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Lecture - 59 Droplet evaporation – Sessile II

In the previous class we started looking at the sessile droplet and we showed that for example, that flow in sessile droplet is created due to evaporation or rather due to the asymmetry of evaporation; that means the droplet tends to vaporize more near the three phase contact line than at the apex. So, based on that we formulated that following (Refer Time: 00:37) that how that such a flow can be understood and how that evaporation phenomena can be understood as well.

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In this particular thing we look at the simple for example, the same droplet where you can see here the maximum evaporation flux comes from the three phase contact line which we already say that that is what creates that capillary flow. So, the evaporation flux however here there is nothing convective about it as we know. So, this is a pure diffusion equation all right.

So, this is diffusion in equation in nature where Cs is basically the vapor layer that surrounds the droplet surface and C infinity is a concentration of the same vapor in the far infinity; A being the surface area of the droplet and R being the contact radius. So and D is basically the water vapor air diffusion coefficient. If instead of water droplet this is something else say for example, it is say ethanol or it is something of that kind then of course, you will have a very different diffusion coefficient ok. So, that value is dependent on the application that you are dealing with all right. So, that is the J ok.

So, Popov what he did was that he gave another expression in terms of the contact angle now. As we said that when the droplet evaporates if you remember there is something called CCR, CCA and mixed mode that we already did in the last class. So, Popov did was that he gave the following expression for the contact angle. So that means, what he did was he basically cos J which is basically the rate of change of volume of the droplet. It was written once again in terms of D and the concentration difference. In addition there is an f theta parameter; where f theta for low contact angle is given in this particular form; for high contact angle it is given in that particular form ok.

So, where M is once again the molecular weight of water; rho is the density and V is the droplet volume. So, if you just plot this kind of a volume flux with time you will find that this is the kind of trajectory that the droplet follows; for all volumes; these are of different volumes. So, their starting points are obviously different. But they show almost like this kind of a linear regression with kind of time except towards the end where it is you start to see the change in the slope and if you look at the time history of the droplet it looks precisely something like this ok.

So, in this particular experiments, for example you can see that the contact angle perhaps remains almost very similar in all the cases; if you look across the board ok. So, these two expressions are important. So, this was given by Popov and there are varieties of other improvements on this equation and this is the standard diffusion equation that is not based on any assumption but just based on that only it is a diffusion driven problem.

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Now in case so, we will come to that in a second so, in order to calculate the droplet volume for example, usually what we take is a spherical cap assumption; that means the droplet is like a cap ok. So, it is like a part of a part of a sphere basically. So, this is how the spherical cap is given and it is applicable for any contact angle droplets as a matter of fact ok. Now if the droplet is sitting on a platform like this; so, far we have seen that it is basically what we call a diffusion problem; that means a water vapor accumulates here and then it diffuses out to the infinity just because of the concentration gradient.

However if there is a flow; that means, there is a convective flow that happens. So, what that flow will do is that it will go over the droplet right. So, it will basically dislodge this saturation water vapor that is accumulating near the near the droplet surface. So, actually fresh stock of air would be brought in. As a result of that you can expect an increase in the J flux that is because simply you are not bound by diffusion anymore. There is an actually a velocity which is carrying a parcel of this vapor that is evaporated to the far field and the process is still happening in the same way; that means, the droplet is still vaporizing in a quasi stead steady kind of a fashion. There is still an accumulation of Cs very close to the surface but this Cs is basically now blown off to the far field ok.

So, basically you get an enhancement of J just because of the Reynolds number effect and this will be particularly useful when you start looking at the scenario in the case of a contact free droplet ok; where the effect is much more prominent ok. But the evaporation mechanism is the same but the transport mechanism which determines the J that is different ok. But it is still given by the by the Clausius-Clapeyron equation that what is the vapor flux or the concentration of the water vapor or whatever vapor that you are dealing with it close to the surface.

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 So, far we have done it up to hydrophilic droplets. If you look at the case of droplets which are hydrophobic in nature; that means, where the contact angle is like this. So, it can be like if you see the lotus leaf you will find that the contact angles are much much higher. So, evaporation here is maximum in this particular plane rather than here. So, it is a complete reversal ok. So, if compared to a sessile droplet what we had was that the evaporation was a maximum here sorry and low in these zones; here it is just the opposite it is maximum here and it is a lowest at this particular point.

