**Convective Heat Transfer**
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**Lecture - 54**
**K – ε model**

So, in the, in the last class, we started doing about this control volume approach and we talked about the K epsilon model.

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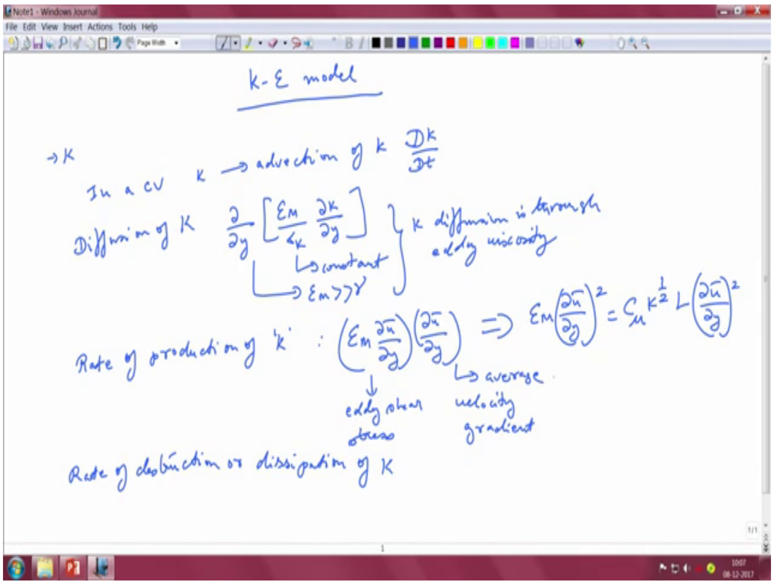

So, K an epsilon model and we defined what K was which is basically the turbulent kinetic energy right. So, in a control volume, we had to write an equation for K basically. K was defined as the advection of K, advection or convection of K was defined as d K by d t then there is diffusion of K which is given by this term. This is basically a constant as you will find that, there are many constants of proportionality that arises not naturally and you are taking that epsilon m is much much greater than your kinematic viscosity ok.

So, that it otherwise it implies that K diffusion is through ok; eddy viscosity only. Once again remember this is not a viscosity per say; this is some way that we have cast it like a viscosity. Then like any transport equation, you will have a rate of production of k which is given by epsilon m d u bar by d y ok. So, this is basically what we call the eddy shear stress and this one is the average velocity gradient ok. So, this is also can be written as epsilon m into d u bar by d y square or in other words if you remember that epsilon m, we already wrote it in terms of k to the power of half L du bar by d y whole square. So, that was the rate of production. Now similarly, there is something called the rate of destruction or dissipation or dissipation of K dissipation of K. So, this is this is akin, the rate of dissipation of K is akin to something like this. So, let us go to the next page to explain this in a little bit more details ok. The dissipation rate is called it is like a you know.

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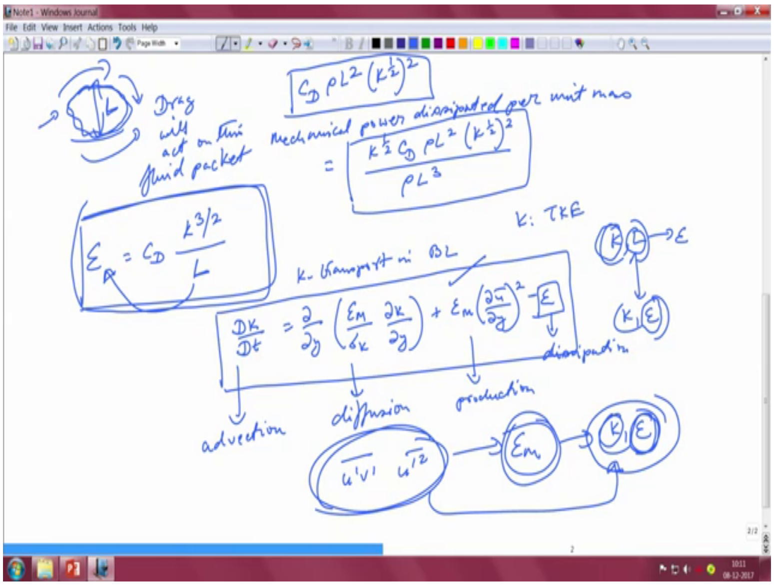

When you actually have a fluid packet of some length scale L right ok, of some length scale L ok, this is there will be some drag and other things that will act on this fluid package on this fluid packet ok.

