Convective Heat Transfer
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Lecture – 50
Heat transfer in turbulent boundary layer

In fact, in the last class, we promised, that we will show the viscous sub layer and the turbulent boundary layer, ok.

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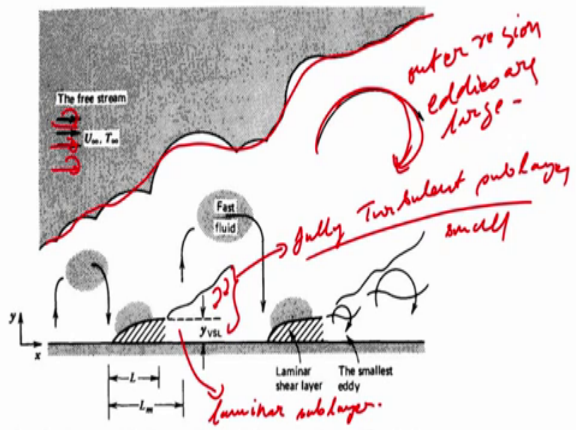

Figure 7.5 Formation of the viscous sublayer as the time-averaged superposition of laminar shear layers with local Reynolds numbers no greater than ~ 10^2.

So, this is basically what you see, this is basically the inner region and this is basically the global structure of the turbulent boundary layer. These are the large eddies; here, the eddies are kind of small. This is a fully turbulent sub layer and out of this, this is the laminar sub layer that we talked about.

So, the formation of the viscous sub layer is the time averaged superposition of the laminar shear layers with local Reynolds number no greater than 100. So, locally in that particular sub layer, the Reynolds number values will not be more than 100. So, after this, you have the fully turbulence up there, where the eddies are small right. And then, you have the fully then you have the full turbulent outer region outer region, where the eddies are large, right. And this is of course, the free stream velocities and the temperature.

So, this is how this is the description of basically the total problem.

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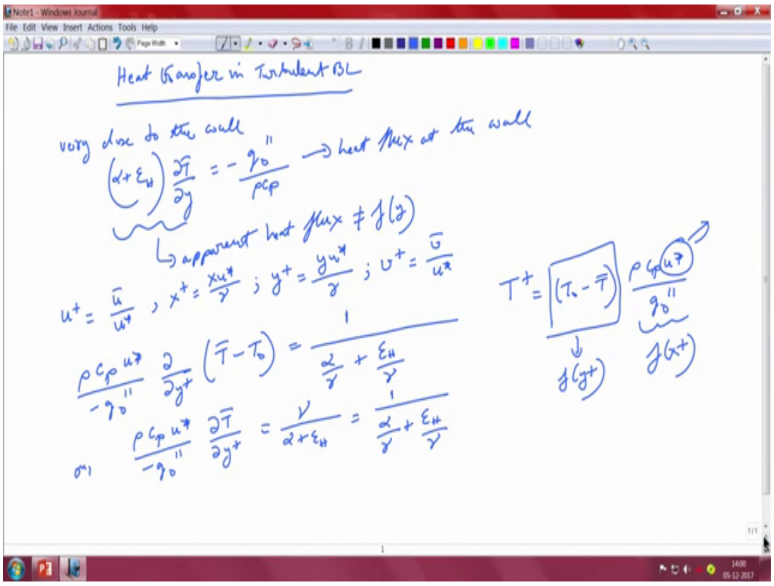

Now, moving on to the heat transfer, we already so, this is the heat transfer in turbulent boundary layer ok. So, we have already said that, very close to the wall, remember we already started doing this. So, very close to the wall alpha plus epsilon H dT by dy in to minus q naught double prime rho CP ok. So, this is the heat flux wall ok.

This we have done by neglecting the convection effect right, very close to the wall. This is what we said, was the apparent heat flux ok. Apparent heat flux, which is not a function of y, similar to our apparent shear stress assumption right? So, once again, we did the same thing ok. So, you define the non-dimensional quantities. So, after putting in the variables rho CP u star divided by minus q naught double prime dy plus ok.

