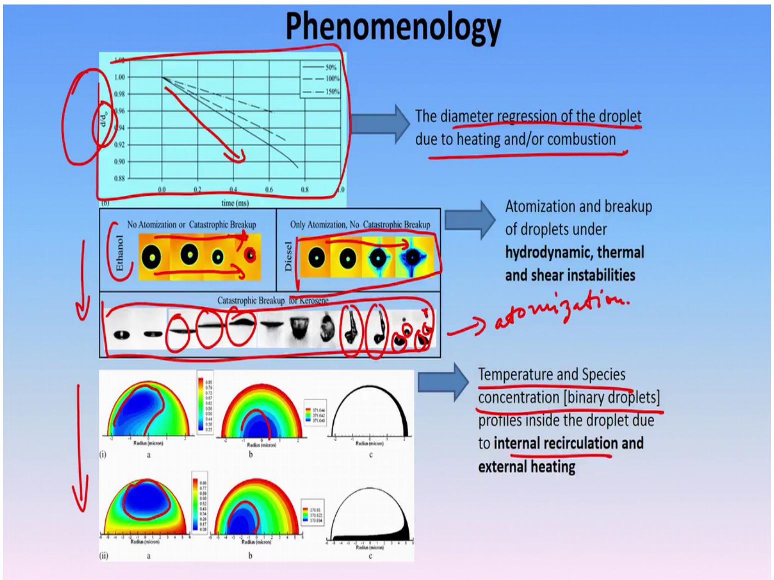
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 60
Droplet evaporation – Contact free

In the lecture before we looked at that these droplets, can actually what is the mode how does it actually evaporate.

(Refer Slide Time: 00:32)



Phenomenology

The diameter regression of the droplet due to heating and/or combustion

Atomization and breakup of droplets under **hydrodynamic, thermal and shear instabilities**

Temperature and Species concentration [binary droplets] profiles inside the droplet due to **internal recirculation and external heating**

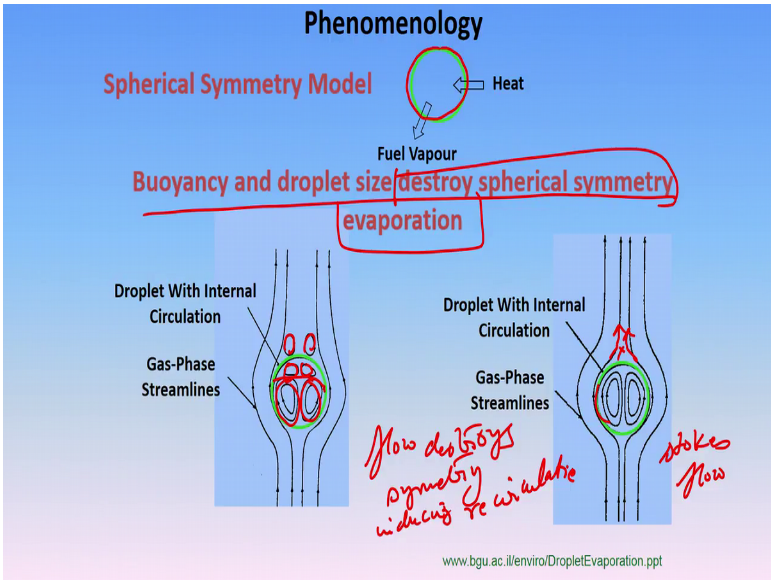
Now, let us look at some of the phenomenology of such a droplet system say for example if you look at this particular plot over here taken from the work of a stork at all it shows that there is a diameter regression of the droplet due to heating and or combustion. So, basically the droplet diameter shows a decay with time, ok. Similarly, the droplets which are in a flow field can actually show lot of shape change oscillations which we are not going to cover; this is called atomization behavior ok.

So; that means, the droplet is a liquid mass right. So, it is deformable. So, as it deforms it can form all kinds of strange structures and these structures might actually atomized in that particular way. These are different modes of atomization. These are called catastrophic breakup, and there can be situations also where it does not show any such thing and it just shrinks in size, ok, just shrinks in size ok. So, as you can see over here this is an ethanol droplet. So, it shows that there is no shrinking there is just a shrinking in size. So, the drop droplet just becomes smaller and smaller and smaller.

On the other hand, the temperature and the species concentration for binary droplets specially droplets which has got two components, ok, because of the internal recirculation they can show very complicated structures. As you can see over here, these are very complicated structures forget about this for the time being this is if you have particulate matter in it, but as a result of the flow you will get this very nice sophisticated patterns that will start to happen within the droplet.

So, it depends on the level of complexity, if you are interested only in this information this is very easy, ok. If you are interested more in the deformation mechanism that is a little bit more difficult if you are interested what goes on within the droplet specially if you are interested in what is happening at the droplet scale your job becomes much more difficult, ok.

(Refer Slide Time: 02:34)



