Convective Heat Transfer
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Lecture – 07
Scaling Analysis – Energy II

Last class we did that, when the thermal boundary layer was thicker than the momentum boundary layer and that was a case when we discussed and showed that, how the Nusselt number can be written as Reynolds number to the power of half and Prandtl number to the power of half.

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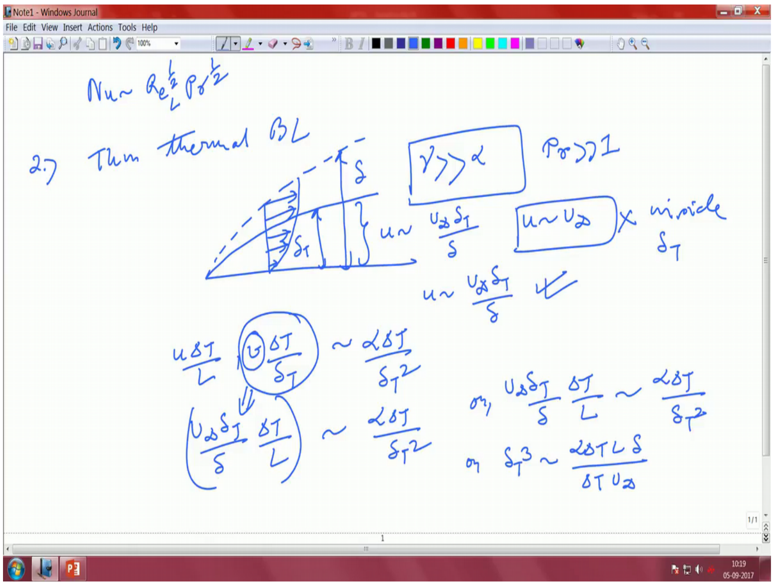

So, that we did. Now, this particular lecture, we will cover with we have thin thermal boundary layer. Now, thin thermal boundary layer means, if you look at this particular drawing, so, this will be your thermal boundary layer. This will be your momentum boundary layer, right.

So, in this particular case, obviously, the kinematic viscosity is much larger than the thermal diffusivity. So, this is the case or in other words, the Prandtl number should be greater than 1, right. So, if you see this particular situation, you will realize that previously, we did Prandtl number much less than 1. This is Prandtl number much greater than 1.

So, the thermal boundary layer is very restrictive; that means, it is a very slender thermal boundary layer. Now, how to analyze a problem like this? Remember our solution now will pertain to the thermal boundary layer only, right? It will not be pertaining to the momentum boundary layer; that means, we have to do whatever we do inside this boundary layer essentially. That is the thermal boundary layer, right.

So, what will be the velocity scale here? So, if the velocity scale u, normally it is U infinity correct, in the momentum boundary layer. This time, let us apply a scale. What we have done is that, basically is like a linear interpolation of u infinity; that means, we have we have divided u infinity by delta a multiplied it by delta T. So, this is a scale factor. Because, inside this boundary layer, if you look at it carefully, here, because, the boundary layer is very thin inside it, the velocity scale cannot be U infinity.

So, u cannot be equal to U infinity. This is a wrong statement inside delta T, right. But, u proportional to U infinity into delta T by delta is the correct scaling because, we have just done a numerical interpolation; that means, we have divided U infinity by delta multiplied by delta T, right. So, that is what we have done to get a scaling factor for this. Now, if we go back to your thermal energy equation, what do you have? These are the terms, right? This particular set of equation remains the same. Now, let us take the first term of this particular series U infinity delta T by delta into delta T by L.

So, that is the first term. By default, the second term will be the same; that is because, now, v will no longer be equal to 0, as we saw in the case of a thick thermal boundary layer. Because, in the thick bound thermal boundary layer, we were outside the momentum boundary layer predominantly for most part of the analysis and because of that, v actually has to be equal to 0. Because outside the momentum boundary layer v is should not have any existence.

So, that is what we showed. But, now here, we are within the thermal boundary layer. So, because we are within the thermal boundary layer, this particular term also should be of the same order as this, from the continuity. So, this is basically proportional to delta T square, got it? So, or in other words, we can say U infinity delta T by delta, delta T by L proportional to alpha delta T square or delta T cube will be proportional to alpha delta T by L delta divided by delta T into U infinity got it?

