Convective Heat Transfer

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Lecture - 53
Turbulent internal flow – III

We saw that, the definition of this M, can be actually, it reduces to a much simpler quantity, if you assume that the actual temperature gradient, is not a function of r. Now next we see that, if for example, if your u bar, that you are considering is a slug profile.

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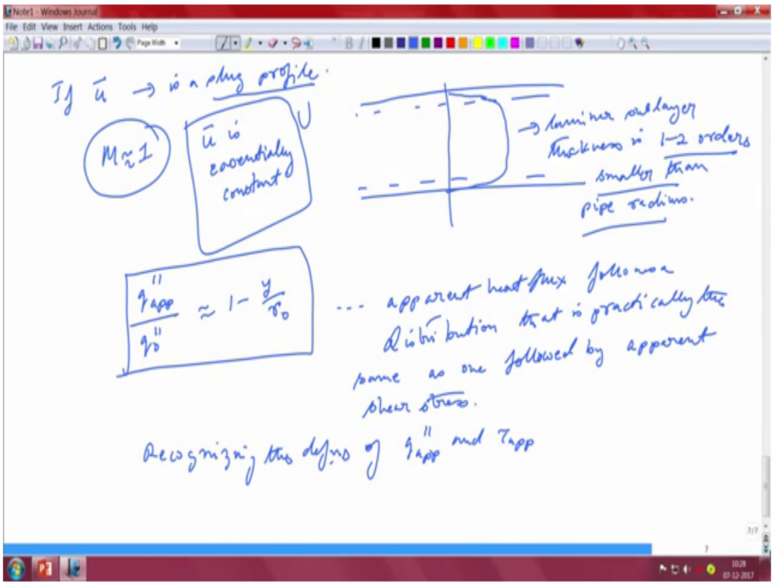

What was a slug profile? The slug profile is something like this, that your flow profile is something; it is a profile like that ok. So, where the laminar sub layer, sub layer thickness, thickness is 1 to 2 orders, ok, smaller, ok, than, the pipe radius, than the pipe radius ok.

So, it is a laminar sub layer thickness, which is 1 or 2 orders smaller than the pipe radius, ok. So, the slug profile means, it is essentially is a constant velocity profile. If you recall, we did a problem in your laminar boundary layer internal flow, where we did that a slug profile is given by a velocity u, right, which is kind of constant, we solved this earlier..

So, when it moment, we say that it is a slug profile and therefore, it is constant. Your M becomes of the value of 1 right, because u ok, is essentially, essentially constant ok, the u is therefore, essentially a constant over here right. So, this laminar sub layer is very small, it is 1 or 2 order smaller than the pipe radius ok. So, therefore, we can get away with this, assumption that u bar is practically a constant in this case. So, moment that happens ok, your q apparent double prime divided by q naught double prime becomes almost equal to 1 minus y over r naught ok.

Because that factor M is now, basically equal to 1. So, in other words, what we can say is that, the apparent heat flux, heat flux ok, follows distribution, distribution ok, that is practically the same ok, that is practically the same as one followed by the apparent, apparent shear stress. There is an important connotation that, it follows basically the same thing, as a shear stress, right, that you did earlier right. So, so, recognizing the definitions, the definitions of, q apparent double prime and your tau apparent ok, that yields gamma plus Ah, epsilon M divided by tau naught d u bar that is equal to C P alpha plus epsilon H divided by minus q naught double prime d T bar ok.

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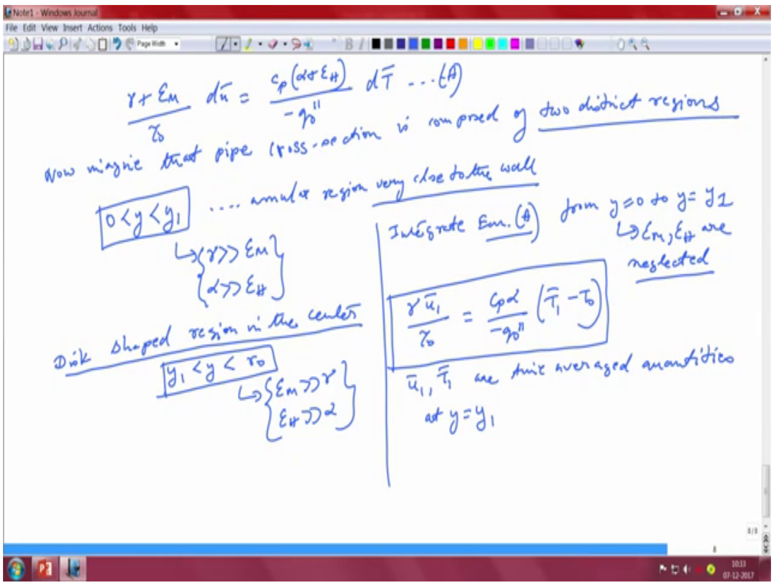

H So, that is the expression that you get, considering that both varies in a very similar way.