Phenomenology

Spherical Symmetry Model

Buoyancy and droplet size destroy spherical symmetry evaporation

flow destroys symmetry inducing recirculation

stokes flow

www.bgu.ac.il/enviro/DropletEvaporation.ppt

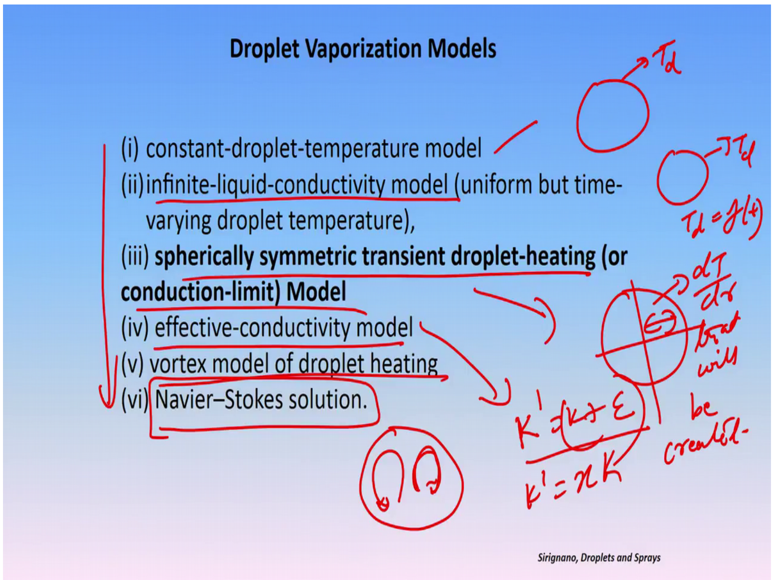
So, in order to attack each of these phenomenological events that actually happens you need a different level of sophistication at each stage, ok. Say for example, normally we assume that the droplet is spherically symmetric, this is what we told in just the previous lecture; when we say that the evaporation flux is the same everywhere. So, that droplet is basically what we call spherically symmetric, that means, everywhere you look around the droplet. So, if this is the droplet and if you go around the droplet you will find the same evaporation flux same gasification rate everything is the same right it vaporizes in the same way also.

But; however, this does not happen. When you actually have you know a flow around the droplet now that flow could be due to natural circulation, it could be due to forced circulation also you are going to destroy the spherical symmetry that is associated with a evaporation we are going to destroy that spherical symmetry like for example, in this particular case you can see that the droplet forms like two toroidal cells when there the when the flow around the droplet does not show any separation. That means, for low Reynolds number flow or in the stokes flow kind of a regime you will find that this flow field does not show any separation in the aft of the droplet.

So, as a result of that your gas phase stream lines show this nice two vortex structure basically so, one single toroidal vortex we are just cutting it across the central plain. So, you are cutting it across the central plain otherwise it is it is like a toroid, right. So, that is what you see over here. So, that is the flow field. If the droplet shows any separation; that means, the flow shows any separation aft the droplet you basically get an asymmetry where is basically you have this four vertical loops it is basically once again two vortex; one is a toroidal here and the other is a toroidal on the top. So, it is basically segregated into two. So, it destroys the symmetry whenever you have a flow. So, flow destroys the symmetry ok, inducing what we call recirculation.

However, in a spherically symmetric model we are not really concerned about all those things the most of the droplet models will actually operate on the spherically symmetric principle, ok.

(Refer Slide Time: 04:55)



Droplet Vaporization Models

(i) constant-droplet-temperature model
(ii) infinite-liquid-conductivity model (uniform but time-varying droplet temperature),
(iii) spherically symmetric transient droplet-heating (or conduction-limit) Model
(iv) effective-conductivity model
(v) vortex model of droplet heating
(vi) Navier–Stokes solution.

Sirignano, Droplets and Sprays

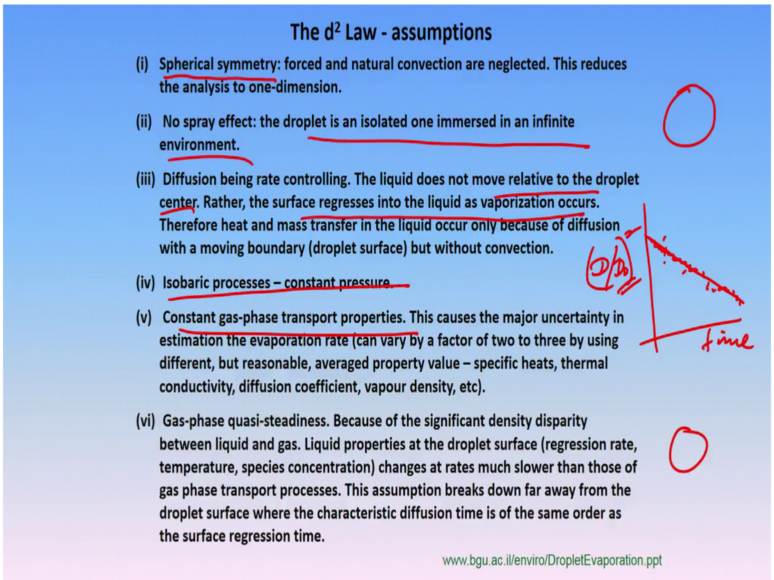
So, droplet vaporization models are of various kinds one is called the constant droplet temperature model; that means, we take the droplet temperature as constant, one is called the infinite liquid conductivity model; that means, that droplet temperature is uniform, but time varying, ok. Then there is the spherically symmetric transient droplet heating this is basically what we call the conduction limit model; that means, you take the temperature gradient within the droplet.

So, in the first case you basically have a constant temperature let us call it td in the second case you also have the same td, but this td can actually now becomes a function of your time ok. On the other hand, in the spherically symmetric case you basically now will have a temperature gradient that will be created at the droplet ok. So, the there will be as temperature gradient within the droplet. So, there will be a dT dr that will be created, ok. Then there is something called the effective conductivity model; that means, in this particular model you take the K that the conductivity and you enhance it ok. So, it is this is basically K plus something else, or K multiplied by some factor x.

Then there is a vortex model that means in this particular model you do take into account the flow field that I showed just a slide before. So, you take into account this particular flow field and you solve for the you take this as an ad hoc flow field and then you solve, ok, for the liquid phase and the gas phase transport and lastly you have the full solution of the Navier-stokes equation that will of course, give you the most perfect result, but albeit with a very high computational cost and specially if you are dealing with lots of droplets this can pose a bit of a problem.

So, in increasing order of complexity you basically do this kind of problems first two and first two are very easy the third one is basically a little difficult, ok. The third the fourth one is variation of the first of the third one basically you just improve on the conductivity limit and then the vortex model basically is somewhere in between a Navier-stokes equation and no flow situation, that means, you incorporate a flow field and try to see the dynamics of that and the last one is basically the full fledged solution of the Navier-stokes equation which is the correct way of doing this thing.

(Refer Slide Time: 07:39)



The d^2 Law - assumptions

(i) Spherical symmetry: forced and natural convection are neglected. This reduces the analysis to one-dimension.

(ii) No spray effect: the droplet is an isolated one immersed in an infinite environment.

