Convective Heat Transfer
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Lecture – 55
Turbulence – Tutorial

So, in this particular class, we are going to see about five problems in turbulence which will give you an idea that how to solve different problems these are simple problems which does not require you to memorize a whole lot of things ok, but ok. So, let us look at the first problem.

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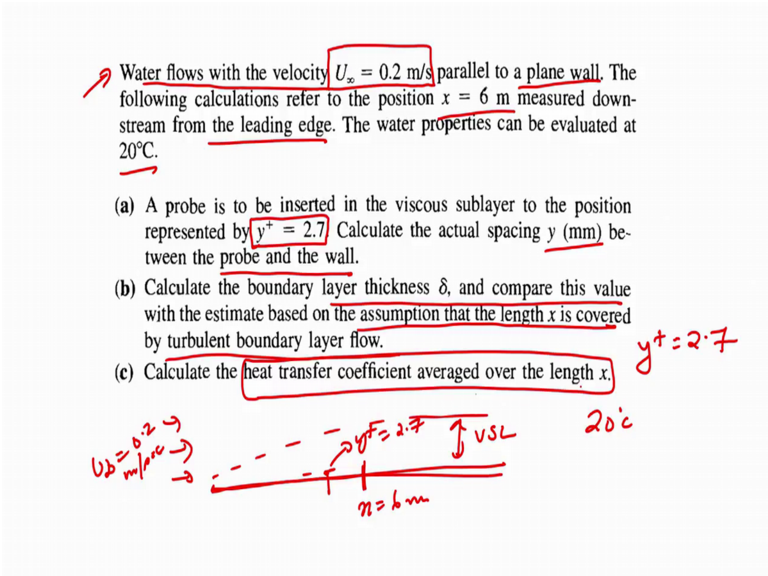

The first problem what does it say is that the water flows if you look at the ppt. So, the water flows with a velocity of U infinity equal to 0.2 meter per second parallel to the plane wall, ok. So, the following calculations refer to the position x equal to 6 meter measured downstream from the leading edge the water properties can be evaluated at 20 degree Celsius.

So, is basically water which is flowing over a flat plate parallel to a plain wall this is water U infinity is about 0.2 meter per second, calculations are carried at x equal to 6 meter, from the leading edge. So, this is the leading edge. So, water properties can be evaluated at 20 degree Celsius. So, what we are trying to do is that we are trying to insert a probe in the viscous sub layer to the position represented by y plus equal to 2.7, if you recall the definition y plus equal to 2.7. Calculate the actual spacing y between the probe and the wall. So, you have placed the probe at a distance which is given as y plus equal to 2.7. We want to know the physical distance all right, we want to know the physical distance and this probe is placed in the what we call the viscous sub layer the VSL, ok. So, that is the first problem that what is the actual spacing.

Next part is calculate the boundary layer thickness and compare the value based on the assumption that x is covered by turbulent boundary layer flow and calculate the heat transfer coefficient which is averaged over the length x. So, this is the problem that we are dealing with here. So, let us see how this problem can be solved.

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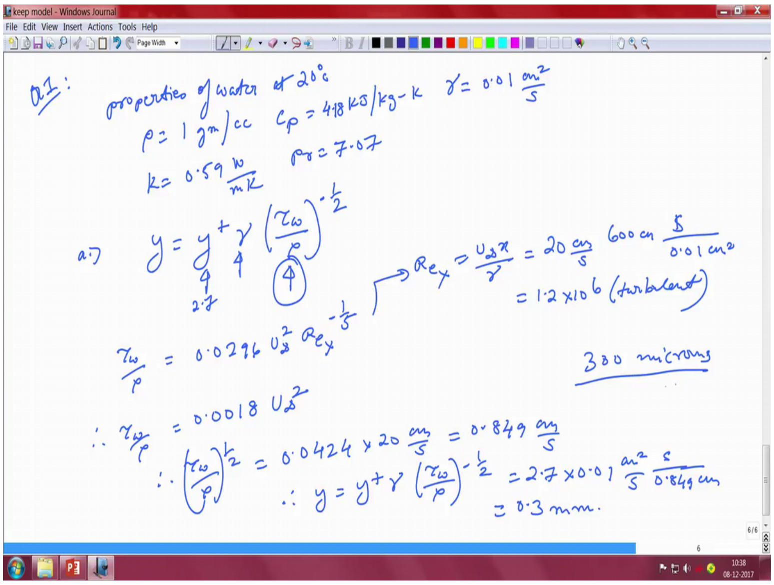

