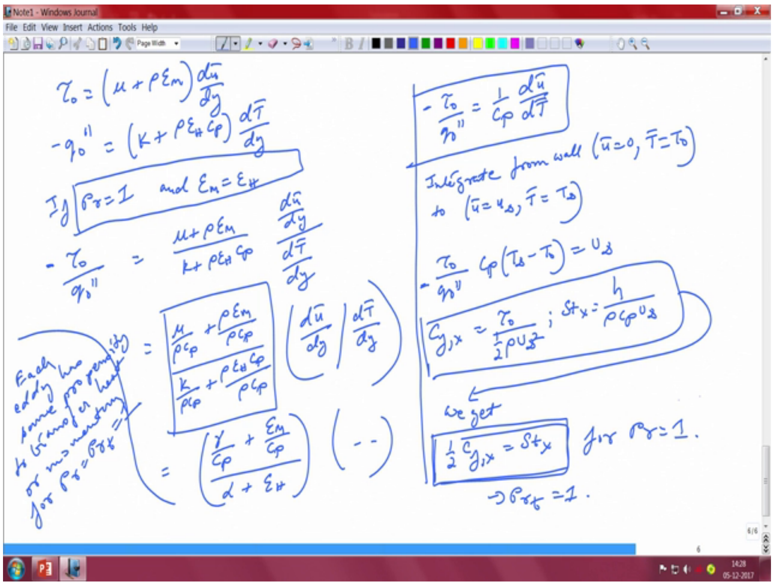
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 51
Turbulent internal flow – I

(Refer Slide Time: 00:22)



Ok so one other way of doing the same type of analogy is basically if you look at. So, your tau naught is basically your mu plus rho epsilon m to du bar by dy and q naught double bar is k plus rho epsilon H into C p dt bar by dy ok. So, these are the two expressions. Now if you are Prandtl number is of the order 1, that is equal to 1 and your epsilon m and epsilon H are the same ok. So, therefore, what you have is basically tau naught by q naught double prime is equal to mu plus rho c m k plus rho epsilon H into C p du bar by dy d T bar by dy or in other words. So, I am not writing the left hand side anymore. I will write the right hand side; rho C p rho epsilon m divided by rho C p divided by k rho C p plus rho epsilon H C p divided by rho C p.

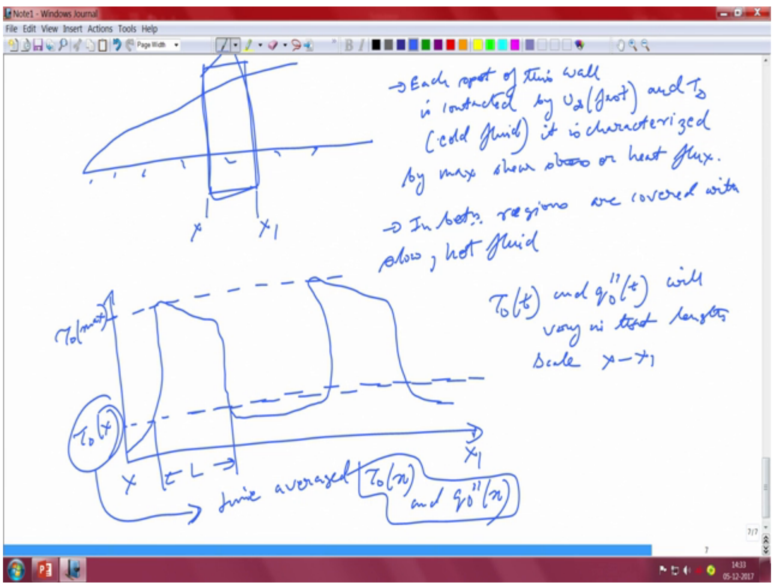
So, du bar by dy divided by dt bar by dy ok. So, again this part I am not writing, I am only going to work on this particular section and therefore, we show C p; C p divided by alpha plus epsilon H. Once again this part you can see what we are going to write; this part I am not writing anymore, minus tau naught divided by q naught square q naught double prime is equal to 1 by C p du bar by d T bar right. So, in other words as you can see over here Prandtl number is equal to 1 and epsilon m is equal to epsilon H right. Based on those assumptions you get this is your final form.