(iii) Diffusion being rate controlling. The liquid does not move relative to the droplet center. Rather, the surface regresses into the liquid as vaporization occurs. Therefore heat and mass transfer in the liquid occur only because of diffusion with a moving boundary (droplet surface) but without convection.

(iv) Isobaric processes – constant pressure.

(v) Constant gas-phase transport properties. This causes the major uncertainty in estimation the evaporation rate (can vary by a factor of two to three by using different, but reasonable, averaged property value – specific heats, thermal conductivity, diffusion coefficient, vapour density, etc).

(vi) Gas-phase quasi-steadiness. Because of the significant density disparity between liquid and gas. Liquid properties at the droplet surface (regression rate, temperature, species concentration) changes at rates much slower than those of gas phase transport processes. This assumption breaks down far away from the droplet surface where the characteristic diffusion time is of the same order as the surface regression time.

www.bgu.ac.il/enviro/DropletEvaporation.ppt

So, the first and foremost what we are going to look at is basically look at something what we call the d square law right. So, d square law this was found by many people including professor C K law so, that means, if you plot the D by D naught square of the droplet with time basically and this time is a normalized time you will get what we call a straight linear flat drop in the profile for any type of droplets because you are normalized it by the initial size, ok. So, for those droplets you find that of course, there is a little spread of the data, but it basically looks rock solid. So, it just decays in that particular way.

Now, d square law assumes inbuilt there are certain assumption many people use the d square law in a very ad hoc basis. So, moment you have something related to droplet you will tend to use the d square law. So, the d square law usually uses a spherical symmetry; that means, forced and natural convections are neglected; that means, it reduces the analysis to basically one dimension, right because it is spherically symmetric right there is no spray effect; that means, the droplet is an isolated one in an infinite environment.

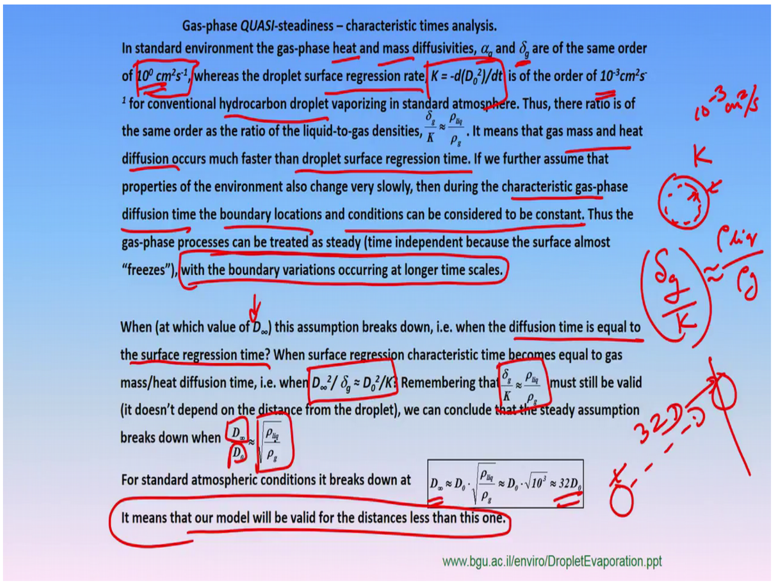
So, that means, if there is another droplet it is so far away that this droplet does not cross talk with the other diffusion is the rate controlling that means the liquid does not move relative to the droplet center rather the surface regresses into the liquid as the vaporization happens. So, the surface slowly and steadily regresses, but there is no flow that is generated within the liquid therefore, the heat and mass transfer in the liquid only happens because of diffusion with a moving boundary there is no convection effect, that means, that regression is slow and there is no flow that is generated within the droplet.

So, there is no question of any convection as I said. So, it is basically a simple diffusion problem with a moving boundary it is an isobaric process. So, it is constant pressure there is a constant gas phase transport properties that means, there is no real change in the specific heats thermal conductivity things like that. The last point is very important this is called the Gas phase quasi steadiness.

So, there is a significant density disparity between the liquid and the gas right is of the order of thousand in high pressure it can come down to about 100 right. So, the liquid properties are the droplet surface; that means, the regression rate temperature and species concentration changes at a rate which is much slower than those of the gas phase transport processes, ok.

So, this assumption of course, is not valid far away from the droplet, but at the droplet surface ok, that means, the liquid properties change in such a slow rate compared to the vapor phase that the we always see what we called quasi steady kind of a phenomena, so, that we will see a later in the in the next slide ok.

(Refer Slide Time: 10:32)



Gas-phase *QUASI*-steadiness – characteristic times analysis.

In standard environment the gas-phase heat and mass diffusivities, α_g and δ_g are of the same order of $10^0\ cm^2s^{-1}$, whereas the droplet surface regression rate $K = -d(D_0^2)/dt$ is of the order of $10^{-3}cm^2s^{-1}$ for conventional hydrocarbon droplet vaporizing in standard atmosphere. Thus, there ratio is of the same order as the ratio of the liquid-to-gas densities, $\frac{\delta_g}{K} \approx \frac{\rho_{liq}}{\rho_g}$. It means that gas mass and heat diffusion occurs much faster than droplet surface regression time. If we further assume that properties of the environment also change very slowly, then during the characteristic gas-phase diffusion time the boundary locations and conditions can be considered to be constant. Thus the gas-phase processes can be treated as steady (time independent because the surface almost "freezes"), with the boundary variations occurring at longer time scales.