So, you get something like this ok. So, now the reason is quite apparent. If you look at this particular situation; here you can see that it forms a wedge or basically a triangular region on both sides of the three phase contact line. So, what happens is that if you just blow it up and see; you will see something like this. So, this is the three phase contact line going up and this is the space.

So, as the water vapor leaks out, so you create this Cs here now this water vapor now because it is a diffusion driven problem has to diffuse out from this wedge to the far field, so that more vapor can now evaporate and occupy this zone. Now because it is a wedge, so you can think of it that when you actually have a wedge anywhere in a system it is very very hard to basically pull out water vapor from that particular system because you have that additional resistance of the constriction which basically resist adds a layer of resistance for this water vapor or whatever vapor that you are dealing with to leak out and go to the ambient ok.

So, automatically what happens is that the vaporization rate is suppressed in this region automatically that happens. So, once again if you look at it so, if this is the wedge region; in this particular region there the evaporation will be suppressed; it is bound to happen. Whereas, if you look at the curvature at the top you can see that it has got now the full free field basically to evaporate right. So, there is no problem now. So, it can evaporate at whatever rate it wants because there is no constriction; there is no wedge there. Now in the case of a hydrophilic droplets it is like you can ask why it does not happen because here is an open wedge ok. So, it is an open cavity; here is a closed cavity. So, that is the reason you have more accumulation of water vapor here and there is no accumulation there.

So, that is why you have a reversal of the J ok. So, J is higher here and lower there; opposite to what you get in a hydrophilic sessile droplet right. Now that creates one more thing. Now in these particular droplets you have seen that there is a flow field which is created because of the pinned contact line and to supply the solvent because the evaporation flux is highest here. In this particular case however, what happens is that because of suppression of evaporation here and an enhanced evaporation in this particular sector ok; you create a small temperature difference; that means, a small delta T ok.

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So, that delta T what it does is that ok; it results in higher evaporative cooling ok; creating a temperature and hence a density gradient along the droplet height ok. So, the maximum evaporation from the top actually leads to a certain evaporative cooling at the top right; whereas, lower evaporation in this sector does not lead to the same extent of cooling all right. So, what happens is that you have naturally a density gradient that is created.

So, the high temperature fluid basically starts to rise towards the top of the surface and therefore, you get to see this kind of a recirculating flow that you see over here. This is in complete contrast to the flow that you would normally see in the hydrophilic case right. Because hydrophilic case you actually have a pink line and here the concentration is the evaporation is the maximum. So, the flow straight away goes there. Here however, the flow the evaporation is minimal here maximal here. So, there is more evaporative cooling in this sector than in this sector. So, naturally the flow starts to rise in a buoyancy driven kind of a phenomenon right.

Now if these droplets also come with a little bit of surfactants or if the temperature difference is large enough that it can create what we call a surface tension gradient right. You can see another type of flow that can happen which is called Marangoni convection right. So, that can also happen in this kind of cases ok. So, the flow that you see over here is buoyancy driven is very feeble because the temperature difference that you create for this kind of small droplets is really small.

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So, it is really small flow ok but however, as we will see if it is driven by Marangoni stress; that is due to the presence of surface tension gradient instead of density variation. So, the velocity field that is induced is given by as you saw here Vm; if that is the Marangoni velocity; it is d sigma by dt; that means, there is a change in temperature which should bring about a change in surface tension divided by delta T by mu. So, in essence delta sigma by delta T is a rate of change of surface tension with temperature. This is the temperature difference and this is the fluid viscosity.

So, in this particular case these velocities are normally very high; it is normally high all right So, Marangoni flow or the flows which varies by surface tension is actually given by this. So, so what happens is that evaporation is still the driver because of the wedge. So, if we recall the steps because of the wedge there is a suppression of evaporation here. So, naturally that leads to a temperature gradient. So, there is a delta T that is created right away ok.

