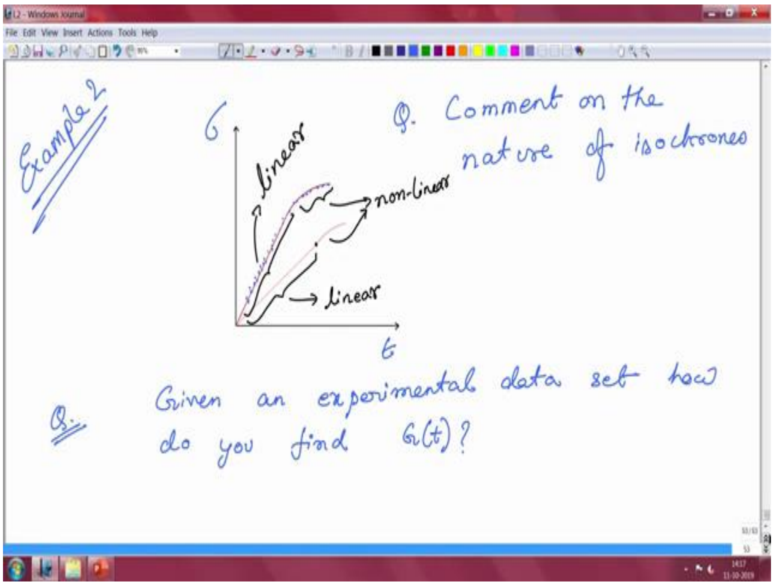

Now, there is another example and this is a rather simple one and okay, so what we have here is an isochrone and the question is comment on the nature of the isochrones. Maybe this is a creep isochron. I have not specified that this is a qualitative problem and the thing that I wanted to point out is that if you look at this graph, these are the two curves. What you see is that there is a part of it, for example, this particular part, this is not a, so this particular part, which is seems to be very nicely linear and then there is this part which is nonlinear.

Similarly, for the other isochrones this part again till here approximately, it looks as if this is linear. And then this other part seems as if it is nonlinear this kind of transition from linear to non-linearity is often seen in experimental data. So, experimental data as I had said before, there is no formal rule, which requires a material to behave in a fashion that you want. There is no formal rule where the linear scaling issues that we had discussed material needs to behave that way.

It is our assumption that if the linear scaling applies, then certain simplifications are possible. And those simplifications allow you to do certain calculations rather easily. So, whether or not such simplifications are possible, or usually only can we evaluated after a successful experiment. And in this case, you have a creep isochrones. So, you look at it, and you make a judgment call on whether a particular portion looks linear or not. Obviously, when you are doing it in the lab, there will be data points that you will have.

So, this line is nobody is going to draw this line for you. So, they will be these data's, data points, and then you might have to do a piecewise fit to the data. So, just because you get the data, do not just take some function and try to fit it. You first make an evaluation whether there is a piecewise, linearity or not, and where the nonlinearities we want to break up and then you go ahead with that.

Okay. So, this is another example. A final other question, and this is related to let us say, this is not exactly an example problem, but it is a good question to ask and that when you are doing experimental data, how do you find Gt? Given an experimental data set how do you find, what do I mean here? What is, what do I mean by saying how do we find Gt?

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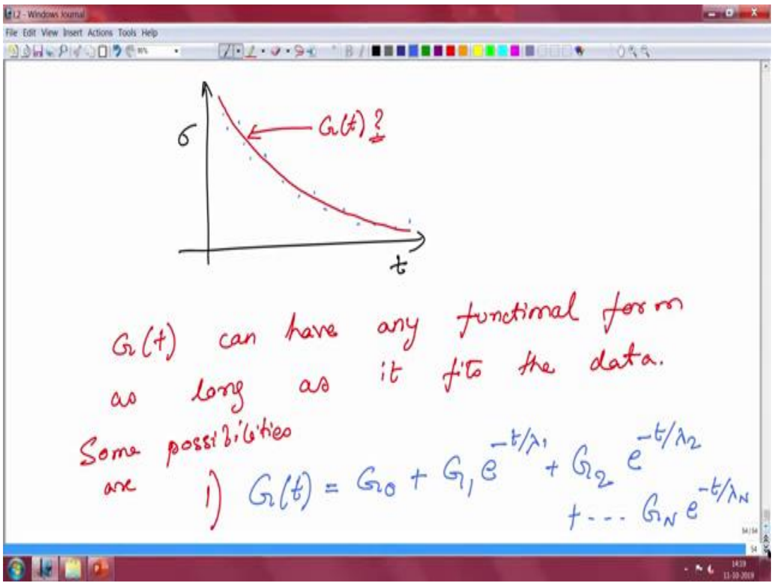

What I am trying to imply is? Let us say you are in the lab, you're doing some experiments. You are doing a creep relaxation experiment. And the data will be, these are the data points. So, how do you determine what is this? So, there will be this functional form Gt is not going to be provided to you. So, the question is how do you find from a given experimental data set what Gt is.