Now, if you integrate this, integrate from the wall which is basically u bar equal to 0 comma T bar is equal to T naught to u bar equal to u infinity T bar equal to T infinity. If you integrate across those twp limits, if you integrate across those two limits ok what you are going to get is T naught minus q naught double prime C p divided by T infinity minus T naught is equal to u infinity. We already know that c f x is basically tau naught divided by half rho u infinity square right and your Stanton number is basically h by rho C p into u infinity.

So, combining these two we get half c f x is equal to Stanton number. So, you basically get the same thing for Prandtl number is equal to 1 ok. This is also a the Colburn analogy and it also is for Prandtl number t is equal to 1. So, in other words what this essentially means is that from that expression that each eddy each eddy has the same propensity, propensity to transfer to transfer heat or momentum; momentum for Prandtl number the turbulent Prandtl number is equal to 1 ok.

So, when this is not equal to 1 this expression starts to deviate ok. That is it starts to deviate from the Colburn's analogy that we have over there right. So, now, now that we have come to this particular form ok. So, let us look at ok, some of the 1 interesting way of putting things together.

(Refer Slide Time: 05:27)



That means, if you have say for example, this is a like a flat plate ok, this is the boundary layer profile and if you take a look at any particular section of this boundary layer profile any particular section. So, this part is X this part is X 1 ok. So, what we can say because of the nature of turbulence that because it is highly oscillating ok. So, each spot, so, so, far we have done assuming that its nice and steady right. Another way of approaching the problem is that to have some temporal fluctuations right. Right now we have said that is all nice and steady. Our equations do not have any temporal variation at all right.

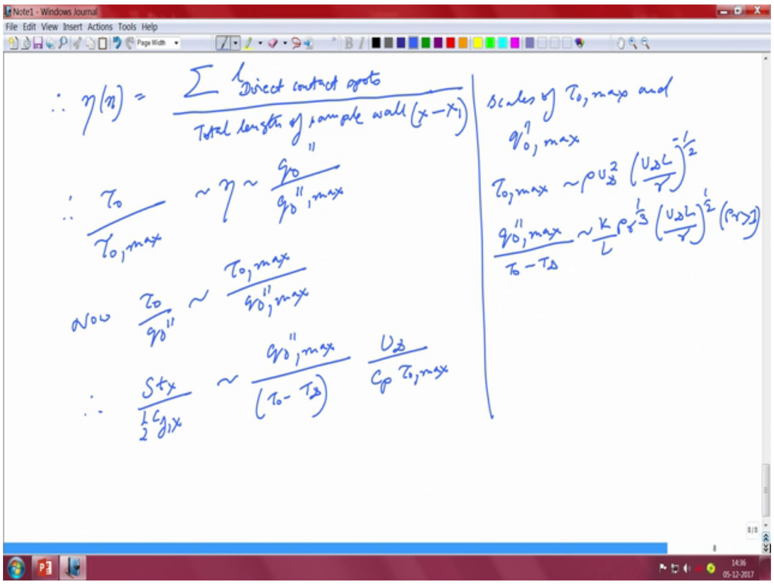
So, one way this is of course, done by Bejan in his book that each spot of this wall of this wall is contacted. So, I am just writing the basic features conducted by u infinity which is a fast moving fluid and T infinity which is a cold fluid, cold fluid ok. So, when they are contact, each spot of this wall is contacted by this fast moving and cold fluid ok.

Therefore, it is characterized; it is characterized by max shear stress shear stress or heat flux ok. So, whenever that happens it is characterized by the max shear stress and heat flux right. In a way that is true because you are bringing in all of a sudden fast moving fluid and cold fluid right; when each spot of this wall right. In between being covered, in between in between the regions are covered with you know slow and hot fluid right and so, in between in between these things you are met with this slope and hot fluid.