When (at which value of D_∞) this assumption breaks down, i.e. when the diffusion time is equal to the surface regression time? When surface regression characteristic time becomes equal to gas mass/heat diffusion time, i.e. when $D_\infty^2/\delta_g \approx D_0^2/K$? Remembering that $\frac{\delta_g}{K} \approx \frac{\rho_{liq}}{\rho_g}$ must still be valid (it doesn't depend on the distance from the droplet), we can conclude that the steady assumption breaks down when $\frac{D_\infty}{D_0} \approx \sqrt{\frac{\rho_{liq}}{\rho_g}}$

For standard atmospheric conditions it breaks down at $\boxed{D_\infty \approx D_0 \cdot \sqrt{\frac{\rho_{liq}}{\rho_g}} \approx D_0 \cdot \sqrt{10^3} \approx 32D_0}$

It means that our model will be valid for the distances less than this one.

www.bgu.ac.il/enviro/DropletEvaporation.ppt

$10^{-3}\ cm^2/s$

K

$\left(\frac{\delta_g}{K}\right) \approx \frac{\rho_{liq}}{\rho_g}$

$32D_0$

So, the gas phase quasi steadiness and let us prove that using what we call a characteristic time scale analysis. Now, in a standard environment what we know is that the gas phase heat and mass diffusivities alpha g and delta g right, let us assume that it is delta g it can be anything are of the same order which is of the order of about 10 to the power of zero centimeter square per second, but thus droplet surface regression rate if you measure; that means, this droplet is actually shrinking right.

So, this rate of this shrinkage is given by k it is of the order of 10 to the power of minus 3 centimeter square per second ok. So, naturally it is three order slower than the corresponding diffusivities either the mass or the heat for a conventional say hydrocarbon droplets is almost of similar order for other droplets as well.

So, that is the ratio if you just take the ratio of delta g that is the mass diffusivity divided by this K factor it is basically scales as your role liquid because of this three orders change by the rho gas correct because they are also of the order 1000, ok. So, that means, that gas phase mass and heat diffusion occurs much faster than the droplet surface regression. So, the time scale that is involved for the droplet surface regression is much higher than the corresponding time scale that is involved for the diffusion to occur, right.

So, we further assume that the property is also vary in the environment also very very slowly. So, during the characteristic gas phase diffusion time the boundary conditions and the locations of the droplet can be almost considered to be constant because it is happening at a 3 order slower time or slower process than compared to the rapid evaporation. So, the gas phase processes from a droplet perspective can be treated as steady, with boundary variations only occurring at the longer timescales. So, this is the whole reason what is what is meant by gas phase quasi steadiness.

Now, the question that naturally comes that at what value of this D this assumption breaks down that is when the diffusion time is equal to the surface regression time because this vapour will diffuse it will continuously diffuse. So, that there will be at a some point where the time that it the length scale or some length scale where the time that it takes for the vapour to diffuse to that length scale should be equal to the surface regression time. So, that is given by if you just equate the two and remembering that this is still given as a ratio of the two liquids.

So, what happens is that your D infinity by D naught is given by the square root of this. So, therefore, if you calculate D infinity it becomes 32 D. So, that means, 32 diameter apart if you consider a location that is the time that it takes for the two timescales to kind of match. So, our distance our model ; that means, the D square model should be valid for distances which are very closed; that means, we are not considering anything close to 32 diameter, right. So, that gas phase quasi steadiness is one of the most important inbuilt assumptions that is there in the D square law, ok.

(Refer Slide Time: 14:02)

The d^2 Law – assumptions

(vii) Single fuel species. Thus it is unnecessary to analyze liquid-phase mass transport (no diffusion term).

(viii) *Constant* and uniform droplet temperature. This implies that there is no droplet heating. *Where all the heat goes?* Combined with (vii), we see that liquid phase heat and mass transport processes are completely neglected. Therefore the d^2 Law is essentially a gas-phase model.

(ix) Saturation vapour pressure at droplet surface. This is based on the assumption that the phase-change process between liquid and vapour occurs at a rate much faster than those for gas-phase transport. Thus, evaporation at the surface is at thermodynamic equilibrium, producing fuel vapour which is at its saturation pressure corresponding to the droplet surface temperature.

(x) No Soret (mass flow because of the temperature gradient), Dufour (heat flow because of the concentration gradient) and radiation effects (*how this effects the validity of the model?*).

www.bgu.ac.il/enviro/DropletEvaporation.ppt

So, some more things ok. So, if it is a single species that means, I have mentioned fuel because it is for a combusting combustible droplet, but it is not need not be it can be valid for anything. So, the constant it is a constant and uniform temperature. So, this is very important. So, the droplet temperature is constant and uniform. So, I mean where does all the heat go, if you ask the question all the heat basically goes towards evaporation. So, that is what happens ok. So, therefore, where does this.

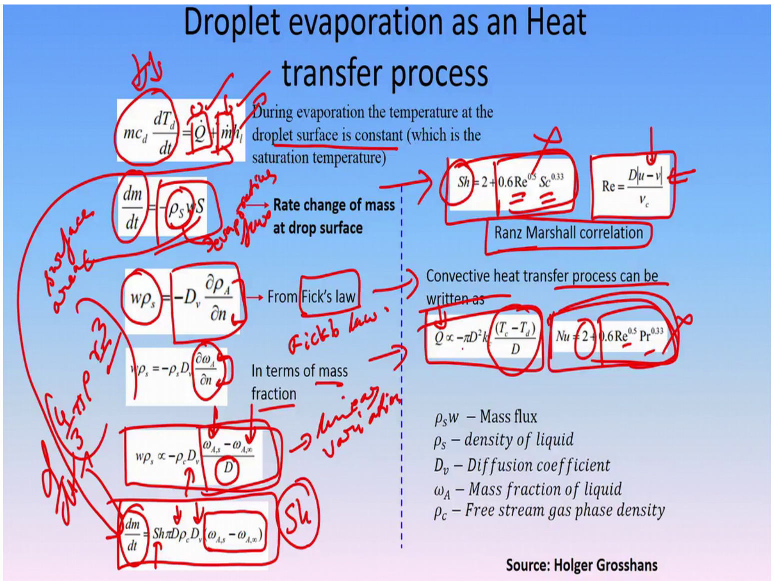
So, the combining with this particular thing we see that a liquid phase heat and mass transport processes are completely neglected because you are assuming it is a constant temperature, right. It is a constant temperature there is no convection within the liquid droplet. So, therefore, where would all this heat and mass go, it just goes towards vaporizing. So, you basically have a very simple equation and we will show that equation where you just take into account this particular physics that you are not solving anything basically within the liquid phase. So, that is a beauty of the d squared model is basically as we can call it is basically essentially a gas phase model right and then the ninth assumption. So, there are ten.

