Convective Heat Transfer
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Lecture - 52
Turbulent internal flow – II

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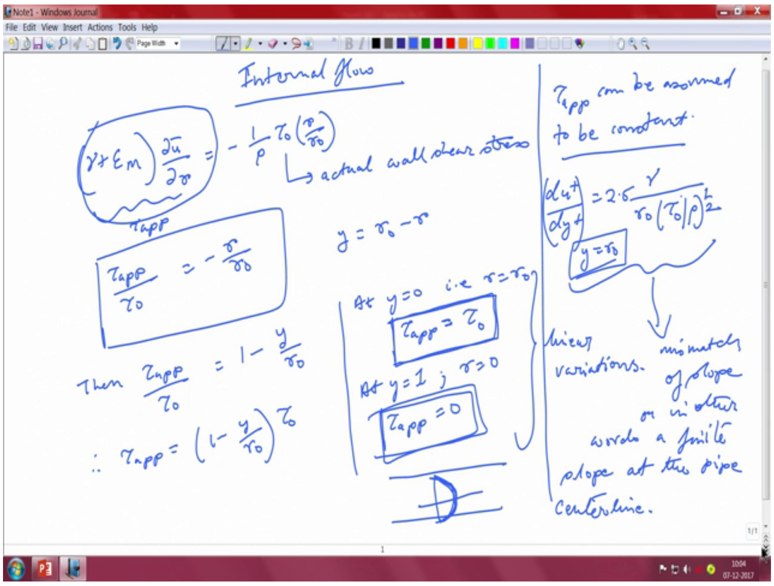

So, we had started this internal flow in the last class ok. So, if you recall what we did in the internal flow? We showed that gamma plus epsilon m du bar by dr is equal to 1 over rho tau naught r by r naught; where this is basically was the tau apparent. This is basically the actual wall shear stress ok. So, tau apparent by tau naught is equal to minus r over r naught.

That is what you can write because of the reason that mentioned we mentioned over here. So, this is your tau apparent ok. Now in other words if you now define a variable as y which is basically r naught minus r ok. Then tau apparent minus tau naught can be written as 1 over y by r naught ok.

Therefore the tau apparent will be equal to 1 over y over r naught into tau naught ok. So, at y equal to 0, that is r equal to r naught. So, it is just a reverse seeing the accesses; so that means, now the y is 0 at the wall basically where r is equal to r naught ok. So, at y equal to 0 the tau apparent is nothing, but the tau wall right. At y equal to 1 ok; where r is equal to 0 that is what it implies that tau apparent is equal to 0 ok. So, therefore, so, these are the two important things. So, at the y equal to 0 ok, you have the tau apparent is the same as the tau wall and at y equal to 1 the tau apparent basically vanishes ok.

So, these are basically what we call the linear variations ok. Now tau apparent can be assumed to be constant to be constant ok. So, that is what we have taken from here ok. So, now since tau apparent is assumed to be constant ok, what can be done is that? We can basically integrate this particular form and we can get du plus by dy plus ok; as 2.5 gamma r naught into gamma naught by rho raised to the power half ok.

So, that is what you get after you take care of this particular integration. After you, now you know that what your profile is going to be all right. Let us push this back. So, that is what you get and this is when evaluated at y equal to r naught, this is the profile that you get. Now based on this agreement this analysis, but however, one interesting feature that you see over here that there is a mismatch; mismatch of the slope right.

There is the clear mismatch of the slope at the center line because it basically tells you that there is a finite slope at the pipe centerline because y equal to r naught is basically that. So, what you have essentially is that there is a mismatch of slope or in other words in other words, a finite slope ok; at the pipe centerline. So, you have a finite slope at the pipe centerline ok. So, that is clearly not correct because if you recall for any pipe flow no matter what the turbulent profile is going to be there has to be some kind of a inflection point at the centerline ok.

So, this is one of the major drawback when you actually have that your tau apparent is the same as your tower wall ok.