Now, this delta T can either lead to a change in density which can lead to a buoyancy driven flow or it can lead to a change in the surface tension giving rise to a Marangoni flow. So, it is the nature of the droplet geometry right. The droplet geometry and of course, there are parameters like thermal conductivity and things like that which basically determines that the flow field that is subsequently created. So, in this case though the evaporation phenomena is strictly driven by diffusion right. The flow field that is inherently created within the droplet that can happen due to Marangoni or it can happen due to the buoyancy for due to the density ok. So, the same diffusion equations are valid for both the cases when you are computing J remember we wrote it in terms of f theta, delta, surface tension etcetera.

So, that remains almost the same except that you factor in the contact angle term. But apart from that if you if you kind of kind of try to understand the dynamics; that means, the flow field that is created, the flow field is very different from that of a capillary from that of a hydrophilic droplet. And this has got very wide implications you know in the case of particle transport whenever you are trying to do trying to custom engineer certain things in different applications right from biomedical applications, biomedical 3D printing ok, then of course, inkjet printing ok, then of course, you have surface patterning big industry ok.

So, lots of these things can actually lots of these things actually do involve calculation of J and the understanding of the internal flow field. So, we saw that J is kind of is a little immune little immune of course, ok, but the flow field is not flow field is very different and this changes with the change in the angle; that means, you create a very hydrophobic droplet with a very sharp cavity or a wedge you will create more the velocity of these flows will actually increase ok. So, those kind of things we have looked at it here. So, this is from the sessile framework.

So, then the sessile framework we have only looked into the diffusion and we have eluded that if there is a convective effect what it will do is that it will enhance the evaporation rate just by blowing away the vapor field ok. And so, it is the same reason why you actually blow air over your tea or coffee. If it is very hot what you do is that you blow air over it; that means, normally if you allow it to evaporate normally it is a diffusion driven problem. As soon as you blow basically you are removing that saturated to region of Cs and you are replenishing it with a fresh stock of air right.

So, naturally the evaporation becomes a lot faster right. So, so, that can also, but when you actually blow air over the droplet it can also influence the flow field inside because you have to still maintain the integrity at the interface; that means, that your boundary condition still has to match ok.

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So, there are different examples of different problems l for example, you can study similar such droplet under the under a convective field. I am not going to go into the details of these equations. There is a companion course which we is also offered on NPTEL which is called heat transfer and combustion in multi phase systems; where we have mentioned all these things. So, basically it means that when you have two phases, you basically write the equations; the continuity equations for the two phases and one for vapor and one for the liquid and these are basically the mass added to from the liquid to the vapor because it is an evaporating system and vice versa.

So one will lose mass; one will gain mass, that is how it happens. Then you write the two momentum equations. So, this is basically a stream of droplets which are being flushed into a cooling tower type of an application or a spray dryer in which an air is actually blown up. So, basically each droplet you know seize a flow field around it ok. So, here of course, we are not concerned with each droplet. We are concerned with a global equation ok. So, this global equation without going into the details there are ways in which you can write volume averaged global equations. The details of which are not within the scope, but just to give you an idea.