Now imagine, now, imagine, that the pipe, cross section is composed of, is composed of two distinct regions ok. So, it is composed of two distinct regions ok. So, in the annular region close to the wall so, one region is y greater than 0 and less than some quantity y 1, which is an annular region, region very close to the wall, the wall ok. So, annular region very close to the wall, where in this particular region, your gamma must be much much greater than your epsilon M ok, and alpha is much much greater than your epsilon H, these are the two things that are valid in that particular region right, in that region ok, and therefore, there is a disc shaped, shaped, region in the center..

So, this is, this is close to the wall, very close to the wall, this is towards the center ok, where your y plus is greater than y less than some r naught ok, where you have your, in this particular region your epsilon M is much much greater than gamma ok, and your epsilon H is much much greater than alpha.

So, this is the disc shaped region in the center, got it? So, this is the disc shaped region, at the center, this is the region which is very close to the wall. So, if you consider that, it is composed of two distinct zones, basically ok. So, what we can do is that, we can take, this equation, let us call this A, integrate equation A ok, integrate equation A, from y equal to 0 to y equal to y 1, ok.

So, in this integration, we are neglecting both alpha H as well as where, epsilon H and epsilon M are neglected, both of these two terms are basically neglected ok. So, therefore, if we do that then what we will get is gamma, u 1 bar divided by tau naught is equal to C P alpha divided by minus q naught double prime t 1 bar minus t naught ok. Let us try to see, what these quantities are. So, u 1 bar and your T 1 bar basically, are time averaged quantities, time averaged, quantities ok, at y equal to y 1 bar, at, at y equal to y 1, I am sorry.

So, this is the first expression, that you get, when you basically integrate equation A, from y equal to 0 to y equal to y 1, where epsilon M and epsilon M H are actually neglected ok. So, therefore, these are the time averaged quantities of the same variables, u and T at y equal to y 1 ok. So, this is the first expression that you get ok, from the near wall integration.

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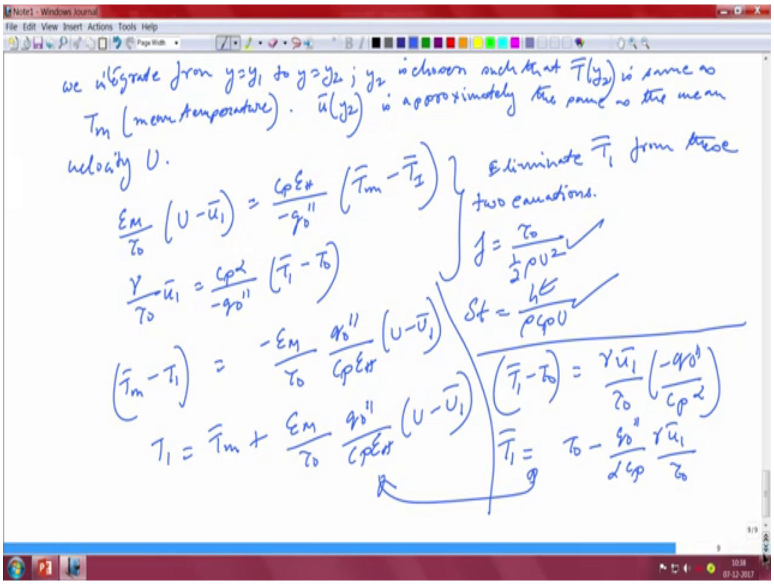

Now next step, we integrate, integrate from y equal to y 1 to y equal to y 2 kind of ok. So, this is chosen. So, the y 2 is chosen, chosen, ok, such that, that, T bar y 2; that means, the average temperature at y 2 ok, is equal is same as another mean temperature, which is T mean, basically the mean temperature, ok. So, equal to the mean temperature T, and u bar at y 2, ok, is approximately, the same as the mean velocity. Remember the mean velocity that we did, u, it is the same as a mean velocity Uok.