So, that will be the nature of the equation. So, let us go to the next page and see what does this mean or in other words, delta T cube will be equal to alpha into L into delta divided by U infinity which leads basically to alpha L by U infinity into L into Reynolds number to the power of minus half.

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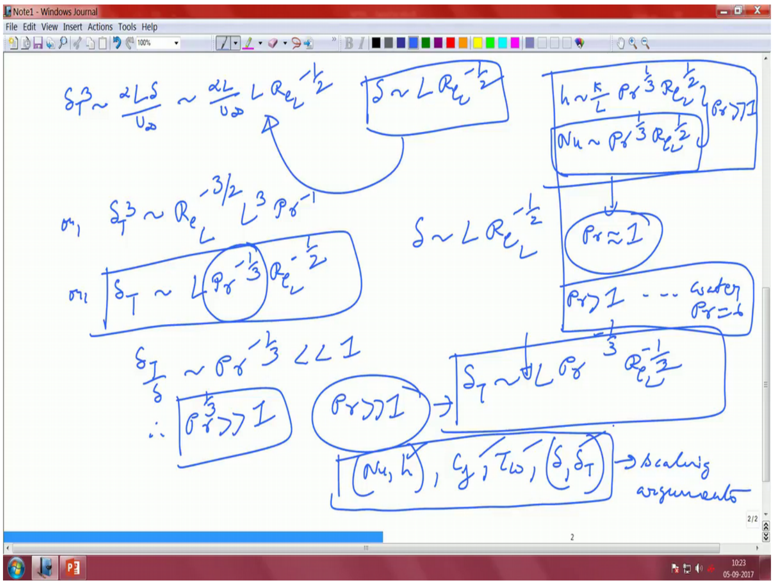

We are what we have done we are substituted delta is, Reynolds number to the power of minus half.

This, we already know from the momentum equation. They are back substituting it here, alright. Or in other words, delta T cube will be equal to Reynolds number to the power of 3 by 2 L cube Prandtl number to the power of minus 1 or delta T will be proportional to L Prandtl number to the power of minus 1 3rd Reynolds number to the power of minus half. So, that is the expression for delta T again, very similar to the expression of delta.

Recall once again, delta was always the same here. Previously, the Prandtl number dependence was minus half. This time, it is minus 1 3rd. Now, delta T by delta is proportional to Prandtl number to the power of minus 1 3rd, right. Just by dividing 1 by the other, this is much less than 1 implying that Prandtl number is much greater than 1 or Prandtl number to the power of 1 3rd is much greater than 1.

So, for Prandtl number much greater than 1, delta T or the thermal boundary layer thickness scales as only for this. Now, once again, this factor as we said earlier, this is a scaling argument. So, that factor that sits in front of it is still basically unknown. For that, we require a proper full-fledged solution of the entire equation. So, similarly, as we know, the heat transfer coefficient was our primary goal of over here, right.

So, that will be given by K by L Prandtl number to the power of 1 3rd Reynolds number to the power of half or Nusselt number can be written as Prandtl number to the power of 1 3rd Reynolds number to the power of half. Both cases, Prandtl number is much greater than 1. Usually, it is seen that, this particular form of the correlation is valid till about Prandtl number equal to 1. So, it is almost valid and Prandtl number greater than 1 normal fluids will be for example, water this is got a Prandtl number of around 6 air has got a Prandtl number close to 1. It is about 0.7 actually, ok.

So, as you can see, most of the common fluids that you will encounter will fall under the Prandtl number greater than 1 category. As I said liquid metals and other things will fall under Prandtl number much less than 1 category like oils and liquid metals. So, using this scaling argument, now, we have got 4 major quantities right, one is your Nusselt number and h, this combination alright.

One is basically your skin friction coefficient C f, the tau wall shear stress. That was one other interesting parameter that we found out and an estimate of delta and delta t this combination right. So, these are the 4 major things that we have unearths out of these analysis using nothing but scaling arguments using nothing but scaling arguments who are able to isolate the effects of these 4 quantities right here.