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 f_i is volume fraction of liquid Liquid v-momentum $\varepsilon_{\ell} \langle \rho_{\ell} \rangle^{\ell} \langle u_{\ell} \rangle^{\ell} \frac{\partial \langle v_{\ell} \rangle^{\ell}}{\partial x} + \varepsilon_{\ell} \langle \rho_{\ell} \rangle^{\ell} \langle v_{\ell} \rangle^{\ell} \frac{\partial \langle v_{\ell} \rangle^{\ell}}{\partial y}$
= $-\frac{\partial \varepsilon_{\ell} \langle p_{\ell} \rangle^{\ell}}{\partial y} + \frac{\partial}{\partial x} \left(\varepsilon_{\ell} \mu_{\ell} \frac{\partial \langle v_{\ell} \rangle^{\ell}}{\partial x} \right) + \frac{\partial}{\partial y} \left(\varepsilon_{\ell} \mu$ ϵ , is volume fraction of vanous $\langle \rho_l \rangle^l$ Intrinsic phase-averaged liquid density $\langle u_i \rangle^l$ Intrinsic phase-averaged liquid velocity in x-direction $\langle v_i \rangle^l$ Intrinsic phase-averaged liquid velocity in y-direction \dot{m}_{vl} ["] mass transfer per unit volume from vapour to the liquid phase due to phase change. $(u_i)^l$ is the phase-averaged liquid viscosity $\langle v_{II} \rangle^I$ is intrinsic phase-averaged liquid velocity at the
interface. $+\langle F_{v\ell,v}\rangle+\langle m''_{v\ell}\rangle\langle v_{\ell,J}\rangle^{\ell}-\varepsilon_{\ell}\langle \rho_{\ell}\rangle^{\ell}g$ F_{vl} is an interactive force between the vapour and liquid Vapor x-momentum or x-momentum
 $\varepsilon_v \langle \rho_v \rangle_v \langle u_v \rangle^v \frac{\partial \langle u_v \rangle^v}{\partial x} + \varepsilon_v \langle \rho_v \rangle^v \langle v_v \rangle \frac{\partial \langle u_v \rangle^v}{\partial y} = -\frac{\partial \varepsilon_v \langle \rho_v \rangle^v}{\partial x}$ $\langle T_l \rangle^l$ Intrinsic phase-averaged liquid temperature $(T_v)^v$ Intrinsic phase-averaged vapour temperature
 $(h_i)^l$ Intrinsic phase-averaged liquid enthalpy $+\frac{\partial}{\partial x}\left(\varepsilon_{v}\mu_{v}\frac{\partial(u_{v})^{v}}{\partial x}\right)+\frac{\partial}{\partial y}\left(\varepsilon_{v}\mu_{v}\frac{\partial(u_{v})^{v}}{\partial y}\right)+\left\langle F_{\alpha_{v}x}\right\rangle \star\left\langle \tilde{m}_{\alpha_{v}}\right\rangle \left\langle u_{v,I}\right\rangle ^{v}$ (\mathbf{h}_1)[,] Instinsic phase-averagea inquia entrindy
 \mathbf{h}_1 _b)^{, In} thristic phase-averaged vapour enthalpy
 $\mathbf{\hat{q}}_1$, I''' heat transfer from vapour to the liquid phase
 $\mathbf{\hat{q}}_1$ _b, ''' heat transfer fr Vapor y-momentum k_n is the thermal conductivity of vapour phase por y-momentum
 $\varepsilon_v \langle \rho_v \rangle^v \langle u_v \rangle^v \frac{\partial \langle v_v \rangle^v}{\partial x} + \varepsilon_v \langle \rho_v \rangle^v \langle v_v \rangle^v \frac{\partial \langle v_v \rangle^v}{\partial y}$
 $= \frac{\partial \varepsilon_v \langle \rho_v \rangle^v}{\partial y} + \frac{\partial}{\partial x} \left(\varepsilon_v \mu_v \frac{\partial \langle v_v \rangle^v}{\partial x} \right) + \frac{\partial}{\partial y} \left(\varepsilon_v \mu_v \frac{\partial \langle v_v \rangle^v}{\partial y} \right)$ 6 $+\langle F_{\scriptscriptstyle{f}_{\scriptscriptstyle{V}}}\rangle+\langle m''_{\scriptscriptstyle{f}_{\scriptscriptstyle{V}}}\rangle\langle v_{\scriptscriptstyle{V}_{\scriptscriptstyle{V}}}\rangle^{\nu}-\varepsilon_{\scriptscriptstyle{V}}\langle\rho_{\scriptscriptstyle{V}}\rangle^{\nu}g$

So, if you write the continuity equation, if you write the momentum equation, the liquid momentum equation, the vapor momentum equation x and y both you should be able to cast this problem and you should be able to solve it. So, this basically takes into account all the interface interactions that takes place between the two phases. Here once again we are not interested in one droplet, but these are thousands of droplets. So we are all more interested in a global volume averaged flow field and mass transfer and heat transfer issue ok. So, convection will actually increase the mass flux, it will increase because of the reasons that I cited already. It will also lead to change in the flow field because of the coupling that happens at the interface level.

