Convective Heat Transfer

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Lecture - 09
Similarity Solution- Energy

After doing the similarity, now for the momentum and we have shown that how the profiles should actually vary. Now let us take the energy equation now.

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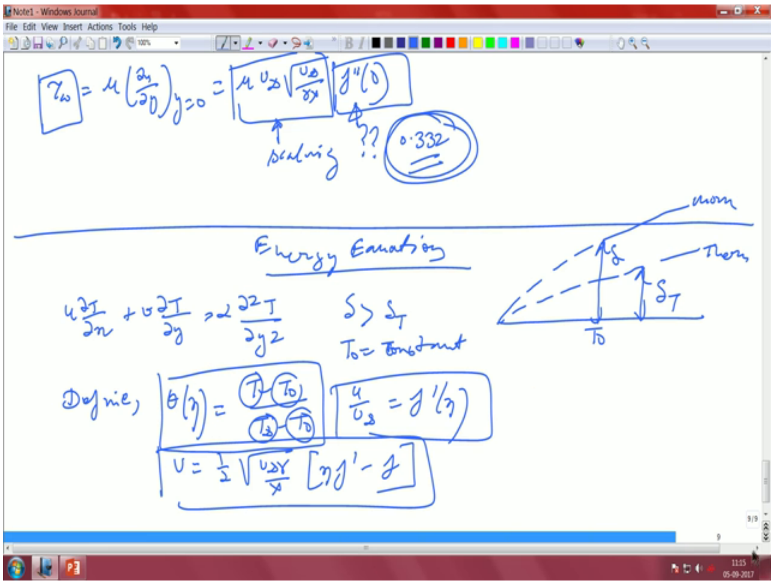

Again, the boundary layer version, see here let us take the case where delta is greater than delta T and T naught is equal to constant T naught is a wall temperature. This is how the profiles are developing the 2 profiles. This is the momentum, this is the thermal. This is delta, this is delta T. That is the premise.

Now, we define the rest of the definitions are already done. The new kid in the block is basically your temperature. That we take as T minus T naught divided by T infinity minus T naught and we already know that u by u infinity is equal to f prime eta and v is equal to half u prime gamma by x into n f prime minus f this we already know. That is v this is u this is your corresponding eta. These are the 3 principal variables that we had earlier also.

Now, let us look at let us go through the motions once again. This temperature this non-dimensional temperature is nothing but the local temperature minus, whatever is a wall temperature, this is the free stream temperature this T infinity and this is the corresponding wall temperature. This is the maximum temperature that is there. The theta will be bounded it should be lower than one. On the other hand, and this is a constant wall temperature case; that means, the substrate is basically not changing it is temperature along the length. Let us now based on this.

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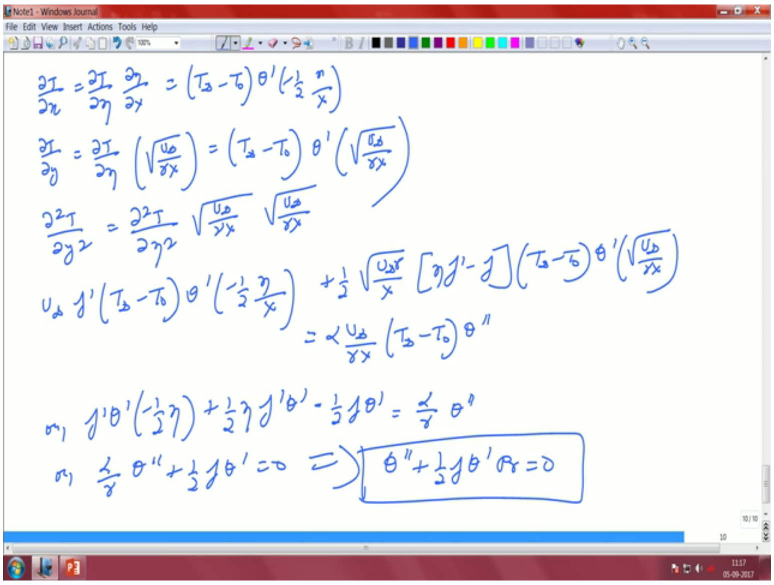

Let us do the next set; that means, let us establish the rules once again. Dt by dx dt eta by dx is equal to T infinity minus T naught into theta prime minus half eta by x. Dt by dy these are once again simple math now we put all these things together in the in the main energy equation. That will give u infinity f prime.