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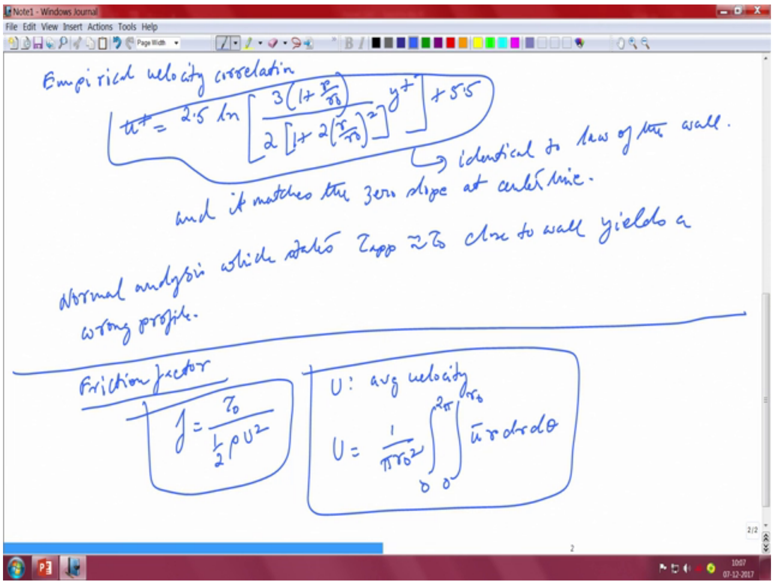

So, this brings about another empirical velocity correlation which says u plus is equal to 2.5 ln ok; 3 1 plus r over r naught divided by 2 1 plus 2 r by r naught square y plus plus 5.5 ok. So, this is like identical to law of the wall and it matches matches the 0 slope at center line ok. So, this is an empirical velocity correlation though whereas, the previous one that we showed that the tau apparent very close to the wall is actually equal to tau naught ok. That produces as we said a slope which is finite ok.

So, the normal analysis, analysis which states you know that tau apparent is equal to tau wall close to wall yields a wrong profile ok. So, that is what we have seen in the last in the in the analysis that we just now presented ok. So, this is important and so, as you find that there is a lot of empirical relationships when you ever you are dealing with this kind of turbulent flow profiles ok. Now let us look at the friction factor. Now that we have done all these things let us look at the friction factor. So, friction factor is like the C f x in the case of an external flow ok.

So, what is friction factor? Friction factor is basically your f is equal to tau naught divided by a half rho U square; where basically U is the average velocity, if you recall and U normally will be given as 1 over pi r naught square; it is a double integral U bar r dr d theta ok. So, that is the definition of friction factor ok. This is the same definition as in the case of a laminar flow right. So, there is no real catch over here.

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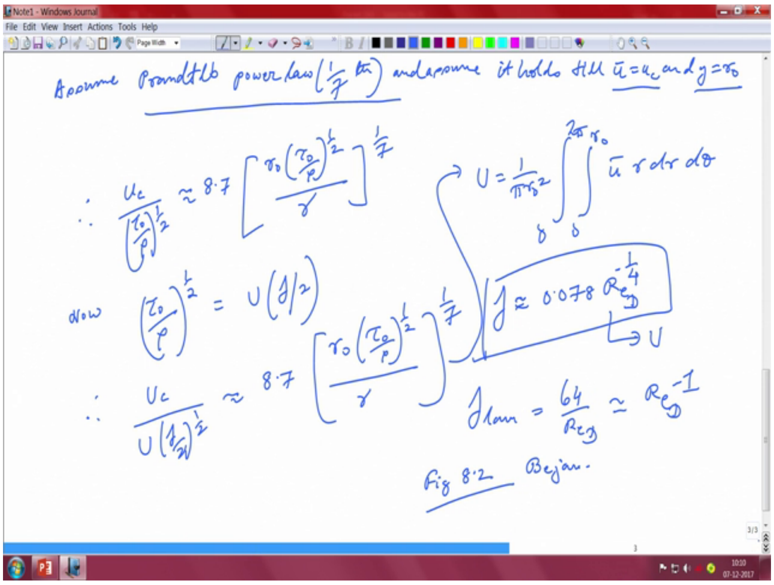