So, in other words that drag force will be given by something like C D rho L square into K to the power of half square something like that ok. So, in other words ah, the mechanical power that is dissipated, power dissipated per unit mass is given as K to the power of half C D rho L square K half square divided by rho L cube ok.

So, the dissipation itself should be written as something like C D K to the power of 3 by 2 divided by L. So, that is the dissipation. So, how we explain the dissipation? Dissipation like is any pack, any pack of fluid which is L is acted upon by the viscous drag that acts about it. So, this is any blob of fluid which has got a length scale of L. So, the total drag force is given something like that and the total mechanical power therefore can be written like this.

So, the dissipation will be this particular term. So, essentially what happens is that, you have the. So, this L which we introduced in the last class as one of the variables which like was like the mean free path ok. So, this L now is related to the dissipation epsilon. So, that is an important thing that we learn out of this. So, the K transport equation therefore, in the boundary layer and therefore becomes if you collect all the terms together, it will be d K by d t ok. So, that is the total K transport equation in the boundary layer ok. So, this is the dissipation, production, diffusion, advection.

So, you get a total species transport or like a transport equation of the turbulent kinetic energy K; K is the turbulent kinetic energy, TKE where we get all these terms ok. So, this dissipation is therefore, some kind of an unknown factor over here. So, because remember we need an equation for K, we need an equation for L. We showed that this can be reduced to an equation for K and epsilon right because L is basically related to the epsilon which is basically the dissipation.

So, in order to connect these two quantities that basically we now have an equation for K, now we need another equation for epsilon basically for the dissipation basically to close the Navier Stokes equation ok. Just recall that we had terms like this and this which we somehow packed it up into epsilon M, then we kind of try to put the same thing in terms of two other quantities.

So, that is the transformation. So, basically this is an unclosed, this was unclosed. Now these two are unclosed; out of that one, we have kind of closed it by writing an equation for K right. Then the other term now has to be closed because we need an equation for actually the dissipation. So, let us write an equation for the dissipation also.

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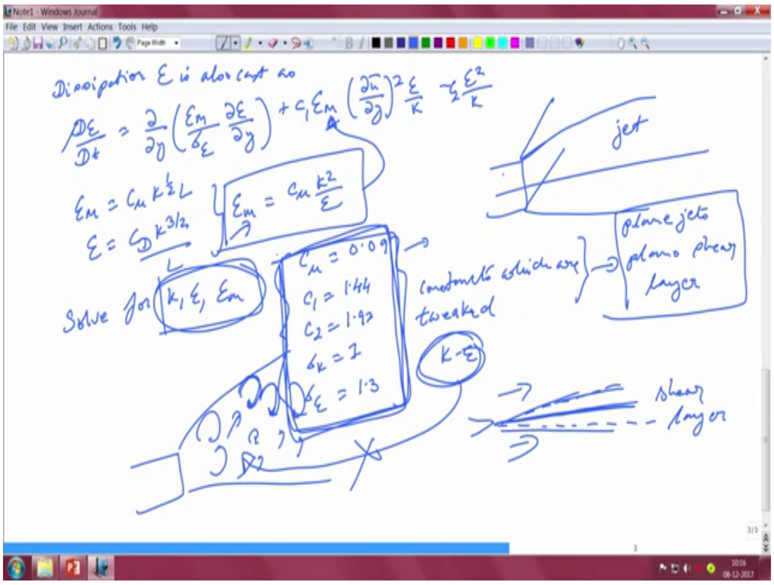


So, dissipation epsilon is also cast as. So, you see a lot of constants which are kind of floating around ok. So, E m is basically C mu K to the power of half into l epsilon is equal to C D K to the power of 3 by 2 by L. This leads to epsilon m should be equal to C mu K square by epsilon.

So, you can see this is now reduced to this particular form ok. Now what the equation boils down. You now solve for K, epsilon and epsilon m; basically where epsilon m is related to K and epsilon. Normally, if you look at any commercial software like fluent and if you do the epsilon k epsilon modeling, you will find that C mu will be given something like 0.9, C 1 will be 1.4 4, C 2 will be 1.92, sigma K will be equal to 1 and sigma epsilon will be equal to 1.3. These are all constants which are tweaked ok.