So, question one ok. So, we can first take the properties of water of water at 20 degree Celsius. So, rho is 1 gram per cc cp is equal to 4.1 one eight kilo joules per kg k you have to take care of the units whatever unit you use you have to just use them properly this is the kinematic viscosity k is 0.59 watt per meter Kelvin and Prandtl number is about 7.07 that is a Prandtl number for water. So, in order to calculate y, so, the first part of the problem is to calculate y. So, we know y is given plus y plus into gamma tau wall divided by rho raised to the power of minus half all right, this is the definition that we know from the during the course of our last lectures.

So, you have to know what about this we already know this is 2.7, and we already know the value of this all right it is given there. So, what will be the value of tau wall by rho ?

So, tau wall by rho therefore, is given from our correlations is 2.296 into U infinity square Reynolds number x to the power of minus one fifth in this particular term Reynolds number x is given as U infinity into x by gamma.

So, therefore, if you start substituting U infinity is about 20 centimeter. So, let us put it at centimeter per second, 600 centimeter is the wall length. So, we are converting it into one particular unit and gamma is basically the kinematic viscosity which is 0.1 centimeters square and second on the top. So, this gives you about 1.2 into 10 to the power of 6 which is basically a turbulent flow all right it is basically a turbulent flow. So, therefore, your tau wall the rho is given as 0.0018 ok, if you convert all the terms U infinity squared. So, therefore, tau wall rho raised to the power of half is about 0.4024 into 20 centimeter per second which is 0.849 centimeter per second.

So, therefore, your y is equal to y plus gamma tau wall by rho raised to the power of half. So, this is actually given as 2.7 in to 0.01 centimeter square per second, second divided by 0.849 centimeter. So, this gives you 0.3 millimeter. So, the physical distance y where you have inserted the probe is about 0.3 mm which is about 300 microns. 100 micron is the diameter of your hair, so, it is about 3 times the thickness of your hair where this probe is inserted just to give you a feel.

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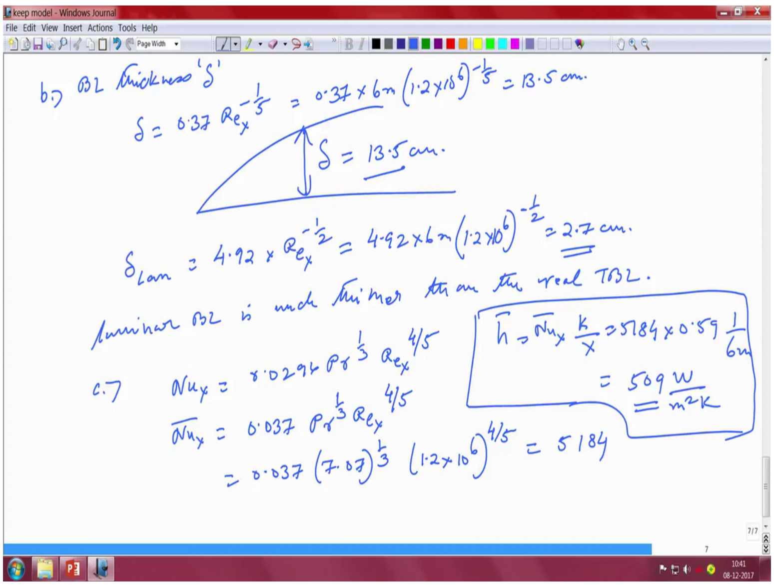

So, similarly now the boundary layer thickness which is delta ok, can be evaluated as this is 0.37 if you recall Reynolds number x to the power of minus one fifth. So, this is 0.37

into 6 meter into 1.2 into 10 to the power of 6 raised to the power of minus one fifth, this gives you about 13.5 centimeter. So, that is the thickness. So, you can put this in perspective the total thickness of this boundary layer is about 13.5 centimeter quite a bit.