So, if you transfer it to a graph I think that would be visually appealing that this is the say X and X 1. These are the, I am only looking at that part of the box right. So, what happens is that this is basically your kind of your tau naught max right. So, what happens is that this is basically your tau naught wall tau naught x, the some kind of a average quantity. So, you see that it goes up like this, it comes then it goes up and then again something like this ok. So, it reaches up to that that particular level and each of these coverage areas is given by L ok. So, basically this is the time averaged is the time averaged quantities which is basically tau naught x and q naught double prime x. So, both of these are basically time averaged quantities ok.

So, now and tau naught t and q naught double naught t will vary will vary in that landscape; which is X to X 1 right. So, that is the emphasis.

(Refer Slide Time: 09:53)

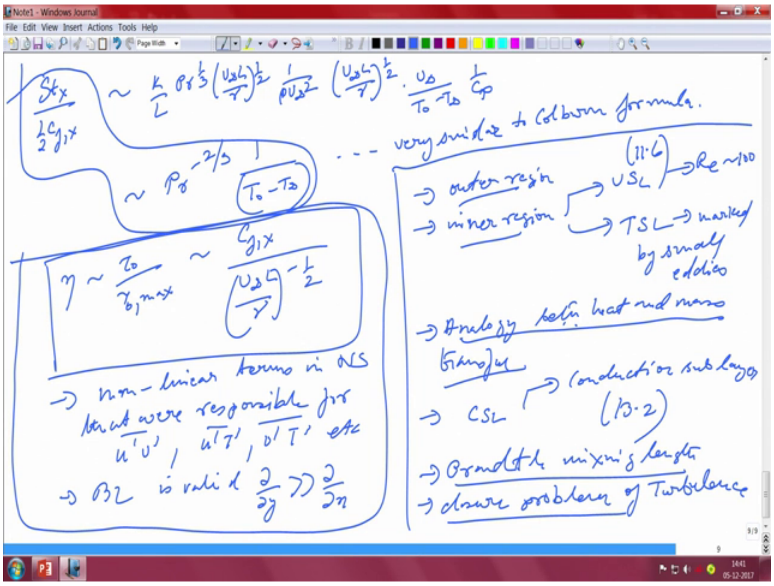


Now, let us now say this eta as a function of x is equal to l; which is the direct contact spots divide it by the total length of sample wall X to X 1 ok. So, this is the total number of those contacts, what is the sum and this is the total length because some spots are never really wetted by this cold or hot fluid cold fluid essentially ok. So, therefore, your tau naught divided by tau naught max should be a function of this eta and this should be similar for this your q naught and q naught double prime max got it.

So, now therefore, your tau naught by q naught double prime; therefore, should scale as your tau max by q double max ok. So, when there is a hot moment of when there is a cold fluid coming, heating at a fast phase, it is also bringing cold temperature with it ok. So, that is the, that is why this scaling actually works. So, this we already know this is of course, Stanton number divided by half of c f x. So, this actually then varies as q double prime max divided by T naught minus T infinity into U infinity divided by C p tau naught max ok.

So, what are the scales basically? Scales of a tau naught max and q double prime 0 max ok. So, tau 0 max scales as a rho U infinity square into U infinity L divided by gamma raised to the power of minus half. This comes from your laminar boundary layer theory, if you recall right because that is what the cold is actually meeting the surface. So, then q double prime max divided by T naught minus T infinity; therefore, scales as K by , Prandtl number 1 third U infinity L by gamma raised to the power of half; all these are valid for Prandtl number greater than 1 ok. So, for this laminar sub layer that covers the direct contact sports this is like a flat plate with length L in the laminar shear layer, that is what we have used over here right.