So, the ninth assumption is basically the saturation vapor pressure at the droplet surface now this is based on the assumption that the phase change occurs at a rate which is much faster than the gas phase transport. So, therefore, the evaporation at the surface is at the thermodynamic equilibrium state. Not really valid if you dealing with extremely rarefied flow and things like that there you can actually have this recoil pressures and things like that.

So, what happens is that we always assume that compared to the gas phase now, this vapor that exists at the liquid vapor interface which you already showed exists at the saturation temperature exist as a saturation that is the saturation pressure. So, that particular phase is always there. So, it is in a thermodynamic equilibrium state, ok. So, basically evaporation happens from the liquid to the gas across the interface it is like I am jumping from the liquid to the gas phase and in the gas phase I am existing at the saturation pressure of whatever is the surface temperature all right ok. So, so, that is what it is and it occurs at a rate which is much faster than the gas phase transport.

Now, Soret, no Soret effect or Dufort effect Dufour effects are basically radiation effects are basically admissible over here now. So, Soret and Dufour these are basically second order effects, ok, you know that your thermocouple is basically operates on these kind of things the second order effects. So, Soret means that there is mass flow because of temperature gradient and Dufour means there is heat flow due to concentration gradient, right. So, these effects are basically neglected at this particular point and also if the droplet is not too hot the radiation effects can also be neglected though this is not always the case, ok. This is not always the case is a very situation dependent problem, ok.

(Refer Slide Time: 17:17)



Droplet evaporation as an Heat transfer process

$$mc_d \frac{dT_d}{dt} = \dot{Q} + \dot{m}h_l$$

During evaporation the temperature at the droplet surface is constant (which is the saturation temperature)

$$\frac{dm}{dt} = -\rho_s w S$$ → **Rate change of mass at drop surface**

$$w\rho_s = -D_v \frac{\partial \rho_A}{\partial n}$$ → From Fick's law

$$w\rho_s = -\rho_s D_v \frac{\partial \omega_A}{\partial n}$$ In terms of mass fraction

$$w\rho_s \propto -\rho_c D_v \frac{\omega_{A,s} - \omega_{A,\infty}}{D}$$

$$\frac{dm}{dt} = Sh\pi D\rho_c D_v(\omega_{A,s} - \omega_{A,\infty})$$

$$Sh = 2 + 0.6Re^{0.5}Sc^{0.33} \qquad Re = \frac{D|u-v|}{\nu_c}$$

Ranz Marshall correlation

Convective heat transfer process can be written as

$$\dot{Q} \propto -\pi D^2 k \frac{(T_c - T_d)}{D} \qquad Nu = 2 + 0.6Re^{0.5}Pr^{0.33}$$

$\rho_s w$ – Mass flux
ρ_s – density of liquid
D_v – Diffusion coefficient
ω_A – Mass fraction of liquid
ρ_c – Free stream gas phase density

Source: Holger Grosshans

So, let us look at the heat and the mass transport of the droplet, enough slightly in some block diagrams because we have described a few things. So, let us first write the evaporation of the droplet. So, what happens during the evaporation of the droplet the surface temperature is constant, which is also the saturation temperature. So, what we see over here is that if we allow also this surface temperature to change with time that may or may not be true it depends by you can take it to be constant also. So, whatever heat that is being supplied this Q dot is either going towards a latent heat and towards the sensible enthalpy, correct. So, latent heat means that it is basically going towards the evaporation.

So, this m dot is basically the mass that is getting evaporated and hl that you see is basically the latent heat of vaporization. So, Q dot is basically the heat that is being supplied to the droplet from whatever means right. So, there is some heat supply that has to happen right. The term on the right hand side that you see is basically takes into account the variation of the droplet temperature it is m c d into the dT by dt. So, that takes into account, if the surface temperature is constant, but essentially it can vary with time. Now, in many of the cases that may not be the case also you can just supply the heat and it goes totally towards evaporating the droplet, ok.

Now, m dot which is given by basically dm by dt that is the rate of evaporation of the droplet is given by this particular form, where as you can see that this particular term is basically the evaporative flux and S is basically the surface area. So, S is the surface area this is basically the evaporative flux all right now the evaporative flux of the droplet now can be written in terms of the concentration gradient either this or this is basically written in terms of mass fraction this is nothing, but the Fick's law is the same as what we have in the case of your thermal conduction also or the thermal diffusion also, ok.

So, the Fick's law is the law that we fall back on when there is a when there is a diffusion problem involved. So, in this particular case is a concentration gradient that is creating it. It is the rate of change of the it is the change of the spatial gradient of the concentration with the distance, right.

Now, this can be also written as in a simpler form if you assume that it is almost like a linear variation it is like a linear variation which in most of the cases will be true this is basically the diffusivity the mass diffusivity and this is basically the surface concentration minus the far field concentration divided by some parameter which is D which is usually taken as one droplet diameter away all right.

So, the same thing now can be written in terms of a composite way which is basically given as this particular expression where you can see two terms; one is a Sherwood number the Sherwood number is basically the Nusselt number for mass transfer. So, this is basically nothing, but is it is it is like the same as the Nusselt number except that it pertains to mass transfer and not heat transfer and then you have the D which is the diffuser, this is the diameter this is the diffusivity these are the corresponding difference in the concentration gradients.