Now, in the laminar region, so, if you calculate the laminar region for this same thing if you recall it was 4.92 into 10 to the in into Reynolds number x to the power of minus half. So, it is a much much stronger dependence so, 4.92 into 6 meter into 1.2 into 10 to the power of 6 raised to the power of half. So, it is basically becomes 2 2.7 centimeter because Reynolds number is in the denominator all right. So, it is as much big it is the same number, but it has been raised to the power of half that is raised to the power of one fifth. So, basically the boundary layer thickness blows up all right because it is a smaller number in the denominator effectively.

So, you can compare one is 13.5, one is 2.7, ok. So, that is the extent that the laminar boundary layer is much thinner boundary layer is much thinner then the real then the real turbulent boundary layer. So, if you use the laminar boundary layer correlations you will be way off that is the whole point that we are trying to make.

So, the Nusselt number on the other hand is given as Nusselt number is given as 0.0296 Prandtl number one third Reynolds number 4 by 5 .

So, Nusselt number x bar is 0.037 Prandtl number one third Reynolds number 4 by fifth 0.037 7.07 to the power of one third 1.2 into 10 to the power of 6 to the power of 4 by 5 this gives you about 5184 Prandtl number. What is your heat transfer coefficient h bar is equal to Nusselt number x bar divided by k by x, all right. So, this gives you about 5184, into 0.59 remember we calculated k earlier, divided by 1 over 6 meter this actually if you take care of the units it would be 5 O 9 watt per meter square Kelvin, got it. So, that is 5 O 9 that is what you get from your from your relation.

So, this completes the first problem which shows that how to for example, use existing correlations and find out these are not correlations per say, but these were problems that we relations that we established earlier and now we are able to use it for solving a real problem ok.

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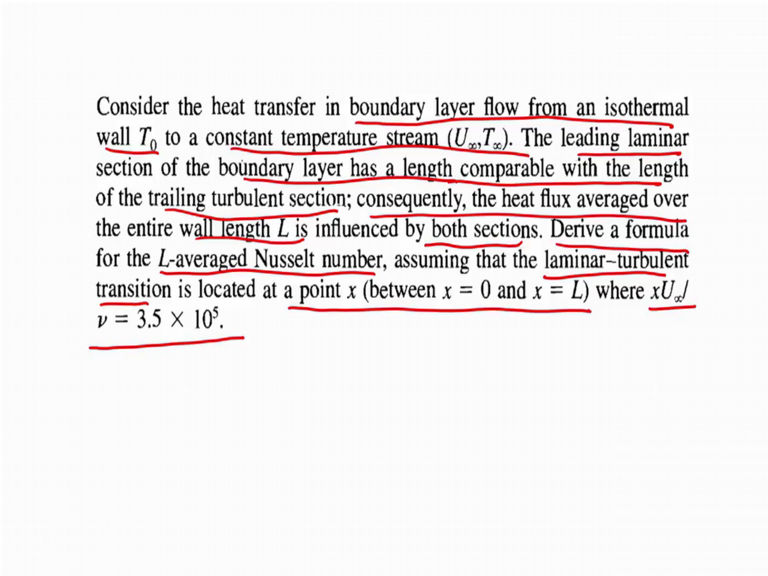

Consider the heat transfer in boundary layer flow from an isothermal wall T_0 to a constant temperature stream (U_∞, T_∞). The leading laminar section of the boundary layer has a length comparable with the length of the trailing turbulent section; consequently, the heat flux averaged over the entire wall length L is influenced by both sections. Derive a formula for the L-averaged Nusselt number, assuming that the laminar–turbulent transition is located at a point x (between $x = 0$ and $x = L$) where $xU_\infty/\nu = 3.5 \times 10^5$.