It is advisable that if you practice the math once at home or whenever you are studying this course online. That you can get an idea that how to solve this equation. Make sure that there are no errors as such alpha by gamma eta double prime plus half f theta prime is equal to 0. This leads to the final form f theta prime into prandtl number is equal to 0. That is the expression that we get ultimately. After doing all these things. That will be the energy equation.

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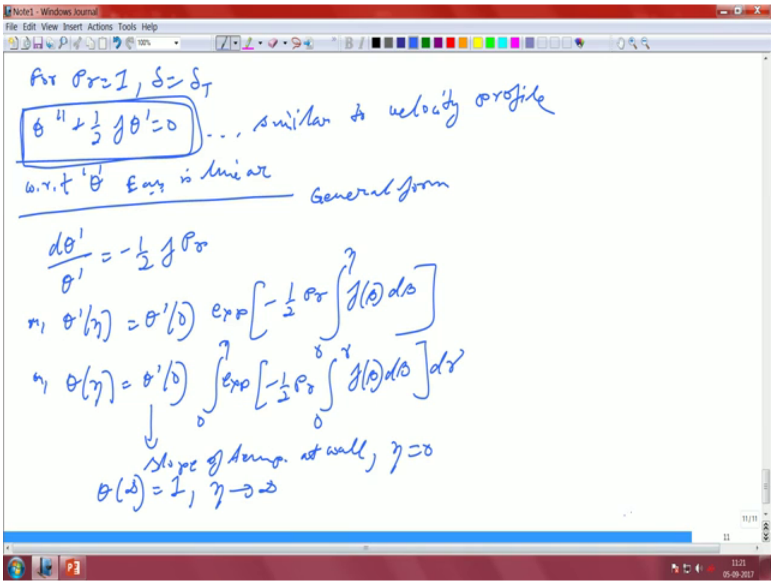

Now, for the case for prandtl number is equal to 1; that means, delta is equal to delta T. This equation will become theta double prime into half f theta prime is equal to 0. This is very similar to the velocity profile the velocity profile now with respect to with respect to theta this equation is linear. Unlike in the case of velocity with respect to f, it was not linear this is linear with respect to theta. Of course, the main caveat comes from the fact that you do not know f; that means, you still need to solve the momentum equation in order to make any guess.

That is why this f becomes very important which I kept aside and I did not show. This f.

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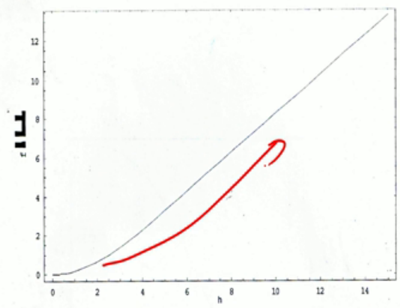

Fig1: $f(\eta)$ as a function of η

Therefore, becomes very important because this is a solution for this is the f and how it actually varies with theta now how it varies with eta. Based on this if you look at it. Here the f comes directly into the picture. You still need to know that what is a variation of f because that is the solution that you are going to use here and if you want to solve it in a very coupled fashion.

Still the difficulty you cannot get rid these 2 equations are coupled; that means, you need to solve the momentum to get an idea of the other energy equation. In this case also the problem remains, but this is also nod as you can see because theta is now a function of eta only and you still require the solution for f and whether at the generalized expression involves prandtl number? Of course, if prandtl number is equal to 1 this is the expression that you get those 2 standard expressions that you get.