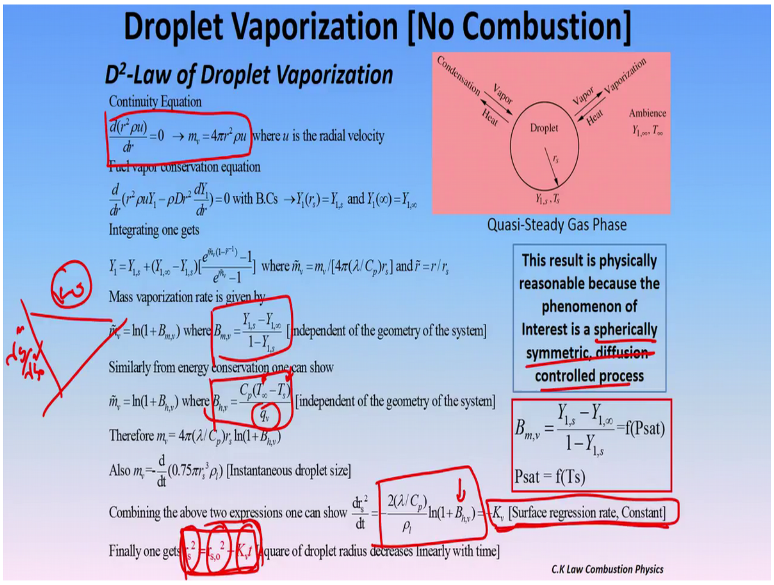
Now in order to so, you can see from this equation you need basically two terms right, one is this term and one is this term, for this term h l is already known right because it is a latent heat it is known and it is a constant temperature. So, you know exactly what the latent heat that you need m dot is something that we need to find out and q dot is something that we need to find out right. So, m dot and q dot so, already m dot we found out using this. So, there is of course, a term like Sherwood number now this Sherwood number can be written in terms of Reynolds number and Schmidt number, where the Reynolds number is nothing, but the difference in the velocity scale if there is a velocity between the gas and the liquid.

If there is no velocity which is the case in the case of a spherically symmetric droplet as in the case of your d square law this particular term will basically vanished. So, it will be given in terms of only two so, but this is also called the Ranz Marshall correlation very popular correlation in the droplet literature. So, the Reynolds number is basically given as the relative velocity between the two, between the droplet and the ambient. So, you can see that it actually enhances the mass transfer which is kind of obvious we told you earlier that the j this is s h is basically like a like a flux term. So, it is basically represents a flux term.

So, therefore, it gets enhanced the convective heat transfer process can be written as therefore, Q dot is equal to this on the other side the Q dot which we need is basically written as k c pi D square T c minus T d divided by D ok, where the Nusselt number is also written in a very similar form once again in terms of Reynolds number and Prandtl number as we know the heat transfer will also get enhanced if there is a flow. So, if there is no Reynolds number once again this goes down you are left with only two.

So, now that you have got Q dot and you have got m dot you can solve this equation and you can evolve that what will be the surface temperature of the droplet to begin with time all right, and you can also calculate that what will be that, but if in the case that you do not in require at all the temperature change if this is constant you can just substitute dm by dt as four third pi r cube dt by dt of that and you can find out what will be the diameter regression and that is exactly what the d square law actually does, ok. But, this is to give you a bird’s eye view that how this problems are solved basically we solve for Q you solve for m dot and then you combine the two and you solve the equation, ok.

(Refer Slide Time: 23:53)



Droplet Vaporization [No Combustion]

D²-Law of Droplet Vaporization

Continuity Equation

$\frac{d(r^2\rho u)}{dr} = 0 \rightarrow m_v = 4\pi r^2 \rho u$ where u is the radial velocity

Fuel vapor conservation equation

$\frac{d}{dr}(r^2\rho u Y_1 - \rho D r^2 \frac{dY_1}{dr}) = 0$ with B.Cs $\rightarrow Y_1(r_s) = Y_{1,s}$ and $Y_1(\infty) = Y_{1,\infty}$

Integrating one gets

$Y_1 = Y_{1,s} + (Y_{1,\infty} - Y_{1,s})[\frac{e^{\tilde{m}_v(1-\tilde{r}^{-1})} - 1}{e^{\tilde{m}_v} - 1}]$ where $\tilde{m}_v = m_v/[4\pi(\lambda/C_p)r_s]$ and $\tilde{r} = r/r_s$

Mass vaporization rate is given by

$\tilde{m}_v = \ln(1+B_{m,v})$ where $B_{m,v} = \frac{Y_{1,s} - Y_{1,\infty}}{1-Y_{1,s}}$ [independent of the geometry of the system]

Similarly from energy conservation one can show

$\tilde{m}_v = \ln(1+B_{h,v})$ where $B_{h,v} = \frac{C_p(T_\infty - T_s)}{q_v}$ [independent of the geometry of the system]

Therefore $m_v = 4\pi(\lambda/C_p)r_s \ln(1+B_{h,v})$

Also $m_v = -\frac{d}{dt}(0.75\pi r_s^3 \rho_l)$ [Instantaneous droplet size]

Combining the above two expressions one can show $\frac{dr_s^2}{dt} = -\frac{2(\lambda/C_p)}{\rho_l}\ln(1+B_{h,v}) = -K_v$ [Surface regression rate, Constant]

Finally one gets $r_s^2 = r_{s,0}^2 - K_v t$ [Square of droplet radius decreases linearly with time]

Quasi-Steady Gas Phase

This result is physically reasonable because the phenomenon of Interest is a spherically symmetric, diffusion controlled process

$B_{m,v} = \frac{Y_{1,s} - Y_{1,\infty}}{1-Y_{1,s}} = f(Psat)$

Psat = f(Ts)