So, they do actually a good job if you actually deal with plane jets or with plano shear layer ok. So, basically what people do is that they basically hand tune this constants. So, plane jets means say a jet coming out of 2 dimensional orifice right. So, that is like a plane jet right ok, flat sleet actually, not an orifice. So, that will be the kind of a plane jet then plane shear layer will be if you have a splitter plane and there are two flows on both sides right ok. You will get this kind of a shear layer which is actually developed right. So, this is called a plane shear layer ok.

So, in these kinds of very canonical flows so, this is a jet, this is a shear layer, this is a mixing problem essentially ok. The k epsilon model does a fabulous job in basically finding out certain statistics right about the flow field but again, remember that when you did an average of the Navier Stokes equation right and when you modeled these terms using K epsilon and things like that or prandle mixing length model, you are basically not resolving any of the eddy scales at all right. So, when we started with turbulence, let me show you this particular plane jet.

So, you have got lots of these eddies of different sizes right ok. So, if somebody asks you the question that using K epsilon model can you predict that how these Eddies are going to grow, how they are going to break up into other Eddies ok, what are the nature of these Eddies and things like that. Those are the questions you cannot answer using this because you are not resolving anything at all right. You are operating on a mean flow field and all the fluctuating and the Eddy part of the problem. You are basically hiding it into this k and epsilon equation right.

So, it does not do any good job at all ok. If you want a more insightful discussion about the nature of the turbulent flow, but however, it can give you good answers regarding; for example, you want to know the shear growth rate or you want to know certain statistics about the flow field that it can do a reasonable job but that also you have to hand tune this constants to meet a particular situation.

So, what normally people do will be that they will you know first do experiments. They will park their model, they will find out these constants and then they will apply the model around the experimental data space right but for example, you cannot use it for example, in a in a flow over an aerofoil for example, ok. So, you cannot use it blindly because these equations are all done with these specific sets of constants ok. So, you have to be very careful when you are using this kind of turbulence model they are not universal in nature at all ok.

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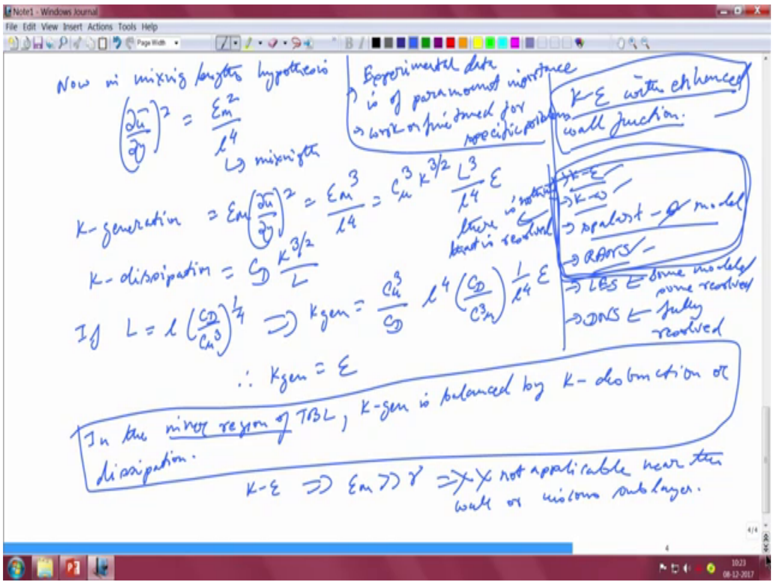

Now in mixing length hypothesis, if you recall your d u by d y square was given as epsilon m square by l to the power of 4 where l was the mixing length ok.

So, the K generation term, if we look at the generation that is epsilon into d u by d y square right, which is epsilon M cube by l 4 which is C mu cube K to the power of 3 by 2, L cube divided by l 4 into epsilon and the k dissipation will be equal to C D K to the power of 3 by 2 by L. And if L, if capital L is equal to small l into C D by C mu cube to the power of one fourth, you can write the K generation term as C mu cube divided by C D l to the power of 4, C D by C cube mu, 1 by l to the power of 4 into epsilon ok. So, therefore, K generation will be equal to K epsilon ok.