Now, let us look at the generalized form. D theta prime by theta prime is equal to minus half f prandtl number. This is the most general form got it? Or theta prime eta theta 0 exponential half prandtl number 0 to eta f beta. That is the dummy variable or theta eta is equal to theta prime 0. These are the dummy variables that we are integrating it. This is basically the slope of temperature at wall eta is equal to 0. That is the slope at the wall because theta prime is once again the same thing it is dt by dy essentially. The slope evaluated at the wall that is required for what because it is required for the heat transfer coefficient.

Similarly, theta infinity is equal to 1 as eta progresses to infinity because that is the temp that is the limit at which the temperature approaches the freestream. That is the that is the 2 expressions. Now, if you work out let us move to the next.

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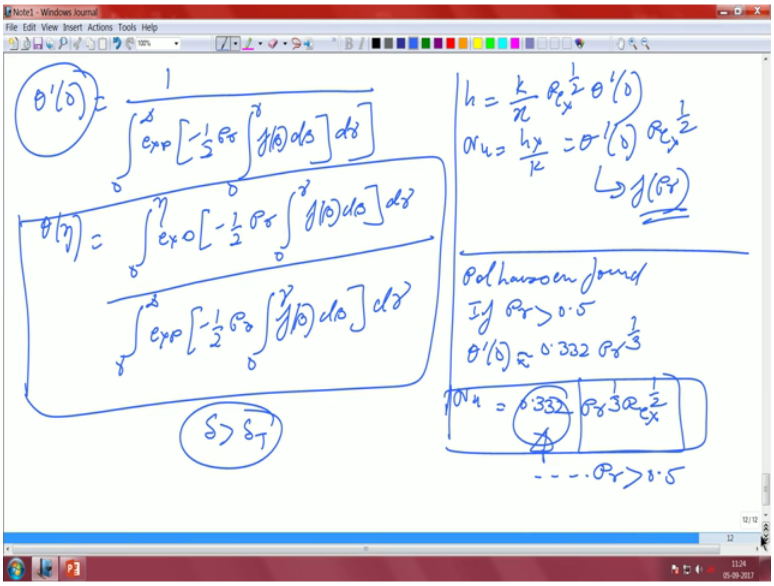

Theta 0 will be equal to 1 by 0 to infinity exponential half prandtl number 0 to gamma f beta d beta and theta eta will be equal to that is the generalized expression for theta.

These are the 2 expressions that you get. As we know that h from our scaling argument we already knew that h is nothing but 2 x Reynolds number to the power of half. If we cast it in terms of x theta into 0 theta prime equal to 0. Similarly, your Nusselt number was h x by k which is equal to this is the local Nusselt number Reynolds number to the power of half. This part is a function of your prandtl number this is what we said. There is no clear-cut way of solving this.

But there are approximate solutions that are given for example, pohlhausen found that if prandtl number is greater than point 5 he actually solved it. Theta 0 is actually equal to 0.332 into prandtl number 1 third. Therefore, the Nusselt number expression then becomes 0.332 prandtl number 1third Reynolds number to the power of half. This is the same form that we got. This part we got it through scaling already this is the form that we got because of that because of the slope.

This is valid pohlhausens method was valid for prandtl number greater than 0.5. That is true because we say that our delta is greater than delta T, all these things that we did. This increase correctly with our scaling argument as well. There is no problem with that. Now on the other hand another scope.