So, if you look at the next problem now so, you consider the heat transfer in the boundary layer from an isothermal wall to a constant temperature T stream all right the leading laminar section of the boundary layer. So, it has got a laminar section and then it has got a turbulent section has a length comparable with the length of the trailing turbulent section.

So, both are kind of comparable you cannot neglect one for the other consequently the heat flux averaged over the entire wall length is influenced by both sections. So, our idea is to derive a formula for the L averaged Nusselt number assuming that laminar turbulent transition is located at a point x which is between x equal to 0 and x equal to L and where x into U infinity divided by gamma is 3.5 into 10 to the power of 5, ok.

So, that is the problem. So, it has got a laminar section and it has got a turbulent section. So, now, let us attack this problem.

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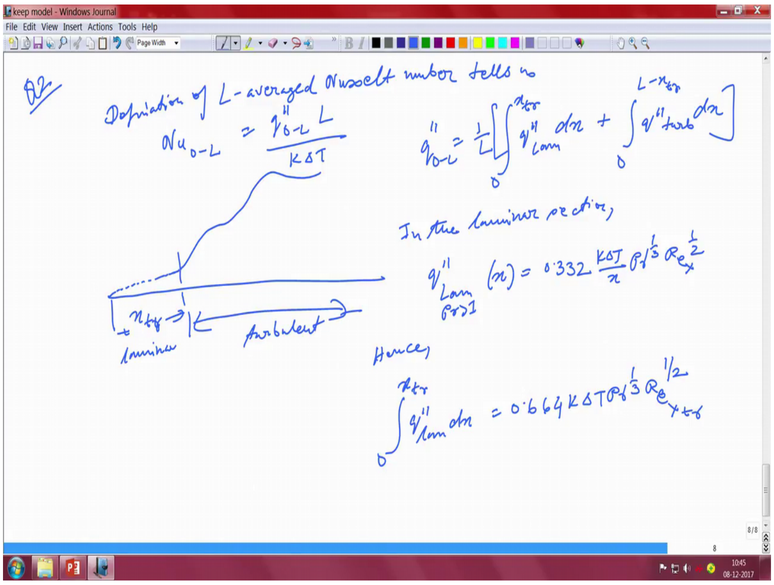

Question 2, ok. So, the definition definition of L averaged Nusselt number tells us Nusselt number 0 to L k into delta T. So, if you look at the problem now there is a section in which it is kind of like this and then you have this particular thing. So, up to this is basically the x transition that point and so, this part is your laminar and beyond this it is basically turbulent all right, beyond this it is basically turbulent.

So, your q, 0 to L double prime 1 over L is basically a sum total from 0 to x transition this is q laminar into dx plus 0 to L minus x t r q double prime turbulent into dx, ok. So, in the laminar section when you write as q laminar for Prandtl number greater than 1 at any point x, if you recall your old stuff it will be this is an isothermal plate, so, this is 0.332 k into delta T by x Prandtl number one third Reynolds number half, ok. So, hence 0 to x t r laminar dx gives you 0.664 k delta T Prandtl number one third, Reynolds number up to the transition is about half ok. So, for the turbulent section we do the same.