So, in the next lecture what we are going to look at or immediately after this is look at the problem of a contact free droplet to begin with ok. So, the contact free droplets are something that many of us have encountered because one example is this spray dryer application that I just showed. So if I take a box around it, I will see that there are thousand such droplets. So, we are interested in how each of these droplets behave ok; that is what we call a Lagrangian viewpoint right. So, let us look into that. So, move to the next ppt.

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So, this is also droplet vaporization. So, let us look at the phenomenology first ok. So, the main idea of doing a droplet vaporization study is that if you look at this droplet over here ok. So, a single droplet basically exchanges heat, mass and momentum with it is gaseous medium; with the surrounding counterpart. So, here T infinity and Y in Y1 infinity is basically the concentration or the mass fraction of whatever is evaporating in the far field ok. So, if it is a droplet of water evaporating here; it would be the concentration of the water vapor in the atmosphere in the far field much away from the droplet. So, what it what a droplet does is that it exchanges mass with this.

So, if it heats it is called vaporization if it condenses it is called condensation. So, at the droplet surface this Y1s is basically the saturation pressure of the vapor phase ok, at the temperature of the droplet surface. So, if the droplet is here and if it is surface temperature is say 320 Kelvin all right; then the Y1s will be whatever is the partial pressure of that particular vapor phase at 320 Kelvin, that means this essentially is the saturation vapor pressure at 320 Kelvin right.

So, that is what it says over here. So, in this case when you have heat, mass exchanges with the gas medium, so there are levels of complications that you can actually have. So, some people may be interested in a very detailed analysis, some people may be interested in just a heuristic type analysis ok. So, the complete analysis will involve basically four interacting processes right. One is that we have to consider the liquid phase transport. So, if this is the droplet what is happening within the droplet?

So, that is your state 1. What is happening within the droplet ok? Then there is a gas phase transport which is 2. So, gas phase transport is that what is happening in the gas phase which is outside the droplet right. So, the gas phase is a second one over here. Then phase change at the interface. So, this liquid is getting converted to vapor. So, there must be some phase change processes that must be happening at the interface and in the case of reacting droplets you can also have a reaction that is happening in this particular phase; specially say for combusting droplets all right.

So, you have to solve three sets of systems basically with the coupling at the liquid gas interface in order to properly anchor this problem right. So, the important considerations over, here is the relative motion between the droplet and the gas phase; Stefan flow due to surface gasification; There is droplet drag and internal recirculation within the droplet. You would not understand many of these things ad hoc ok. So, first and foremost let us try to put a little bit of a context to it. So, when a droplet is actually injected; these droplets are usually in a flow field. Like for example, that spray dryer application that our cooling down application that I showed. So, this droplet has got a flow field surrounding it all right.

So, when there is a relative motion between this droplet and the flow field that is very important because as you know relative velocity also leads to droplet drag. So that means, if develop friction and the droplet surface. It also leads to internal recirculation; that means, the liquid inside the droplet will also start to show some motion and this will also affect and this droplet is actually evaporating droplet.

So, you can actually have the gasification effect that I say to you just a few moments ago that the blowing effect ok. That will be also part of this ok. So, we have to look all these things in a kind of a holistic manner, if you have to kind of you know understand this problem from a very you know at different levels. So, there are models; there are very complicated models there are very simple models also which tries to attack this problem in a multitude of ways.

So, this problem is not very different from a sessile droplet except that there is no substrate over here anymore right. But the mechanism of evaporation as I as it is shown over here is still the same ok. Now if you do not have a flow that diffusion or the J flux is also the same; except that J over here; that means, the evaporative flux over here then will be isotropic; that means, it will be the same everywhere because there is no substrate effect at all right. However, when you actually have a flow that happens over the droplet this evaporative flux will start to change right because of the, because when you actually have flow over a droplet is like flow over a cylinder right.

So, you can have wake regions that can actually develop and then you can have complicated motions within the droplet. So, the situation will be a little bit different. So, what we are going to do in the immediate lecture that follows is basically we are going to look at a couple of this problems from a very heuristic point of view. So, that you get an idea that how to attack this problems in a multi phase system ok. This is strictly not a multi phase flow class. This is just to give you some birds eye view about what are the different multi phase problems; typical multi phase problems involving droplets can be.

Thank you.