So, that is the expression or rho CP u star dy plus. So, that is the form that you get. So, your T plus is T naught minus T bar then, there is a rho CP u star divided by q naught double star. So, this particular factor, as we know is a function of y plus obviously right. And this particular guy is a function of X plus because, u star is a function of x plus right ok. And then, you have the q naught which is the wall, heat flux.

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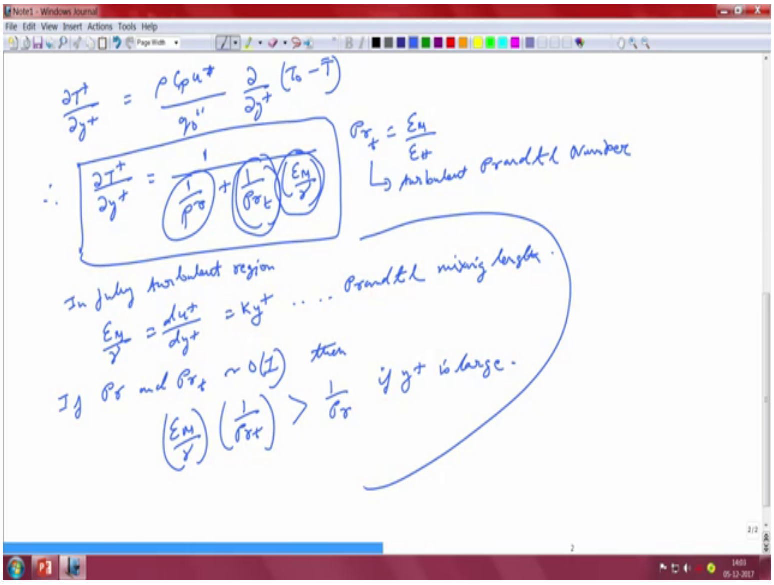

So, now, therefore, given as CP u star divided by q naught double prime minus T bar ok. So, therefore, where this is called the turbulent Prandtl number, this is called the turbulent Prandtl number. So, as you can see, it is a combination the profile the way that we have written, it is a combination of the one over the Prandtl number, one over the turbulent Prandtl number and the ratio of the eddy viscosity and the corresponding kinematic viscosity right ok.

So, this is a fully integral form of the equation ok, except that in fully turbulent region, epsilon m by gamma K y plus right. So, that is your that is we already proved it from where the Prandtl mixing length right.

Now, if your Prandtl number and your turbulent Prandtl number, both are of the order 1 ok, both are of the order 1, then, epsilon m by gamma into 1 over Prandtl number T right. This guy will be greater than 1 over Prandtl number, if your y plus is large right, if your y plus is large, this will be true right. So, using this therefore, we get a 2-stage solution for our temperature.

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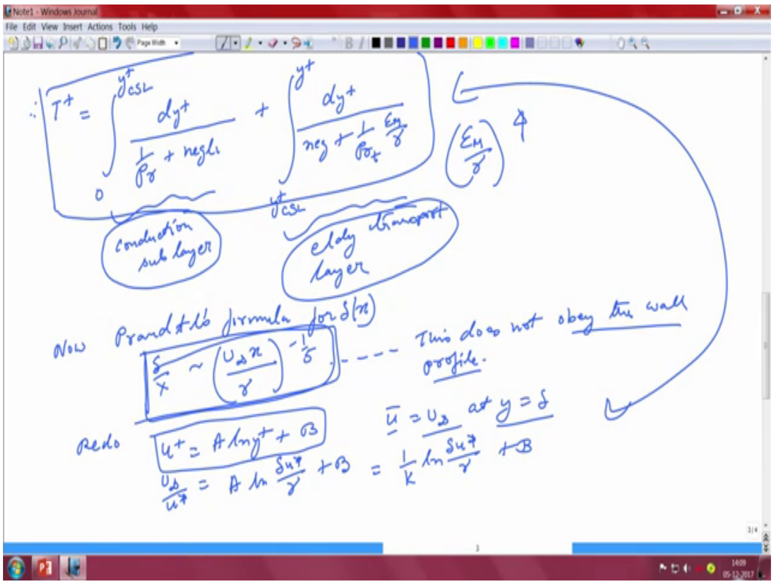

So, those that 2-state solution will be, so, T plus from 0. So, this is what we call the conduction sub layer the upper limit ok, dy plus divided by 1 over Prandtl number plus some negligible terms right. So, this is what we call the conduction sub layer right, plus you have from the sub layer see yourself the way up to any y plus. So, this will be dy plus divided by some negligible terms because, the Prandtl number is now kind of low compared to this epsilon m by gamma. So, this is what we called the eddy transport right, got it?