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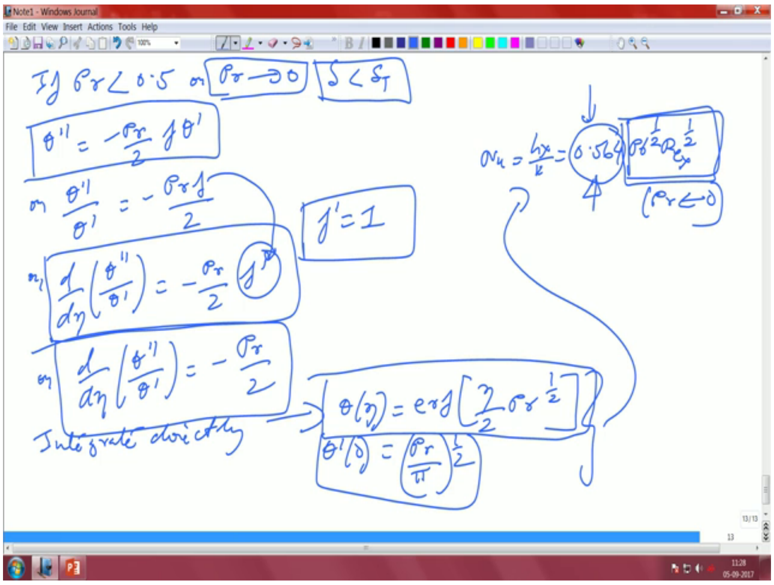

That if prandtl number is less than 0.5 or the cases where prandtl number approaches 0. We already know that delta is become smaller than delta T. Correct becomes smaller than delta T.

Your expression becomes this or in other words theta double prime by theta prime becomes equal to prandtl number into f by 2 or in other words d by d eta theta double prime by theta prime minus prandtl number by 2 into f prime. Here, the interesting part that we get is that. We have basically differentiated this guy with respect to eta here we can see this f prime is equal to 1 because of the thinness of the velocity boundary layer for all practical purposes your f prime is equal to 1 correct.

Because of that reason this becomes d by d eta theta double prime by theta prime is equal to minus prandtl number by 2. You can integrate directly. In other words, you get your theta and eta becomes equal to an error function eta by 2 prandtl number to the power of half. This makes you evaluate theta as 0 is equal to prandtl number pi to the power of half.

Based on these 2 expressions now, you can get your Nusselt number will be equal to which is basically hx by k is given by 0.564 prandtl number to the power of half Reynolds number to the power of half as prandtl number approaches 0. This also agrees with our scaling argument this part agrees with our scaling argument. What part is that we have plugged in? Is this 0.564.

Let us recapitulate that recall the steps where pandtl number is less than 0.5 or prandtl number approaches 0. Your delta is much less than delta T because, delta is much much less than delta T it means that it is a thin boundary layer. What we have done is that we have taken the same expression we have just differentiated it with respect to eta as soon as you differentiate with respect to eta, f becomes now f prime correct. As the f becomes f prime now we know that f prime is equal to 1 because for all practical purposes we are outside the velocity boundary layer.

Because the velocity boundary layer is very thin. In other words, it offers that you can integrate this expression directly because, previously we had f. There was a requirement that you needed to know what f is you can integrate here directly as you integrate you get an error function for theta and if you evaluate the slope.

Because theta prime 0 is nothing but the slope which is required for evaluating h or Nusselt number that becomes prandtl number by pi to the power of half. Once again, this particular parameter becomes 0.564 whereas, we retain the functional form as prandtl number half Reynolds number to the power of half. This is perfect that makes sense and that is the kind of obvious thing that we get out of this exercise.

We have divided the problem into 2 once again in the same way that we did the scaling. Greater the thermal boundary layer greater than velocity boundary layer or less than the velocity boundary layer. In both ways one was given by pohl housen where he found that it obeys with our scaling argument the other one can be more integrated easily and we found that that is given by the error function and we have evaluated and found that our scaling argument is exact.

This offers us now, some people what they did was after this like for example, churchill correlation.