So, that makes it an interesting problem that now we know that how for example, the flow the shear stress the Nusselt number all these things varies as a function of Reynolds number and Prandtl number right. So, as I said earlier that there is a flow dependence because this is convective heat transfer there is a flow dependence of on everything. So, here the flow dependence comes through the Reynolds number right the property dependence comes through the Prandtl number correct. So, these are the 2 major parameters that you need to know in order to identify what will be your heat transfer coefficient right ok.

But even then so, this gives you a nice back up the envelope calculation; that means, once your Reynolds number increases your Prandtl number should your Nusselt number should increase I mean those kind of things you can infer from this relation right, but if somebody asked you for an exact value of your Nusselt number, for that you need to pursue this the more formal quantitative analysis which we are going to do next, but the argument from scaling comes from the fact that there are several things that we found that delta by L. As we found, if you look at this particular relationship is Reynolds number to the power of minus half, right? That is the expression that you already saw

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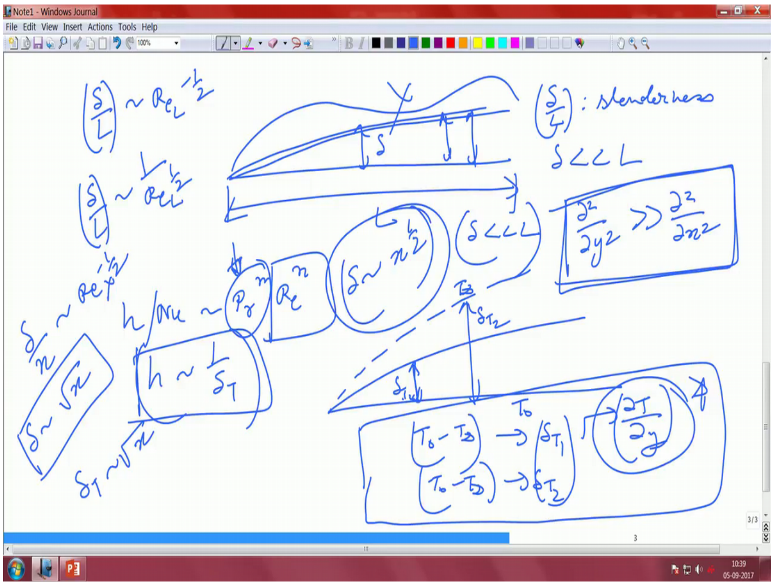

So, if I try to plot this particular thing that not plot, but basically see the dependence right Reynolds number to the power of minus half. So, that is the relationship that we have initially right. So, this as I say, it is nothing but the slenderness ratio so; that means, if this is the boundary layer that is growing right, this is delta this is L right L or x right..

So, delta by L is essentially like a; slenderness right; obviously, we know that delta is much less than L right. So, it is like a slenderness ratio correct so; that means, how slender is this boundary layer compared to the overall dimension of the plate right now as you can see, so, delta by L less then Reynolds number to the power of half right. So, as Reynolds number goes up quite a bit.

The ratio between delta and the delta by l actually becomes a very small number right you can just plug in some values right put Reynolds number equal to 10 thousand put Reynolds number equal to 10 to the power of 6. As you see, as we progressively increase the Reynolds number, what will happen is that, this delta by L ratio becomes smaller and smaller or in other words what it means is that the boundary layer the slenderness ratio actually increases in a way right; that means, delta becomes much smaller than L progressively right.

So; that means, when you actually have a large Reynolds number flow your boundary layer thickness is very small compared to the overall dimension of your plate correct that is an important argument over here because, that that we will see is of paramount importance when you apply the boundary layer equations. Because, when we can apply the boundary layer equation, recall the boundary layer equations. We are able to apply when we say it that this is much greater than that right. When we dealt with the viscous terms correct that is because, we said that the slope in the y direction that is a transverse direction has got a very high value compared to the variation in the x direction; it is only possible when delta is a very small number.

So, ideally this solution becomes more and more accurate as your delta goes down compared to L right. So, that is a fundamental importance over here if you are dealing with a very low Reynolds number flow in which that delta becomes comparable with L you strictly cannot apply this boundary layer equations. So, it becomes progressively more and more approaches the more and more you know exact limit as your delta progressively becomes small got it. So, that is of high importance now.