So, if you assume once again assume Prandtl's power law which is one-seventh if you recall and assume that it holds till u bar is equal to uc and y equal to r naught ok. So, assume that it holds till u bar equal to uc and y equal to r naught. So, the Prandtls power law profile ok. So, then what you are you going to get your profile will be uc uc being the center line velocity ok. So, tau naught divided by rho raised to the power of half; this is the profile tau naught by rho half by gamma raised to the power of 1 by 7 th. Now tau naught by rho raised to the power half is equal to u f by 2 ok. Therefore, U c by U f by 2 raised to the power of half; it is 8.7 r naught tau naught by rho half divided by gamma raised to the power of one-seventh ok. .

Now if you put now this expression back ok; this expression now if you put it back to the original form of your U, which is equal to 1 by naught square. So, you put it back in this particular form you get your f to be almost equal to 0.078 Reynolds number based on the diameter to the power of one-fourth. This Reynolds number is based on your U, compare it with the laminar counterpart you will find that it is 64 by R e D if you recall. So, this is basically scales as Reynolds number to the power of minus 1.

So, as you can see this is a weak dependence on the Reynolds number, this is the same type of dependence that we saw in our previous case as well ok right? Where we saw that the Reynolds number dependence was kind of very mild ok. So, you can take a look at your figure 8.2 of Bejan, where you can see that this gives you basically the friction factor with the Reynolds number ok.

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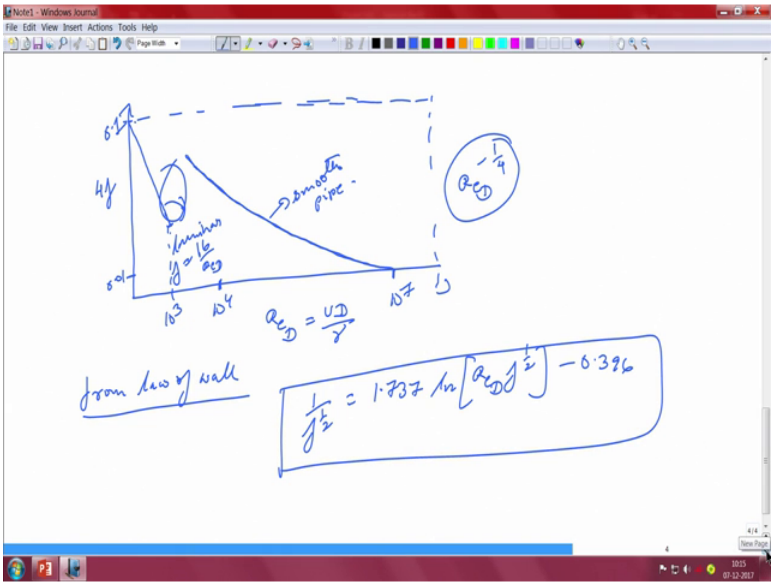

So, for the laminar counterpart you will have if I try to redraw it kind off. Let us try to do that. Of course, you can still take a look from your book this is UD by gamma right, that is the Reynolds number and this is basically written as 4f ok. It does not really matter what you write ok. So, it is basically plotted against f versus the Reynolds number. So, it is about 0.1 it starts ok, this is about point o 1 ok.

So, your laminar profile is going to come down like this ok. So, this is the laminar profile; as you can see the drop is very sharp is f is basically 64 by R e D. So, this scale because it is plotted with 4 f, this will be with respect to 16; 16 by R e D right ok. So, that will be the velocity profile. Now this is basically the Reynolds number is about 10 to the power of 4 right around here. Well it is a little bit moved from here; let me not just it is around here, 10 to the power of 4.