(Refer Slide Time: 13:13)



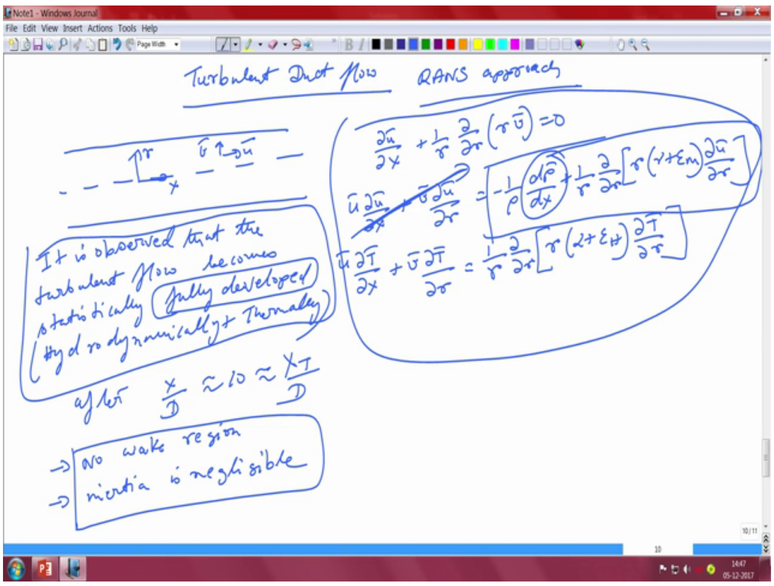
So, Stanton number X to half C f x is therefore equal to K L Prandtl number 1 third U infinity L by gamma raised to the power half rho U infinity square L by gamma raised to the power of half multiplied by U infinity T naught minus T infinity into 1 by C p. This therefore, if you do all the motions this will be Prandtl number two-third into 1 minus T naught minus T infinity. This is very similar to the Colburn. You just do the map, do the steps you will get this Colburn formula; very very similar to the Colburn formula ok. Also we get your eta, actually equal to C f x divided by U infinity L by gamma raised to the power of minus half. This is also you get ok, ok.

So, we got a lot of detailed outcome for the temperatures and we also got a lot of detailed insights ok, into the external flow right. For the turbulent external flow we can summarize and say that there is an outer region right. Then there is an inner region; inner region is composed of two viscous sub layer and the turbulent sub layer right. In the viscous sub layer ends where the Reynolds number is approximately 10 ok. Then the turbulent sub layer is marked by small eddies ok. We have established the analogy between heat and mass transfer ok. For the temperature profile there exists something called CSL which is basically conduction, which is basically the same as the viscous sub layer. So, conduction sub layer. This is of the order of 11.6; this is of the order of 13.2 right. In addition we have learned something about Prandtl mixing length, the closure problem of turbulence right and where it originates. We learned that it is the non-linear terms; terms in Navier stokes equation that were responsible, for the for U prime, V prime and then U prime T prime, V prime T prime etcetera ok. We saw that the boundary layer approximation is valid; that means you still have your dy much much greater than your dx right got it.

So, there are numerous such things that we have found out using a lot of these relationships got it ok. So, the closure problem of turbulence and the Prandtl mixing length everything we have been able to provide analogy between heat and mass transfer, the inner region the outer region. So, we have covered a lot of materials ok, in especially in the external flow part.

Now what we are going to do is that we are going to look at the turbulent duct flow right. Because turbulent duct flow is the one that is going to take us to the almost towards the end of this particular work, particular lecture ok; I mean let us start with the turbulent duct flow and let us see how much we can do in this class.

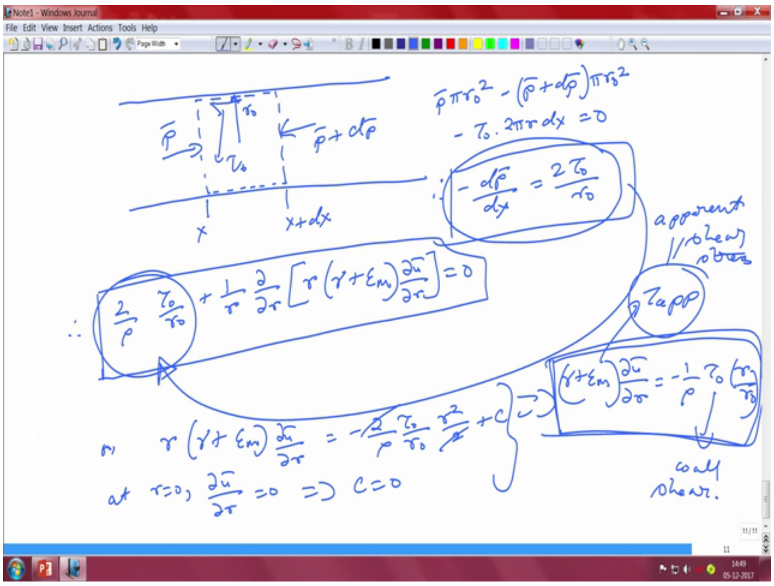
(Refer Slide Time: 18:04)