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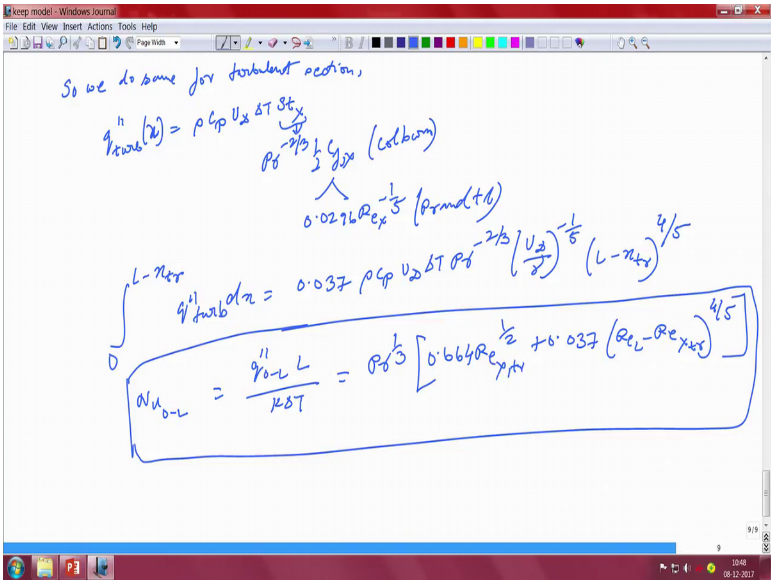

Do the same for the turbulent section for turbulence section so, q turbulent double prime x rho C p U infinity delta T Stanton number x, ok. So, now this particular expression if you look at the different types of expressions for this ok, you have of course, there is the Prandtl two third half C f x thing which is once again this is given as 0.0296 into Reynolds to the power of minus one fifth is Prandtl if you recall this is basically Colburn once again if you recall ok. So, this is the not this whole thing, but just a standard number part ok. So, 0 to L minus x T r q turbulence dx is equal to 0.037 rho C p U infinity delta T Prandtl number minus 2 third U infinity by gamma to the power of minus one fifth, L minus x t r 4 by 5 ok.

So, in conclusion your Nusselt number from 0 to L is basically given as q 0 to L double prime L by k into delta T which will be Prandtl one third 0.664 Reynolds transition to the power of half plus 0.037 Reynolds number L minus Reynolds number x t r to the power of 4 fifth, all right. So, this gives you the total expression for your Nusselt number.

So, it is basically an addition of two quantities basically one is the under Reynolds number part one is the laminar part and one is the turbulence part. So, that is a very simple way of and we have used two additional relationships one is the C f x which is Prandtl and one is basically the Stanton number relationship which is from Colburns analogy, all right.

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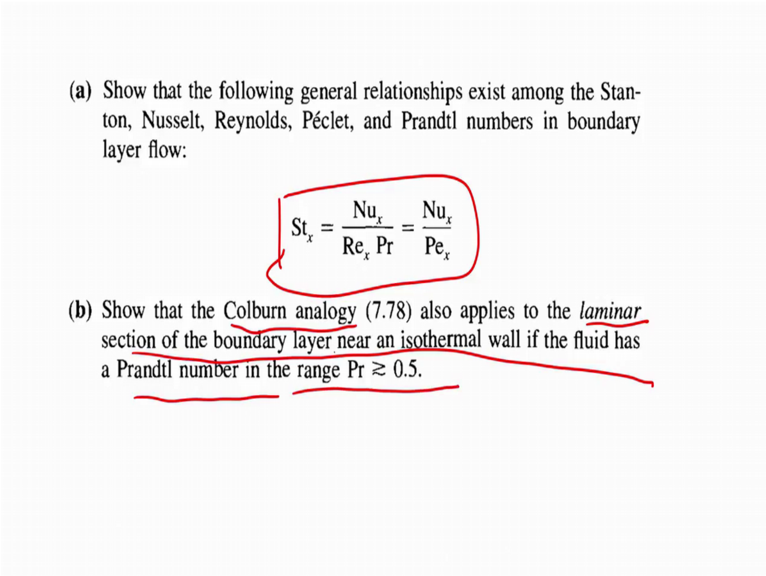

So, let us look at the next problem. This is this problem is even simpler. So, you can see that the general relationship that exist between Reynolds number and Prandtl number is given by this this we already did multiple times. So, it will be a nice thing to work it out once again and show that the Colburn analogy also applies to the laminar section of the boundary layer for an isothermal plate if the fluid has got a Prandtl number which is greater than 0.5. So, far has been we have been doing it for turbulent flow only let us look at it that whether Colburn analogy is also applicable for laminar flow.