So, in the inner region, in the inner region of turbulent boundary layer, if you recall what the inner region was all right; so the K generation is balanced by K destruction or dissipation right so and so, this is one of the most important thing that you can take from here in the inner region only ok. So, the k epsilon model as we said also implies that epsilon M is much much greater than gamma; so therefore, it is not applicable, applicable ok, near the wall, near the wall or viscous sub layer ok. So, what do people do? They use K epsilon with; you will hear this kind of models with enhanced wall function because you cannot predict the viscous sub layer with the K epsilon model.

So, you will use K epsilon with enhanced wall function and similar stuff like this ok. So, that would be the way that people approach most of the problems that K epsilon model if you want to do it ok, you use it with enhanced wall function. So, there will be lots of models K epsilon, then there will be K omega ok. There are splattered alarmers model, then of course, there comes the RANS set of models which is 5 equation models essentially, then you have the LES and the DNS. DNS is of course fully resolved. LES is some is modeled, some modeled some resolved of all these models ok, there is no, there is nothing that is resolved actually ok. So, interesting or not, so whatever models that you are using and Prandtl mixing length is one of one above this. These are non algebraic models because basically, you are writing the equations for K an epsilon ok.

The equations are exact, but how you write the equations and the constants that you used that depends on the model argument that you are trying to put forward. So, K epsilon, K omega, splattered alarmers and then RANS, these are all the different types of models that you will find different industries are going to use. You cannot say one is better than the other. Two equations, then 5 equations it does not mean that the results become any more sophisticated.

T just gives you more constants to basically fine tune your answer. Basically, it is like a like a control panel where you can have different knobs and you can basically turn the knobs to match your results to it with whatever that you are expecting ok. So, all of these things requires experimental backing; that means, you need to validate it using very stringent experiments. If you do not use experiments, then none of these models are actually good and it has to match within a certain experimental space ok.

So, no matter what you use; you use splatter alarmers or K epsilon or K omega or RANS, it does not really matter. You have to use all the models with a pinch of salt basically that you have to know that how this models actually works. So, experimental backing or experimental data is of paramount importance for all these models and they work for specific scenarios, work or for our fine tuned, fine tuned for specific problems, specific problems. So, do not think that they can be used universally ok. So, there are no universal models as such ok.

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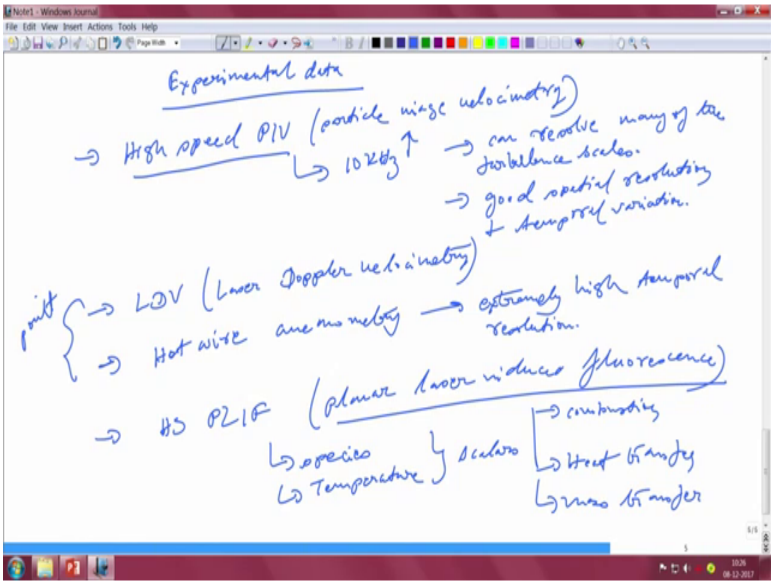


And when you are talking about experimental data, so you will find and we this is of course not experimental measurements technique course.

So, experimental data how they can be earned is that these days we have high speed particle image velocimetry. I mean these are the different types of measurements that can help you, velocimetry. So, if you take a measurement course. So, this can actually go 10 kilohertz and higher. So, you can resolve ok, can resolve many of the turbulent scales, turbulent scales. So, that is good ok. The advent of cameras also enables you to have both good spatial; spatial means the pixels of the camera is essentially what the spatial resolution is all about. So, good spatial resolution and plus temporal resolution. So, you have high speed cameras these days which can get you very high temporal resolution also.