Now, Gt it turns out can have any functional form as long as it fits the data. Some examples are, some possibilities are and say some possibilities are Gt can be expressed maybe in the form of a series where each of this and there can be perhaps N terms in there. When we discuss the Maxwell's model will in particular focus on this e to the power minus t by lambda form will see that it comes out very naturally for Maxwell model. This is one possible functional form.

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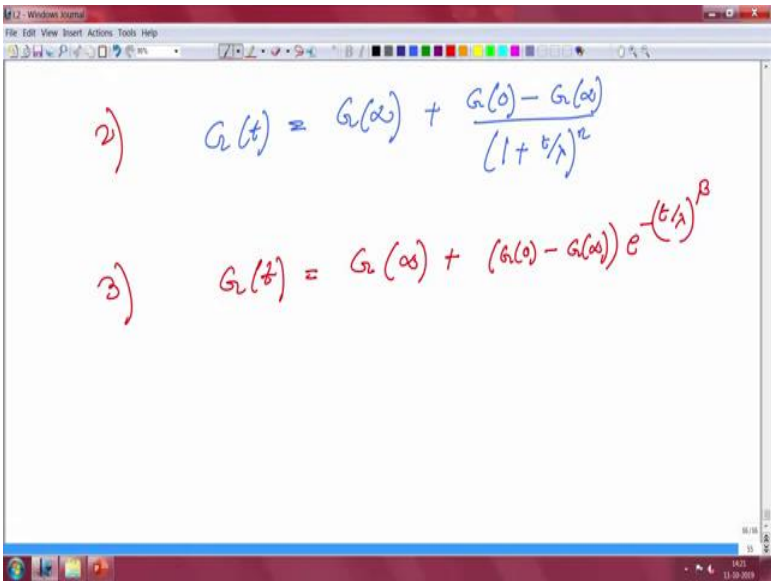

Another possible functional form is a rational polynomial form where we say Gt is equal to some G infinity plus G naught sorry. So, G0 means G at t equal to 0 and G infinity means G as t tends to infinity, 1 plus t by lambda to the power some n. And this lambda can be, this is a fitting parameter that you can determine.

Another possibility can be is, this is G infinity, this is G of 0 minus G of infinity this extra bracket is not required. This is e to the power minus t by lambda some alpha okay. Sorry, okay I think this alpha looks almost like infinity I will just replace this with another constant beta. So, these are all different fitting constants that you have to evaluate from the data and that will give you an idea of what, so, these are the, so, these are not the only ones these are some of the commonly used ones, okay.

So, to summarize what we are going to do now, so, we looked, we had started off with the continuum mechanics aspect of viscoelasticity. And what we did is we looked at the historical aspect in the beginning where we looked at some ancient timescales natural timescales, that people had identified and how that builds in very nicely with some of the old experiments special specifically the 1927 pitch drop experiment and then we also looked at the definition of Deborah number as given by Rainer in his, I believe 1960 somewhere around 1960s paper.

Then we started looking at this term called viscoelasticity, okay. And we saw that there are different tests, one dimensional tests by the way, those are all 1D tests, by which you can evaluate the material response. And we particularly looked at two important responses, which is the creep and the stress relaxation phenomena. Once we did that, then we looked at material linearity, which we discussed was a combination of two aspects which is one being the linear scaling, and other being a superposition of responses.

And the two put together makes a material linear and we hope the material behaves that way because there is nothing that may that is requiring the material to behave in a nice and simple linear fashion. And then we also discussed aging materials. And finally, we discuss some mechanical analogs and some sample problems.

And in the next class we will just start off with a totally different way of looking at viscoelasticity or such kind of material, soft materials, which is the atomic view point, okay. So, with this we will end today’s class.