So, this will be roughly around it continues up to about 10 to the power of 3 and a little above right; which is standard as you know around 2000 Reynolds number of around 2000 the laminar profile this kind of reasonably valid right. Then of course, you have the Carmen Nikoradas type of relationships base based on what is the roughness factor, but if the roughness is kind of pretty low. The profile will look something like this. Let me draw it, something like that. So, this actually kind of merges around 10 to the power of 7 ok. So, this is about 10 to the power of the 4, this is about roughly 10 to the power of 3 ish ok. So, you know around 2000 it will start to go. So, there is a transition zone right around here and then the profile is kind of a lots smoother something like that. Of course, as you increase the roughness this profile takes different values, so for kind of a smooth pipe ok.

So, that is the kind of friction factor that you will normally get which gives you roughly a Reynolds number of minus one-fourth kind of our dependence. That is why the slope is kind of lots smoother compared to the minus 1 slope that you would not only have in the case of your Reynolds number ok. So, based on surface conditions and other things you can have a lot of differences, but this is in a nutshell which actually tells you that when you actually have a turbulent flow there basically this is what actually happens ok.

Now, you can device the same relationship from law of wall. Remember there are two ways of finding the friction factor. In 1 case we can find out the friction factor, but just by looking at the assumed velocity profile which was what was done by Prandtl. Now you there is any another alternative by which you can actually look at the same profile, the same friction factor profile from the law of the wall right. So, that will be the second way.

We did the same thing in our flat in our external boundary layer ok. So, there what we found is that the friction factor would be something like this; 1.737 ln Reynolds number D minus 0.396. This comes from the law of the wall ok. Ok? So, these are the two interesting things that we can observe ok. So, in two ways we have basically determined the same thing.

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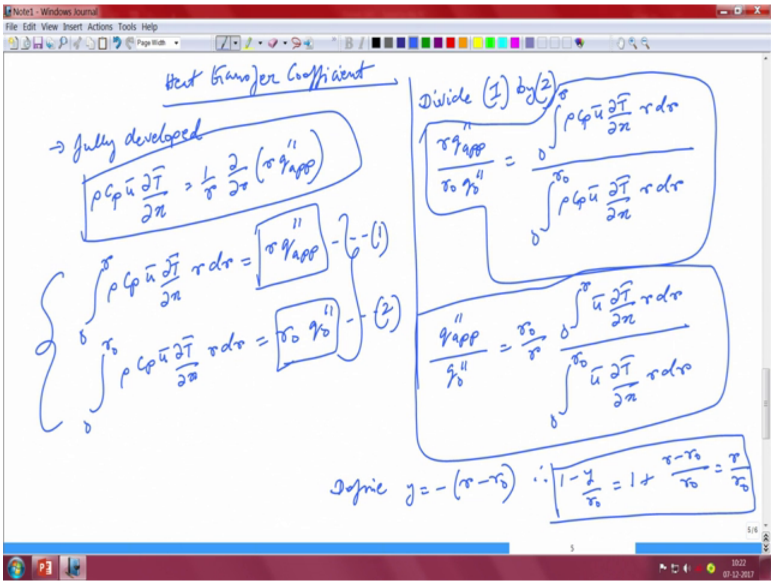

Now, we go to the heat transfer coefficient argument. So, you first go to the stage of fully developed ok. So, once again we start with the fully developed kind of a flow profile right. So, the position will be rho C p u bar dt bar by dx is equal to 1 over r d by dr r q apparent double bar right. So, this is basically the fully developed profile. Once again we have substituted it by the q apparent right because that is if you recall that is epsilon h plus alpha right that was what it was. Now if you integrate it up to any r; this is rho C p u bar dt bar by dx r d r is equal to r q apparent double prime. If you integrate it up to r naught you would get rho C p u bar dt bar by dx rdr r naught into q naught double prime right because if you are doing it the integral up to the up to the center line right?

So, it will be naturally equal to the q naught double prime. This is the wall; heat flux this is the apparent heat flux that we can decipher right. So, what we do is that if you take the ratio of the two right, these two quantities Let us call this 1, call this 2. So, what we do is that we divide 1 by 2 ok. So, in other words you have your r q apparent double prime divided by r naught q naught double prime ok. That is equal to from 0 to r rho C p u bar dt bar by dx r d r divided by to r naught rho C p u bar dt bar by dx r d r right.