So, therefore, and here of course, we are neglecting the gamma, and the alpha, right, because, epsilon M is greater than both those quantities. So, when you integrate this, this becomes epsilon by tau naught U minus u 1 bar is equal to C P epsilon H divided by minus q naught double prime t M t M bar, whatever you call it T 1, right ok..

So, now, what you can do is that from this is the, this is the second equation. Let me write down the first equation for the benefit, tau naught u 1 bar is equal to C P alpha minus q naught double prime T 1 bar minus T naught. So, these are the two expressions that we have; so, you eliminate, eliminate t 1 bar from these two equations, ok, from these two equations, we eliminate the T 1 bar, and we use the definition of friction factor, friction factor, as we know is tau naught divided by half rho U bars U square right, and Stanton number definition.

So, use two non dimensional numbers now, h by rho C P into U, these were the definition of the two numbers alright.

So, if you eliminate T 1 from these two expressions ok. So, then basically what you can see is that, you can get a more revised form of this equation ok. So, you can do it like this, say for example, from the first equation T M minus T 1 T M minus T 1, you can do it like epsilon M by tau naught into q naught double prime divided by C P epsilon H into U minus U 1 bar ok. So, basically your T 1, will be equal to T M plus epsilon M by tau naught q naught double prime c p epsilon H into U by U 1. So, that will be from one expression, from the other expression you can get, so, this I am just doing a rough scheme, just to show you guys what happens.

So, T 1 minus T naught there, is equal to gamma U 1 bar by tau naught, q naught double prime divided by C P into alpha. So, T 1 bar is equal to, gamma or basically T naught minus q naught double prime alpha C P gamma u 1 divided by tau naught ok. So, now, it is very easy, you subtract one equation from the other ok, and then apply the definition of Stanton number, and the, heat transfer, and the and the definition of heat transfer coefficient, written in terms of the mean temperature, as we already know.

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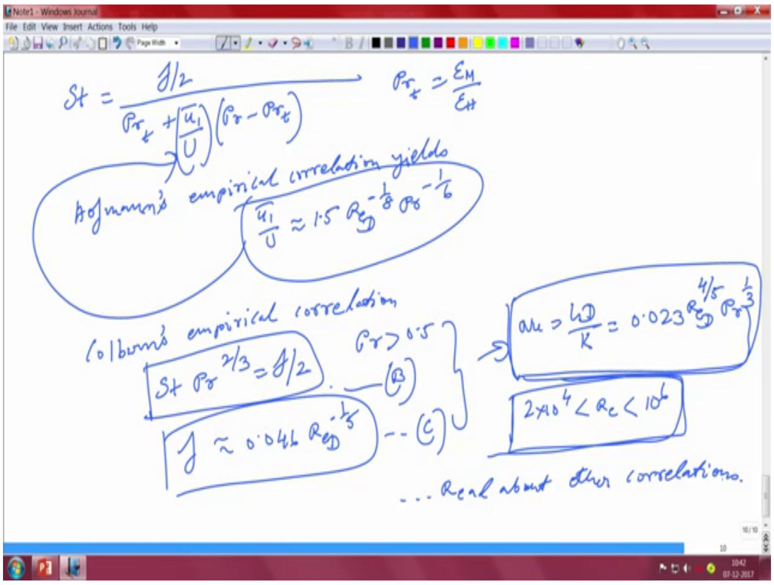

So, if you do that, do that part of the part, you will get that your Stanton number will now come out to be, f by 2 divided by Prandal number t plus u 1 by U Prandtl number minus Prandtl number t ok.

So, this turbulent Prandtl number, is basically, as we already know, this is epsilon by epsilon H, just in case you have forgotten it right ok. So, that is the, that is the whole thing. Now Hoffmann, Hoffmanns correlation empirical 1 second, correlation yields u 1 bar by U approximately equal to 1.5, Reynolds number D to the power of minus 1 by 8, Prandtl number to the power of minus 1 by 6 something like this ok..

So, this now, can be substituted, if you want in this particular expression, and of course, we have the, Colburn’s, burns empirical correlation, ok, Colburn’s, empirical correlation, which actually tells you that, Stanton number into Prandtl number two third is basically equal to f by 2 rho, and it holds for Prandtl number greater than about 0.5 ok.