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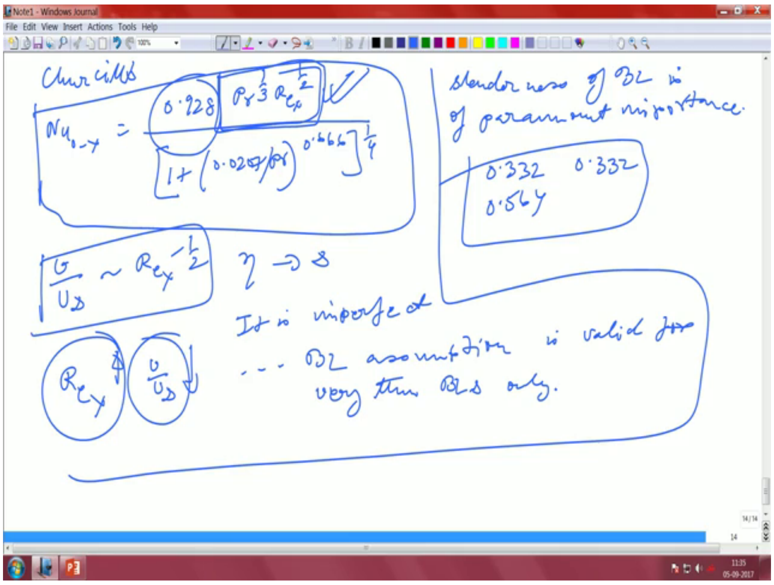

They provided 1 composite correlations for the whole thing. Of course, these correlations are only useful for engineers who want to you know quickly find something. 1 plus that is called churchill correlation churchill correlation. There is like a composite correlation.

That is valid across a large scale of prandtl number and Reynolds number. This this is basically data fits complicated data fits. While retaining the you know the variation of this because that comes from a functional analysis. The rest are all kind of you put together all the data and you maintain this variation and then you do a curve fit and find out the leading order coefficient. That is what has been done before we wrap up this similarity transformation let us put a small discussion. What we said that we will do see the nature of this v; that means, the vertical velocity or the y component of the velocity. V by u infinity scales as Reynolds number to the power of minus half. As eta goes to infinity you can see that this becomes equal to this.

Now, you can see that this is imperfect because of the simple reason that it is never 0. Because, whatever is the value of Reynolds number it may be a small quantity, but it existsm it is imperfect. Imperfect to begin with, but; however, as we can see when Reynolds number goes up v by u infinity comes down. Very easy to see from this expression. This particular boundary layer assumption, what we said earlier is valid for very thin bills only.

Because you want this v by u infinity factor to be very small because your vertical component of the it has to disappear. Otherwise you have a problem that only can become small when your Reynolds number becomes very high and as we said earlier that your Reynolds number becoming very high implies that your delta is very small that we already said earlier.

More slender the boundary layer is more perfect the boundary layer solution becomes. That is a very important point that you should try to mention maintain in your head. That slenderness of boundary layer is of paramount importance and this becomes a solution becomes more and more perfect and more and more exact. As you actually move to a very high Reynolds number flow higher; that means, if you have a very low Reynolds number these solutions will be kind of imperfect.

We have covered a lot of materials in which we have shown very clearly that we are in a situation that we have been able to able to cast. That using similarity variables the similarity transform was correct and we are able to show that both the heat transfer and the momentum boundary layers can be exactly evaluated and we have put forward those terms like 0.32, 0.56 for these are the prefix coefficients that sits in front and somehow, we have been able to say that what will be the values of this heat transfer coefficient. Using the methodology as has been detailed out in this particular case.

However, there is 1 problem, the problem lies that in these cases you have been able to find out the similarity transformation. Is there a better way of you know basically doing these problems? Which is somewhere in between a scaling analysis and the full exact solution of course, you can do a full-fledged numerical simulation of the of the flat plate boundary layer and you can find out that how the flat plate boundary layer actually the exact values of this without resorting to the boundary layer assumption, but that would be like that would carry no sense. Because, you would not be able to get these physical arguments coming out of it.