So, this is the two divisions that we have done. So, now what will happen is that q apparent double prime divided by q naught double prime is equal to r naught by r; 0 to r u bar dt by dx r d r to r naught. So, this is just a basic division that we are doing ok, ok. So, at this point once again we define define y is equal to minus r minus r naught right; y equal to min[us]- that is the same way that we defined it in our velocity case. So, this will lead to 1 minus y by r naught should be now equal to 1 plus r minus r naught by r naught which will give you basically r over r naught right ok. This is what you get.

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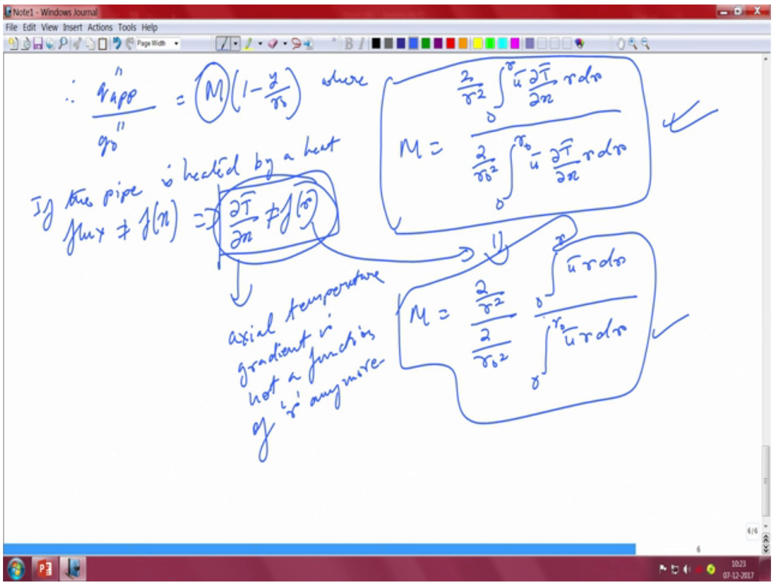

So, therefore, your q apparent double prime divided by q naught double prime is given by some factor M multiplied by 1 minus r over y over r naught. So, where M is basically given as 2 by r square 0 to r u bar dT bar by dx r d r divided by 2 by r naught square 0 to r naught u bar dT bar by dx r d r. So, this is basically that ratio of the two terms that we have written as M. Now if the pipe is heated by heat flux, heat flux which is not a function of so, it is heated by a heat flux such that that heat flux is independent of x ok.

So, that heat flux is independent of x that is not a function of x ok. So, therefore, this leads to your d T bar by dx is also not a function of r in that particular case. Or in other words what happens is that this term that you see over here this basically all cancels out this dT by dx from on both sides because this is an integral which is performed with respect to r only.

So, therefore, your M based on this assumption if you apply this assumption basically boils down to 2 by r square by 2 by r naught square right, integral 0 to r u bar r d r divided by 0 to r naught u bar r dr right. That is what we get. hmm So, these two this is a reduction in the profile based on the fact that your axial temperature gradient; temperature gradient gradient is not function of r anymore got it. So, that is an interesting fact ok. So, so, right now in what we will do in the immediate next class is that we will see that how this profile can be kind of adapted and it can give us some pretty interesting results at the end but you understand the steps once again to recap it very quickly ok.

So, what we have done is basically after we had done with the with the friction factor, we have just written the equation for the fully developed fully developed flow ok. And we have just integrated it into two parts: one from 0 to r and from 0 to r naught; one is q apparent, one is q not double prime. We have taken that ratio of the two, we have defined another variable substitution basically and then we have defined that this variable is a function of your M and this M is basically given by this particular quantity over here. Then we argued that your actual temperature must have been not a function of r.

In that case is just boils down to something like this ok. Remember the integral limits are different ok. So, in the next class we will see now how this can be further simplified, so that we can get some insights about what is going on in this kind of a pipe flow ok.

Thanks.