C.K Law Combustion Physics

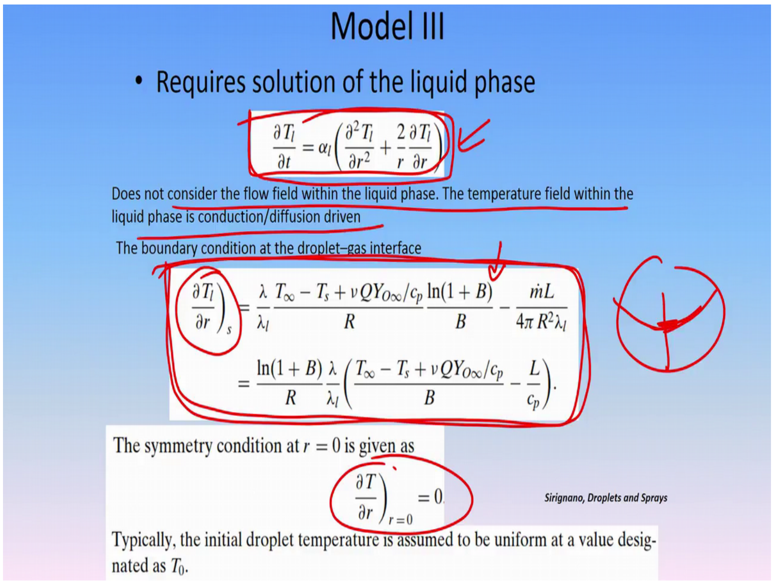
So, the D square law going very fast so, we from the continuity you already showed that these are the as I say d square law is only about the gas phase there is almost nothing that is solved on the liquid phase. So, you can go through the motion of this there are a bit of complications which I really do not you do not have to really go through it in that kind of a detail because I have already said, what are the key things, ok. It is physically reasonable because the phenomena of interest is a spherically symmetric diffusion control process that is exactly what we are looking at over here.

So, all that people need to do is to solve for the equations in the vapor phase that is all that is required, right and here you see that some of these terms that you see over here these are called the heat and the mass transfer numbers again not going into the details of all these things, ok. Ultimately it has focus on the answer that the regression of the droplet diameter is given in terms of the initial diameter and some factor K, which is called the surface regression rate which is constant.

So, this basically if you plot it you will get a curve which is basically r s square by r s o square where the slope of this curve will be given by K v and K v is basically the slope is basically given as an expression like this where this is called the Spalding heat transfer number this B h v is basically given here it is C p t infinity minus T s which once again is the infinite temperature minus the droplet surface temperature and Q v is basically whatever heat that is actually supplied to the droplet. It has got a very similar form as what we saw the saw earlier. So, this is the surface regression rate that you can solve and you will find that it actually scales linearly with the square of the droplet diameter.

So, therefore, this is the D square law which takes into account that only the gas phase and no liquid phase equation is at all solved.

(Refer Slide Time: 25:59)



Model III

- Requires solution of the liquid phase

$$\frac{\partial T_l}{\partial t} = \alpha_l \left(\frac{\partial^2 T_l}{\partial r^2} + \frac{2}{r}\frac{\partial T_l}{\partial r} \right)$$

Does not consider the flow field within the liquid phase. The temperature field within the liquid phase is conduction/diffusion driven

The boundary condition at the droplet–gas interface

$$\left.\frac{\partial T_l}{\partial r}\right)_s = \frac{\lambda}{\lambda_l}\frac{T_\infty - T_s + \nu Q Y_{O\infty}/c_p}{R}\frac{\ln(1+B)}{B} - \frac{\dot{m}L}{4\pi R^2 \lambda_l}$$

$$= \frac{\ln(1+B)}{R}\frac{\lambda}{\lambda_l}\left(\frac{T_\infty - T_s + \nu Q Y_{O\infty}/c_p}{B} - \frac{L}{c_p}\right).$$

The symmetry condition at $r = 0$ is given as

$$\left.\frac{\partial T}{\partial r}\right)_{r=0} = 0.$$

Sirignano, Droplets and Sprays

Typically, the initial droplet temperature is assumed to be uniform at a value designated as T_0.

If you however, are doing model III which is basically this ones the spherically symmetric transient droplet heating; that means, in that particular case unlike the d square law you have to solve for the liquid phase for the liquid phase equation that you are solving is basically nothing, but nothing, but what we call a simple transient heat conduction equation, ok. So, it does not consider any flow and the temperature field is conduction driven within the liquid. So, all you need to do is at the boundary you need to match the temperature slope and the boundary and the symmetry condition at the droplet center.

Because at the droplet center the temperature should that temperature gradient should actually go to 0. This is because you have basically it is a symmetric droplet, right. So, all you need to do is solve this extra piece of equation and you need to saw adjust the boundary condition accordingly do not bother about these terms. So, much because these are basically a little bit more complicated all you need to do is match at the droplet gas interface, ok.

(Refer Slide Time: 27:08)

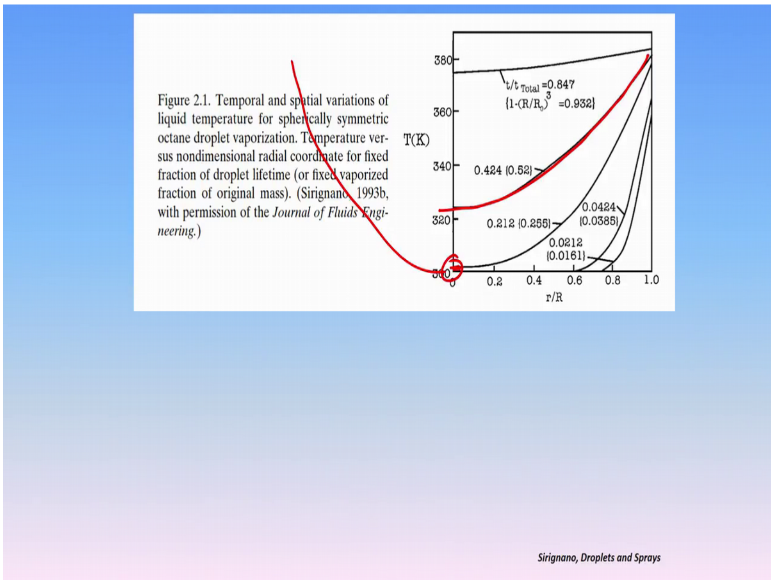


Figure 2.1. Temporal and spatial variations of liquid temperature for spherically symmetric octane droplet vaporization. Temperature versus nondimensional radial coordinate for fixed fraction of droplet lifetime (or fixed vaporized fraction of original mass). (Sirignano, 1993b, with permission of the *Journal of Fluids Engineering.*)

Sirignano, Droplets and Sprays

So, this is for example, some of the temperature profiles that you get when you actually solve the spherically symmetric problem. So, you can see it comes like this and you can see the slope goes 0 over here, because it will go like that on the other side of the interface, ok. So, this was the work done by professor Sirignano.