Also, let us see that we have said let us look at the physical insights that are coming out of this. So, delta T, we said that it is basically Prandtl number to the power of some m if I have to write it properly right, not delta T. Let us put Nusselt number right or the heat transfer coefficient whatever you call it right it is some Prandtl number to the power n Reynolds number to the power n right.

Now, as we increase the Reynolds number right and we also said that h is proportional to L over delta T. Do you recall that particular argument that h is always proportional to L over delta T right? So, that means, as we increase the Reynolds number right as we increase the Reynolds number, what happens to your delta T? The delta T goes down right? The delta T goes down, the thermal boundary layer goes down. You just look at the expression that I wrote in the previous slide.

You look at this any one of these expressions you can see that delta t right as we increase the Reynolds number goes down right it goes down because this is inverse dependence right. So, it goes down. So, that would mean that your boundary layer becomes very small very slender and since h is inversely related to delta t h actually shoots up does it make sense yes it does that is because if your thermal boundary layer thickness is small the slope; obviously, will be larger..

Let's take these 2 examples is one thermal boundary layer this is say another thermal boundary layer let us call this delta 1 T 1. Let us call this delta T 2 2 situations alright. Now, the temperature from t naught to T infinity that remains the same. So, basically you are having a variation from T naught minus T infinity or T infinity minus T naught over a distance of delta T 1 the same variation you are now having over a distance of delta T 2 right.

So, naturally, if you from common sense you can see the d t by d y type of term alright which determines what is the slope right, at the surface of the plate. So, this will be naturally, whichever one has got a lower delta t that slope will be higher right. The profile will show a sharper slope because; it has to change within a shorter distance.

So, the slope has to be higher, right delta T by delta y. So, it has to have a higher slope right. So, because it has a higher slope; that means, the slenderness of the boundary layer actually leads to a higher slope in the temperature right. Since it leads to a higher slope in the temperature right, the h the heat transfer coefficient has to be more high, alright? That is straightforward, reason, right? Because the original definition of heat transfer coefficient if you recall it was K dT by d y divided by delta T right.

So, the same delta T is varying. Now, over 2 distances and if delta T 1 is it is less than delta T 2; obviously, the slope corresponding to delta T 1 will be more. So, naturally, if the slope is more h is high; that means, when we increase the Reynolds number, we decrease delta T, as we decrease delta T we increase the slope at the wall, which leads to an increase in h or the heat transfer coefficient and the Nusselt number. So, it makes perfect sense.

So, whatever scaling that we did actually make sense, right. That at high Reynolds number flow the boundary layer is thin because it is thin the thermal gradient is high. Because the thermal gradient is high at the wall, the h or the Nusselt number is more, progressively as we increase the Reynolds number. But remember the dependence is not a linear dependence it is actually a root over dependence right. So, it is not a linear dependence that well if I increase in Reynolds number by 100, that will change the heat transfer coefficient by a same amount, it is not like that.

So, it has got that root over dependence. Similarly, when you look at the Prandtl number situation; that means, the Prandtl number either it is half or one third, in any case it is a positive dependence on the Prandtl number right. So, if the Prandtl number goes up, we can see readily that there is an increase in the heat transfer coefficient, right. In 2 different ways obviously; that means, depending on which boundary layer, which factor is more important; that means, the Prandtl number is greater than 1 or less than 1 we have a dependence in a certain way on the Prandtl number, right.

So, that is the property part that is where the property part comes out comes into the picture right. So, in this way, we now have a heuristic understanding, right? That from the scaling argument all these things makes perfect sense. Boundary layer has to be thin, the thermal boundary layer can be thicker it can be thinner, but essentially there is a scaling between the thermal boundary layer and the momentum boundary layer usually happens through the Prandtl number value, right? And we have seen that y where the boundary layer assumption should hold; that means, the delta by L has to be a very small number. And that is automatically satisfied, as we increase the Reynolds number more and more right, the slenderness part and we have also seen that, how this will actually slenderness will actually lead to an increase in the heat wall heat flux.

So, based on this, we finish the scaling argument for this external flow part right. So, some points to recap for the external flow part is that across momentum and thermal boundary layer, there is for a flat plate at least there is no pressure gradient within the within the momentum boundary layer right..

Let us get this. Because, the flow outside is oil arian and we argued that that flow field or that pressure field is actually imposed inside the boundary layer.