So, turbulent duct flow is a typical example of an internal flow right. So, you consider this as your duct. So, this is your r, this is your x, this u bar, this is your v bar. So, I am not going to redraw it once again. This is what we are going to carry forward in the next class also. Say again we are trying to do the same thing like RANS approach. That means, we are doing the Reynolds averaged, Navier stokes equation ok. So, for this particular duct what we have? The equations are is equal to 0; that is the continuity for u ok. Similarly, so, it is observed it is observed that the turbulent flow becomes statistically fully developed developed ok.

So that means, Hydrodynamically plus Thermally ok, after X by D of approximately 10 which is basically the same as X t by D ok; one is the thermal boundary layer thickness, one is the velocity boundary layer thickness ok. So, these are the basic equations. It is observed that it becomes statistically fully developed; does not mean that all the fluctuations are gone. It is statistically fully developed and on the top of that no wake region and of course, inertia is negligible. No wake region and inertia is negligible ok. So, this is r and x, this is u and v; u bar and v bar rather ok. And so, the this is the expression we have ok. So, we will do force balance now.

(Refer Slide Time: 21:56)



So, if this is the pipe or duct whatever. You take a control volume along this. So, this has got the pressure p ok. So, there is the pressure on the other side which is basically your p bar plus d p bar ok. So, this is the center line, this is that r naught; that is that this is x, this is x plus dx right. So, what you have you do a force balance on this right. You will have your p bar into pi r naught square minus p bar plus dp bar pi r naught square minus tau naught into 2 pi r dx is equal to 0 right. Of course, you have the shear which is acting here that is your tau naught right. So, this is just a control volume around the control volume what are the forces that are applicable you take that there is no inertia. So, there is no component of inertia that is acting. So, it is only a pressure and the shear stress balance right.

So, therefore, what you get is minus dp bar by dx is equal to 2 2 tau naught by r naught; this is quite clear right this is how you get it. So, this solves a major part of the problem because if you look at your previous expression you find that this term is obviously, not really useful right because that is inertia. We needed something for dp by dx right. We needed to know what is the nature of dp by dx. Now we have found out what will be the value of this dp by dx. So, that that is nothing, but the wall shear stress. So, therefore 2 by rho tau naught by r naught plus 1 or d by dr ok; where the substituted for dp by dx. That is all that we did took it and substitute it here right ok.

So, this forms the basis. Now we can quickly do the integration. So, r gamma plus epsilon m dr is equal to minus 2 by rho tau naught by r naught r square by 2 plus c ok. So, at r equal to 0, du bar by dr is equal to 0. So, therefore, this leads to c is equal to 0 right. So, together what we get is gamma plus epsilon m du bar by dr is actually equal to 1 over rho tau naught r by r naught ok. So, that we get through our standard because this r and this r one r cancels out. So, this is the form that we get and this 2 and this 2 cancels out. So, this is the form that we get ok.

So, out of this we already know is a tau apparent. This is the apparent shear stress got it. This is the apparent shear stress and on the right hand side we have the actual wall shear stress; the wall shear right. So, in the next class we will see how we can handle this problem from this part onwards right. So, this is very easy. We have now got the apparent shear stress is equal to the wall shear stress multiplied by these factors ok. So, we will do in the next class, that how this problem now can be handled.

Thank you.