(Refer Slide Time: 27:25)

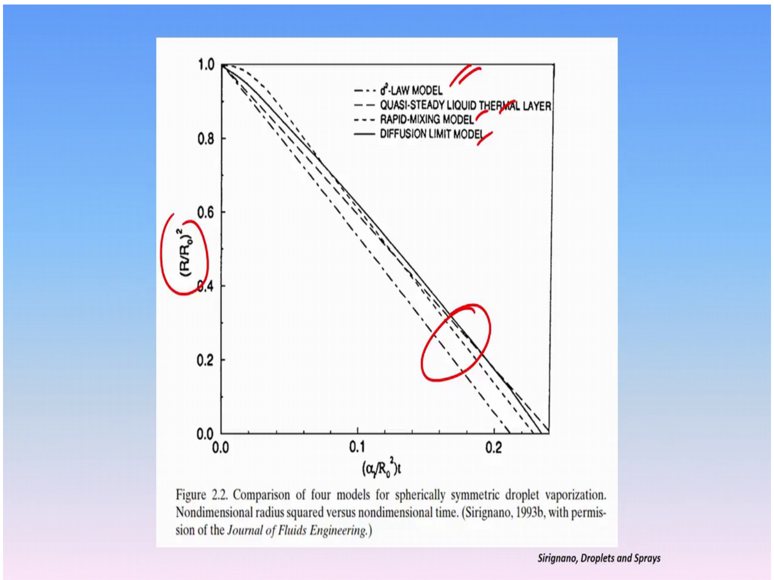
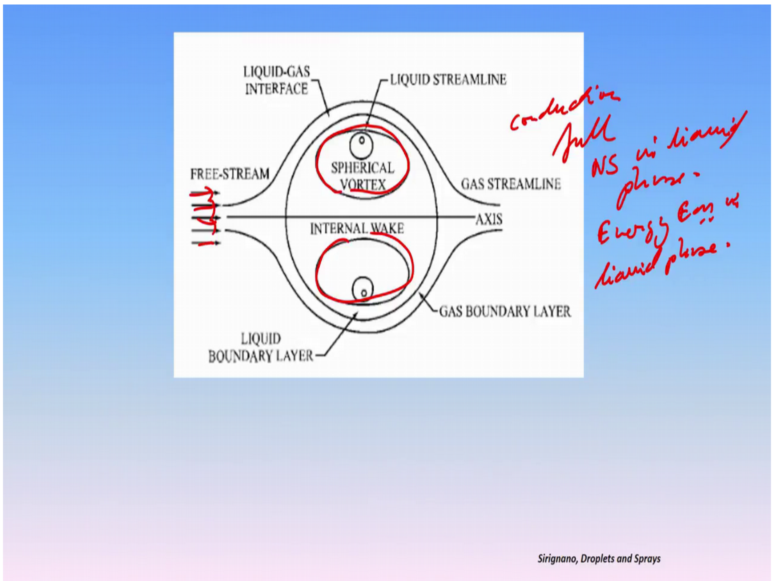


Figure 2.2. Comparison of four models for spherically symmetric droplet vaporization. Nondimensional radius squared versus nondimensional time. (Sirignano, 1993b, with permission of the *Journal of Fluids Engineering.*)

Sirignano, Droplets and Sprays

If you compare the four models that is the quasi-steady, rapid mixing, the diffusion limit and the d squared law, you will find it in the r squared scale all of them kind of show more or less similar stuff, that is why your d square law is a very good assumption to begin with if you just are interested in knowing the decay of the droplet diameter, but if you are interested in something more than that then it becomes a little bit of a problem.

(Refer Slide Time: 27:52)



conduction

full NS in liquid phase.

Energy Eqn in liquid phase.

Sirignano, Droplets and Sprays

And, lastly if you want to solve for the vortex method then there is the spherical vortex that you have to solve within the droplet along with the free stream and you have to take into account not only just the liquid conduction equation, but you have to write the full fledged the full Navier stokes equation in the liquid phase. If you assume the velocity profile then you can just do away with just the energy equation in the liquid phase in liquid phase, ok. So, this is more complicated and you are strongly recommended to look at the other course on this where it is covered in much more details, again how is it exactly covered all those things are there mentioned.

So, with this particular thing we basically come to the end of this convective heat transfer course. So, what we have done is basically just a small recap, we have basically studied the entire spectrum of canonical convection problem. We emphasized more on the scaling argument, we emphasized more on the analytical solutions and the integral solutions and once those integral solutions have evolved we have to started to you know look at some of the interesting insightful discussions related to those problems which has given as a lot of insights into forced convection as well as natural convection as well as turbulence and as well as now a little bit on the multi phase systems, ok.

So, we have covered basically forced and natural convection for laminar flows in details turbulence flow we just gave you some insights of what happens in the turbulence flow and of course, we did a little bit on what we call the multi phase systems in just a couple of lectures to just give you a brief overview of what happens.

We also did a lecture on the diagnostic techniques specially TLC and IR Thermography just to give an idea that all these things that we say that we have measured heat transfer coefficient, we have measured the mass transfer coefficient, we measure the surface temperature how these things can be done, experimentally because you need experimental validation for everything. So, that was done using these kind of analogies the, this kind of this kind of tools not analogies.

So, we did that in quite a bit of details and there were some homework problems that we also solved just to give you an idea that how to solve attack typical problems in this particular area, but the main takeout takeaway points from for this particular work from this particular course is basically you have to look into details that what are the key factors, right, what are the key factors that effects convective heat transfer. So, you have to have an insight about fluid dynamics, you have to have an insight in heat transfer and you have to have some insights in how these equations are coupled, how they actually go from one stage to the other, and what are the different scales that are relevant concepts like boundary layer, concepts like fully develop profile, what does it mean in terms of temperature etcetera.

So, I think if you study all these things in details and follow the lectures you should have a very clear idea of what convective heat transfer is. This is the first course so, nothing advanced, ok. So, you can have a reasonably good idea of what it is before you move into more advanced system specially involving multi phase. So, we end the course here and wish you guys all best of luck.