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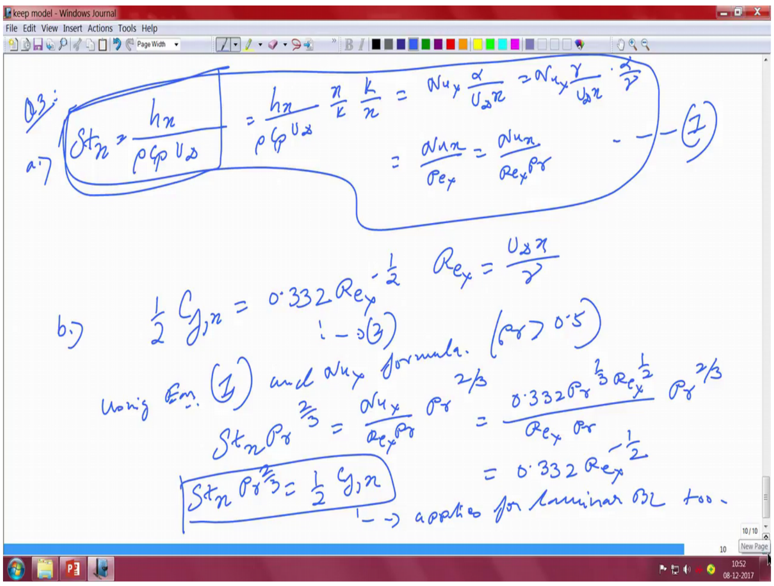

So, question 3 this is easy enough question. So, your Stanton number if you write it from the definition is h x by rho C p into U infinity. So, if you do it h x divided by rho C p U infinity x by k into k by x ok. So, this will be Nusselt number x into alpha by U infinity into x which will be Nusselt number x into gamma by U infinity into x divided by alpha by gamma. So, this will further lead to Nusselt number by Peclet number which will give you Nusselt number divided by Reynolds number into Prandtl number it is very simple math. So, if you show it this is how this thing was done just starting from the basic definition of your Stanton number.

So, the b part of the question. So, this is the a part ok. So, the Colburn analogy is 1 by C f x is equal to 0.332 to establish the Colburn analogy for the laminar, let us write the C f x term also for the for the laminar boundary layer where your Reynolds number x is given as U infinity into x by gamma all right. So, the left side can be estimated of this Colburn analogy can be estimated using this first form of the equation which is basically what we did here, ok. So, this is the second ok. So, using equation – I and the N U x formula formula greater for Prandtl number greater than 0.5 we get Stanton number x Prandtl number two third is equal to Nusselt number x Reynolds number x Prandtl number Prandtl number two third.

So, now, we can substitute for the C f x part. So, this is Prandtl number one third Reynolds number half divided by Reynolds number into Prandtl number Prandtl number two third. So, where using the Stanton number expression, ok. So, this once again gives rise to 0.332 Reynolds number to the power of minus half see in other words this clearly shows that Stanton number Prandtl number to the power of two third is equal to half C f x this is applicable also for laminar applies for laminar boundary layer too interesting revelation all right, it is applicable for laminar boundary layer too, got it, ok.