So, high speed PIV, then there are systems like LDV; Laser Doppler Velocimetry, Laser Doppler Velocimetry, which can also measure. Then you have something called Hotwire anemometry. Both of these are point measurement techniques, point measurement techniques. So, this can give you extremely high, extremely high temporal resolution which is good ok. You can get extremely high temporally. All these things are commercially available. So, that is a good part; that means, these days the experimental measurement techniques has improved so much, all of these now can be used as data for validating many of the turbulent models ok; the efficacy of the different turbulent models ah, even for alias measurements and things like that all this can be used ok. So, that is an interesting tool all right.

So and even for species, we have these days high speed PLIF which is Planar Laser Induced Fluorescence, laser induced fluorescence. So, this can also go up to a high speed these days which can be used for species and in some cases temperature too. So, scalars basically, so that also makes the job very important for heat transfer as well as for combustion and these applications; applicability in combustion and in heat transfer, mass transfer too. So, this enables us to once again validate some of the experimental data, some of the experimental data is used for validation of different ah models.

So, in a nutshell, these are the different techniques about which you can read, but in a nutshell that we are now finishing up the turbulence part of the course that we have given you a very ah birds eye view of the turbulence spectrum and we have said a few things which you should take like a gospel. First and foremost in turbulence, it is basically an initial condition and a boundary condition problem that is where the all the all the complications arises and it is the nature of the Navier Stokes equation or basically the nonlinearities of the Navier Stokes equation to amplify small differences and basically to blow them up ok.

So and turbulence is also a length scale issue that is because of the disparity of the length scale between the smallest Eddies and the largest Eddies, as well as the time scales that are associated with them. It makes very very difficult for any complicated flow problem to be or heat transfer problems to be solved by solving the full Navier Stokes equation; that means, the DNS class of solutions that we are talking about is very very complicated ah, even for very simple geometries. Though people have done it with the advent of modern supercomputers, you can actually solve many of these problems using DNS.

But that is very limited and for complicated geometries, particularly in problems where there are lot of other things say multi phase combustion, multi phase flows and things like that. You not only have to take care of the flow field but also of the species and the chemical kinetics and the mass transfer, heat transfer, such a whole gamut of things and they are inter coupled with each other at various length scales and time scales. So, that makes the job very very difficult ok. And it might take a lot of time basically to solve those equations and that is not physically a very interesting thing. That is because industries or whoever wants to use these data they cannot do that it is simply because it takes too much amount of time. It is more than the lifecycle of their product probably.

So, in those cases people have sought to understand the problem more from a modeling, from an ad hoc point of view and that is what we covered throughout the class. That what is Eddy viscosity originates from the non-linear terms but people have thought why not couple it as a viscous term, coin it as a viscous term and still be able to answer some of the basic questions like what is a wall shear stress, what is the wall heat flux, what is the heat transfer coefficients and stuff like that how the laminar sub layer will look like.

So, it is basically makes people more engineers and they have put some thought process that there may be a laminar sub layer, maybe there is a turbulent sub there and maybe there is this full blown turbulence outer region and you can do simple scaling arguments in these two layers and try to find out still that what is the heat and mass transfer rates and things like that and people have used analogies which we talked about like colburn analogy, linking heat and mass transfer with the skin friction coefficient.

So, all these things I have been able to give you some answer and they match with some of the experimental data which is the good part but it does not give you a complete understanding and it is very limited to a particular type of application. So, it is very applicable to for very canonical problems. You cannot universally apply it for any problems that you want ok.

And we also saw that if we make the process a little bit harder, we can go to this k epsilon suit of equations which are no better no words essentially but it just gives you a little bit more flexibility in playing with your constants and basically parking your simulation to a particular experimental data set. That is how these things actually proceed and we are able to get some answers like this heat transfer coefficient and skin friction which is relevant to the industry because that is all that they want to do ok.

So, at this point we finished the turbulence. In the next class, we are going to look at some of the problems like some kind of a homework problem in turbulence and that will wrap up the entire turbulence series and then we will look at more interesting ah, I mean give a little bit of introduction to some multi phase problems in which convection also plays a very important role.

Thank you.