So, so, what we can do is that, now we can use different equations like 8.29 of course, this is from the book, this is equation 8.29, but using this equation, let us call it B, ok. And, an equation, of f which is equal to something like 0.0046 or it can be 0.078, depends on what we are doing ok. So, for higher Reynolds number, this is the expression. So, this is like C. So, combining these 2 expressions, you can find out Nusselt number, which is H D by K, that will be roughly given as 0.023 ok.

Reynolds number to the power of 4 by 5 and, Prandtl number to the power of one third ok, wholes in 2 into 10 to the power of 4, Reynolds number is greater than this, and Reynolds number to the power of 6, that is where it kind of approximately holds there, may be lot of other correlations as well ok. So, one of them is given by Dittas and Boitler..

So, there are tons of other correlations that you will find in different books, in different chapters right, and so, in essence, the Nusselt number is of this particular form right ok. And, you can read the book for, varieties of other, types of correlations that will exist, in this particular, particular case right ok. So, this pretty much; so, there are read about the other coalitions, read about other coalitions ok.

So, we have concentrated mostly on the few things in turbulence, one is of course, that we have considered that,,, using a simple Prandtl mixing length type of a model, which is basically a very ad hoc kind of an analogy, we were able to do some insights, right. About this law of the wall, we saw that, very close to the wall, there would be this laminar sub layer, then there will be a kind of a stable turbulent sub layer, and beyond that you have the outer region, where turbulence is kind of very, where the you have the large eddies and all these things ok.

So, there are of course, you hear, that there are lots of. So, how can this turbo, but this is all this, analysis has got some ad hoc nature to it, that we were able to neglect some quantities in certain ranges, we were able to work out, some other quantities in some other ranges. So, this might give you some picture of the wall, heat transfer, or the friction factor..

And there are correlations, which are very ad hoc, which assumes a velocity and temperature profile, but we were able to get some idea about, how to handle this problems. What is; however, not really clear that, what are the other ways, by which the same problem of turbulence can be handled. So, one is that as we already know, that you can, always have, what we call a, full DNS solution of the whole thing; that means, you take the whole Navier-Stokes equation and you basically solve it ok.

So, that would be one way of tackling the problem, that you take the whole Navier-Stokes equation, other than that if the Navier-Stokes equation, is not possible. There are methodologies, by which you can, there are large eddy simulations, and things like that which are more complicated, but there are like different types of model equations like for example, you have heard the name of K epsilon model, similarly there are Reynolds averaged 5 equation models, and things like that. So, these are all ad hoc models in their own way, but they serve certain purposes somewhere, especially in the outer region of the flow ok. So, one of these models is, basically what we call the K epsilon model.

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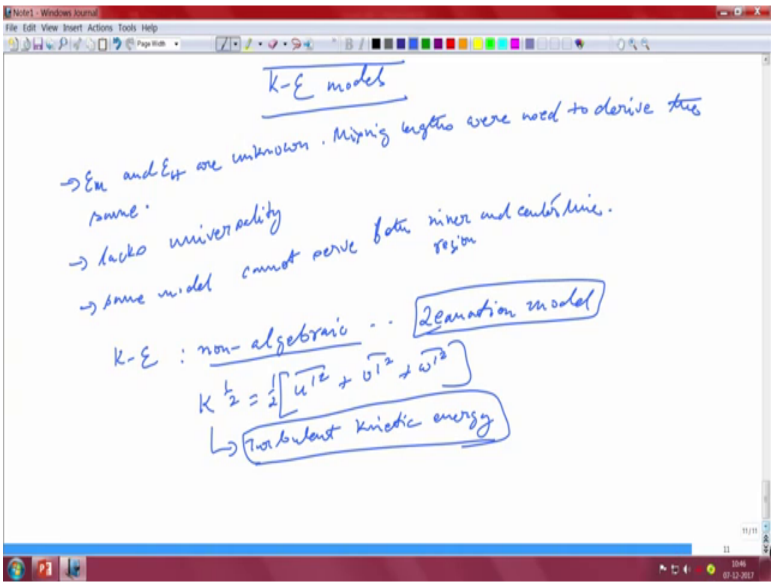

So the K epsilon model is one of the model ok. So, we already know that your epsilon M and epsilon H are unknown right, this we already knew, ok, and mixing lengths were used to derive, use to derive the same ok. So, they; obviously, lacks universality, it is not universal at all, which is quite obvious. Then the same model, model cannot serve ok, cannot serve, both inner and inner region basically inner region and centerline, it is impossible to do that.