Because, the flow outside does not have any variation with respect to x or y it is basically uniform flow field. There is no pressure gradient that exists, within the boundary there. That makes our job a whole lot simpler right? V that is, the velocity in the y direction within the boundary layer is not 0. Contrary to some, you know, some misconceptions that you might have it is not 0. It is a small number definitely, but it is not 0. And that we have established again and again throughout the course of these scaling arguments. That it is not a 0 number, right.

So, never be under the under the misconception that v within the boundary layer is a very small; I mean it is 0. It is a small quantity no doubt about it. But v velocity outside the boundary layer should be equal to 0, right? That is how we analyzed the thick thermal boundary layer problem, got it? Now, since now we have established that, this is the nature of v. So, that is how we should vary, inside and outside the thermal boundary layer. We will come we will show that the boundary layer equations honestly speaking does not satisfy this criteria that v is equal to 0 outside the boundary layer.

So, that is a problem which you will address in the next few lectures, but v for all practical purposes outside the boundary layer should be equal to 0, right. And this delta is basically an arbitrarily fixed quantity; ideally delta value should be equal to infinity, because the boundary layer normally should stretch right up to infinity because if you are looking at how the u velocity goes to the free stream value is an asymptotic variation. So, it should actually be become equal to U infinity at you know y equal to infinity essentially, right.

But that is not the, I mean for all practical purposes we cut it off at say 99 percent of U infinity that we take as delta you can take it as 0.999 you can increase the number of decimals as you want. For most engineering purposes 99.999 percent is good enough or 99 percent is good enough right for 0.9 is good enough of. So, these are some of the takeaway points from the scaling argument.

So, remember when you do a scaling argument, we have done an order of magnitude analysis, we have not made a concrete you know what we call a full-fledged solution of the whole problem; that means, we are still missing certain things, but we are able to capture the essential physics of the problem that what is the flow as we started that h should depend on flow convective heat transfer is a different problem it does not depend on the properties you cannot have a farm now you can see why we cannot have a firm number for each, right or C f skin friction coefficient right.

Because, these are the 2 things we say that the engineers want, right, the drag and the Nusselt number or the heat transfer coefficient. You cannot have it because you need to know what is your Reynolds number your Prandtl number this kind of situations right.

So, there is no one no fixed answer to this particular question. Even if you specify the system for us and of course, here we have taken that the flow is fully laminar there is no separation. So, we have not considered any separation of the flow. So, that will complicate issues a little bit more. Remember, the boundary layer assumption holds away from the leading edge of the plate right. So, we are at a certain reasonable distance away from the leading edge.

So, that we can you know discount for the edge effect also. Remember, one other thing this is an established solution; that means, it is a steady kind of a solution over here right. So, initially when the flow encounters the plate there will be a lot of other flow complications which you are not covering it over here, before the flow stabilizes to a steady state configuration. So, the flow will this is after the flow has kind of you know attained some kind of a steady state that is what we have d1 over here another thing to note if you look at the thing before we go to the next one that delta.

Obviously scales as x to the power of half; that means, your boundary layer thickness increases as you march along the plate length right. And it marches as x to the power of half. So, that is why whatever we drew earlier that this kind of a profile right. And not this kind of a profile, that is wrong. We have drawn this kind of a profile that is mainly because of this. How do you establish this because delta by x is proportional to Reynolds number x to the power of minus half, if you now open up the terms you will get that this will be root over delta x.

So, that is what you would get. So, this is very important that shows that is boundary layer monotonically increases as we increase the as we march along the plate length. So, this is another important parameter, and same thing happens for the thermal boundary layer also right, it scales as Reynolds number to the power of x to the power of half. So, in both cases the thermal boundary layer and the momentum boundary layer grows with increasing distance right; that means, the gradient becomes shallower and shallower in a way. It becomes shallower and shallower as we march on right with respect to x, got it?

So, we finish this lecture over here, we have given a lot of insights into what happens, now next we will take up the portion in which we will try to see that if we now try to solve this equation in a proper way, and then do the results look in different. Or what we have done is it correct or not it made common sense; obviously, it explained a few things of course, it matches with experimental data as well this kind of variations like delta x and other things. But let us see that whether we can get a more concrete kind of a solution using some analytical tools as a matter of fact.

Thank you.