But in complicated problem say for example, if you want to do it over a very complicated geometry you have to resort to either experiments or numerical simulations. Your scaling can give you some idea, but it cannot be used for the full-fledged understanding of the whole thing. But; however, is there a middle ground by which we need not solve all these things because all we needed to do if I put it correctly was to find out the values of 2 constants. One of those constant was 0.332.

The other constant was 0.564 correct actually 3 because to 0.332 1 was 0.564. These are the 3 constants that we found out after doing all this exercise because the front portion was already known this part was already known. All we need found out was that something like that. Is there I mean people can argue that whether it is I mean advisable to do all these things? Just to get those values of the constant or is there a better and quicker way without going through all this math, without going through all these integrations. Is there a better way by which we can get a constant value which is close to these values? may not be exactly those values and we can live with it.

Say for example, instead of 0.332 if the constant value that we determined by some other method we do not know what that method is, but if say by some other method we are able to determine the value of that constant to be say 0.6, 0.36 only it will be off by a factor of say 8 percent 10 percent if you can live with that; that means, it depends also on your application. In many of the application 10 percent error is kind of its. If your 10 percent error is acceptable for your application.

Once again, the catch is that what it what your application is actually doing? What is the sensitivity? What kind of accuracy do you want? But in most of the cases if you want decent accuracy without going through all this motion of you know evaluating complicated integrals using shooting scheme to solve the equations. Is there a better way? By which you can use some of these some approximate techniques not necessarily numerical techniques. Some approximate techniques by which you can find out that what will be the values of this constant and maybe incur an error of something like 10 percent. If that is acceptable, then there are several ways out.

Now, you might say flat plate looks I can still bear with this much amount of math because the math is honestly speaking not very complicated to begin with. It is just a lot of transformations, but you are missing then you would be missing the point because the main idea of introducing the scaling and this canonical problems was not that this is something that you are going to do in your research or in your industry or wherever you are. This is just to give you that whether you could use this methodology for analyzing the problem which may be far more complex than this. If the problem is very complex like for example, as I say it flow over a turbine blade for example, they are doing a similarity transformation and solving the whole thing maybe a mathematically a pretty arduous task. It can be very complicated.

The scaling can still be done, but scaling would not give you the exact answer. You can be off by approximately 1 order. All within our order you would be correct. It is good for knowing the functional variation those things are correct. But the map can be pretty onerous if you actually want to do the whole thing using a similarity transformation because you it may might a very, very complicated task. Can you devise some other way by which you reduce the portion of your math and suggest some approximate techniques which perhaps does not require that much this much complexity?

Because all we need to do is find out the values of 3 constants. Because you are not really concerned about the whole temperature profile or the whole velocity profile you are only interested in the slope of that particular profile at a point. So, if you do not care that how do my profile look like and if I see that all these profiles looks kind of parabolic in nature. If I can somehow incorporate the y variation as a parabolic curve or some other polynomial curve and then find out the value of that constant or the slope at that particular point and if those values kind of fall within 5 to 10 percent error bar, your job is done. That is what we are going to do next.

We are going to suggest an approximate methodology which is basically called the integral solution. Integral solution is strictly approximate it is not a solution that that is exactly like the similarity well similarity is also not fully exact, but within the boundary layer assumptions. This banks on the fact that basically your variation of your u temperature with y. If you approximate it by a curve without doing the actual math without knowing the exact variation.

If you can approximate it with a curve fit of some sort. Which validates still the boundary conditions then perhaps you can have a better way of you know quickly finding out what is the value of that slope going to be and if that value of the slope kind of falls very close to your actual exact simulation. Maybe it is all worth your time to invest in that particular direction.

That is what we are going to do in the next class. We are going to find out that by using what we call the pohlhausen von karman integral method. How we can actually approximate the u and the temperature profiles. Using a some kind of an integration technique and then try to see that how this constants this theta prime theta double prime and f double prime. How do they look like; and whether they kind of fall close to the exact solution that we did. See you next class.