So, K epsilon model was basically a non algebraic model, basically you devise 2 mode equations, it is called a 2 equation model, one equation for K, one equation for epsilon, where k basically stands for the turbulent kinetic energy, it is basically if you remember the matrix that we did so, this is basically what we call the turbulent kinetic energy, is the turbulent kinetic energy. This is a non algebraic model 2 equation model in fact, the k epsilon ok. So, it is basically, the sum of the variance of the fluctuating velocity field ok. So, that is what is calculated as the turbulent kinetic energy, right ok. Next, I mean, the turbulent kinetic energy is not all.

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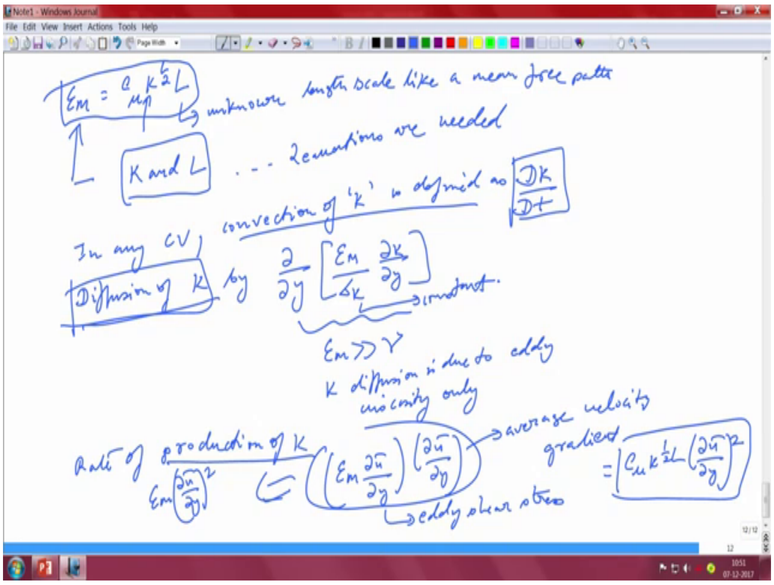

So, if we look at, the concept of eddy diffusivity ok, we can write this as C mu K to the power of half into L where, L is basically an unknown length scale, like a mean free path ok. So, in order to know epsilon M ok, we need to have a knowledge of K and L basically right. So, epsilon M is written in this particular way, this is the turbulent kinetic energy, this is some lengths scale, which is akin to the Prandtl mixing length ok..

So, this two needs to be solved, if we have to solve, for the if you have to close, because, we need to close epsilon M, basically right, in order to solve, that is what we did so far in the class right. So, basically you use, basically 2 equations to do this right, 2 equations are needed, because your epsilon M is a function of 2, variables, the 2 new variables that you have included, you have suddenly brought out. So, naturally in order to solve this equation, you just need 2 more additional equations right, to solve that ok.

So, in any control volume approach, normally in a full fledged turbulence course, you would actually solve for this equation. So, the convection of K, ok, is defined as, DK by DT ok, a diffusion of k, k by so, basically you write transport equations, that is all that you are doing, sigma K d k by d y ok. So, here of course, the assumption is that epsilon M is much much greater than gamma ok, K diffusion is therefore, it is due to eddy viscosity only ok. So, this is basically a constant ok. So, these two should be clear, one is basically the advection, and one is basically the diffusion of K right.

So therefore, there could be a rate of production also, of K, which is given as epsilon M, it is a transport equation. So, you should have all these kind of equations over here, y and d u bar by d y right. So, this is eddy shear stress, and this is some kind of an average velocity gradient.

So, this is the production term ok, ok, or in other words this is further written as, this entire thing is subsequently written as, epsilon M d u bar by d y square right, which in other words, I am writing it here, C mu K half L d u bar by d y whole square, something like that, ok. Now of course, there is now so, there is a advection, a convection, is a diffusion, there is a production. Now, there is only other thing, that is remaining is dissipation..

There should be a dissipation of K as well. Now we will see in the next class, how that dissipation equation, is actually written ok, because once we are able to write the dissipation equation, then we can write the full transport equation of K, and then see how that equation can be solved ok..

Similarly our equation, can be written for the, for the L also ok, equivalent of L, that we will do in the next class. To just you, give you an idea that, how the same turbulence problem, that same eddy viscosity, and other things can be solved ok, in a little bit more complex way, ok, by using 2 more equations, not necessarily, that is, this is any better, it just gives you, a little bit more freedom, in juggling the quantities ok.

Thank you.