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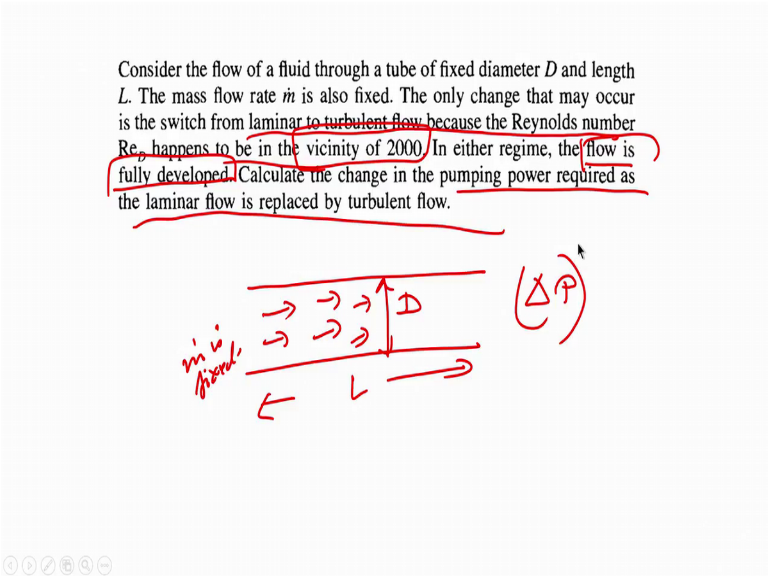

Now, so, that takes care of the third problem. The fourth problem; so, there is a fluid that is flowing through a tube which has got a fixed diameter of about D there is a fluid flow through the tube it has got a diameter of about D and length is about L m dot is fixed the only change that can happen from laminar to turbulent flow because Reynolds number happens to be in the vicinity of 2000, there is for internal fluid flow that is the transition Reynolds number in either regime the flow is fully developed.

So, this is an important assumption that is given. Calculate the change in pumping power required as the laminar flow is replaced by the turbulent flow. Pumping power is basically given by the how much pressure that you need to put and correspondingly how much mass you are pumping because of that ok. So, let us look at this problem, it is a simple problem.

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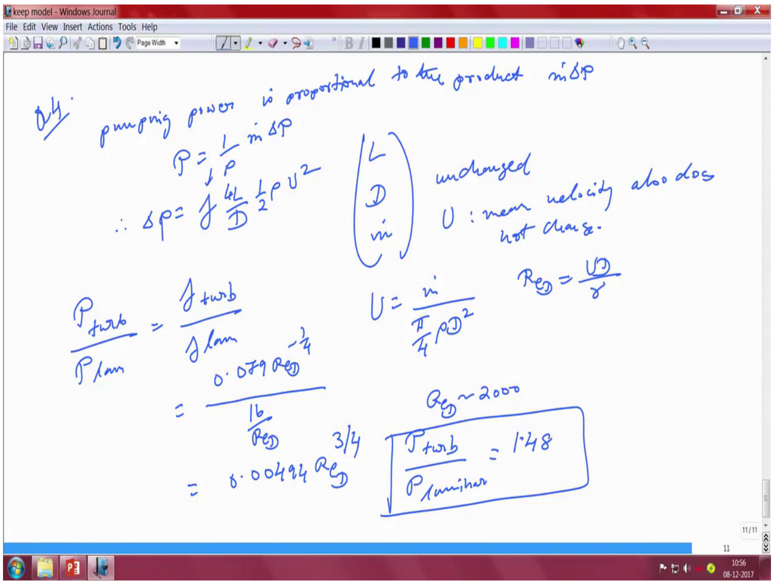

So, question 4, so, the pumping power is proportional basically proportional to the product m dot delta P. So, P is basically equal to 1 by rho m dot delta P, where your delta P is the one that actually is the, is a key player because it is f by 4 L by D into half rho U square this we know, where f is the friction factor.

Now, all these other terms this L, D, U and the m dot these are kind of unchanged all right, these are unchanged, ok. So, the mean velocity also does not change U which is the mean velocity also does not change ok. So, U is basically what mean velocity is m dot by pi by 4 rho D square where your Reynolds number D is given as U D by gamma all right. So, the only thing that can change is the friction factor all right, ok.

So, the pumping power turbulent divided by the pumping power laminar is basically given as f of the turbulence which is basically the friction factor and f of laminar because rest of the things are kind of the same. So, for the turbulent friction factor it is given as 0.079 to Reynolds number D to the power of minus one fourth, if you recall your notes and for laminar it is 16 by R e D.

So, the total thing that we get is 0.00494 into Reynolds number D to the power of 3 fourth. If the Reynolds number is of the order of roughly about 2000, ok. The P turbulent by P laminar these are not pressure these are basically the pumping power is about 1.48. So, the pumping power of the turbulent flow or the pumping power to maintain the turbulent flow is about 1.48 than the corresponding laminar counterpart all right.

So, this is one of the most important another interesting short problem and we have done.

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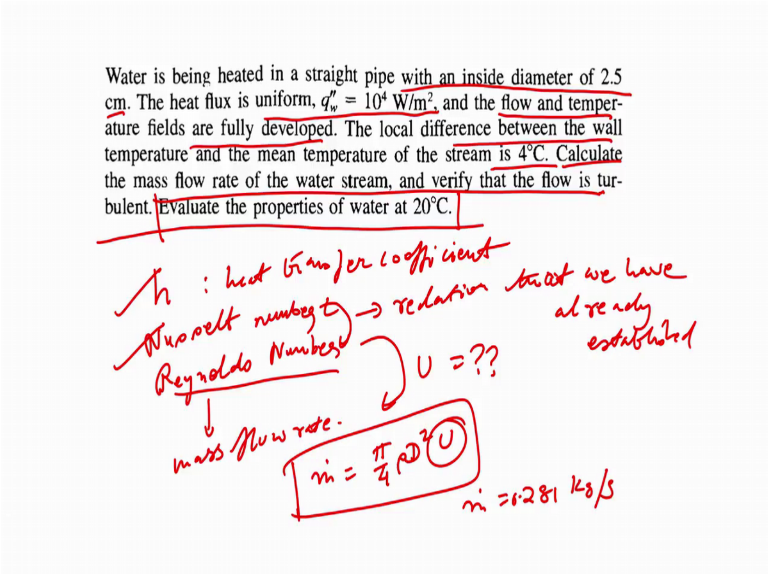

The last problem in this particular series is that water is being heated in a straight pipe with an inside diameter of 2.5 centimeter. The heat flux is uniform and the flow and the temperature fields are fully developed. The local difference between the wall temperature and the mean temperature is about 4 degree Celsius. Calculate the mass flow rate of the water stream and verify that the flow is turbulent evaluate the water properties at about 20 degree Celsius, all right.

So, in order to do this ok, so, I am not going to solve this problem in total, but what I can give you are some hints. So, that you can work it out it is not a homework problem per say, but this is some problem that will kind of give you and we will post the solution a little later ok, but you should know how this problem actually works. So, in order to calculate, ok, I am showing you the steps h needs to be calculated first which is the heat transfer coefficient that needs to be calculated first, then you need to calculate the Nusselt number, the Reynolds number and lastly the mass flow rate.

So, Nusselt number, Reynolds number and the heat transfer coefficient all can be calculated using the standard correlations and the data that is given over there. So, once you calculate the Reynolds number you can calculate U and from U you can calculate what will be going to be the mass flow rate and check what will be the velocity of U.

From U you can find out what will be the mass flow rate because m dot is nothing, but one forth rho D square into U.

So, you can once you know U you can calculate. So, for knowing U you need to calculate the Reynolds number all right and for knowing the Reynolds number you need to know the heat transfer coefficient and a Nusselt number. Nusselt number and Reynolds number are connected through the correlation or the correlation by the relation that we established that we have already established. Do this problem and try to see if you get a good match I can give you the answer m dot is about 0.281 kg per second. Try to see if you can match that ok.

So, with this we finish basically our turbulence small session in which we have shown you how to work out certain problems some numerical problems, and it will also give you an idea how to use the relationships why and how they should be used. Also give you a good idea about how the turbulence physics is in the limited time that was possible and what turbulent convection is all about, ok.

So, in the next class, we will look at some of the other problems in the series which is basically going to deal with droplets, sessile droplets and normal contact free droplet us and try to see that how we one should evolve a methodology for solving those problems.

Thank you.