So, what we have done over here is that, we have divided now the problem into 2 parts ok. The first part of the problem is, basically we found out that one part was dependent on your Prandtl number while the other part was dependent on your eddy viscosity right ok. Using those 2 expressions ok, we have been able to separate the 2. So, we are initially considering a conduction sub layer and then an eddy transport layer knowing the fact that, epsilon by gamma actually goes up with y prime y plus ok.

So, now, the point is that ok, so, these are the 2, 2 layer the information that we have ok. Now of course, of course, Prandtl's formula for delta x right. So, now, for delta by X, what Prandtl assumed was U x U infinity x to the power of minus 1/5th. If you recall, this is what we did for the overall boundary layer thickness ok.

But, this does not obey. We told earlier also. This does not obey the wall profile, got it? This does not obey the wall profile right. This was original Prandtl's Prandtl's are mixing layer Prandtl's are formula for delta X not mixing length. So, we redo this using the original expression for from the law of the wall, if you recall the logarithmic form ok, if you extend it to the edge of the boundary layer to the edge of the outer region, let us see what we get.

So, we can redo this whole thing where, you know that your u plus is A ln y plus B right. So, your U infinity by u star ok. That is where it becomes actually the same as where your U bar becomes equal to U infinity at y basically equal to delta. you have A ln delta u star by gamma plus B, which gives you one over K ln delta u star by gamma plus B ok.

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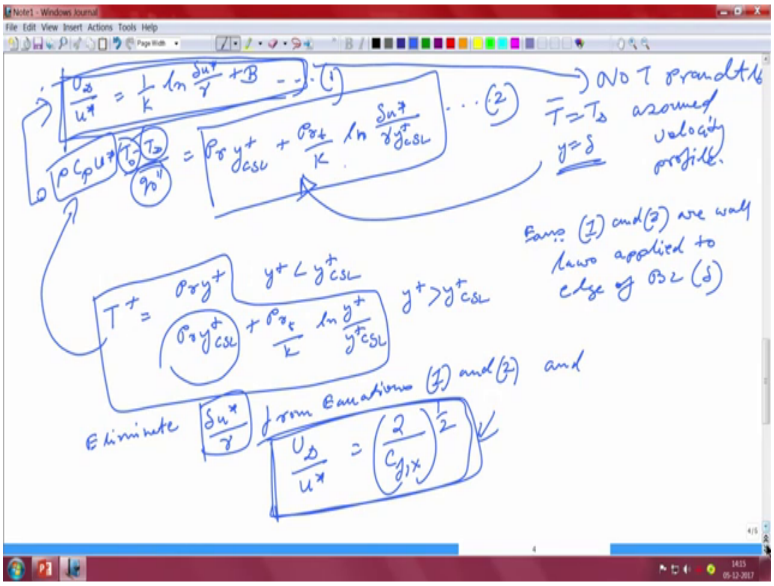

So, this therefore, leads to u star gamma plus B right. So, we have this expression as your 1; you have rho CP u star T naught minus T infinity by q naught squared this is the other expression CSL plus Prandtl number T by K ln delta u star divided by gamma y plus CSL right. So, this is your expression 2 ok. So, these are the 2 equations that you get ok, just by substituting.

So, what we have done is, basically you get the 2 temperature profiles right. You get the true temperature we you get the 2 temperature profiles over here, when you actually integrate like if you look at it here, the conduction sub layer the total temperature profile right, but when we in order to find out what will be the velocity profile, we could have used Prandtl's form.

But, the problem here is, this Prandtl's form does not obey the wall profile. So, you cannot take a temperature profile, which obeys the law of the wall right and you take a velocity profile which is does not obey the law of the wall. So, what we have done is in essence is, that we have taken the temperature profile we have taken the velocity profile and we have extended it to the outermost edge of the boundary layer to the outermost edge of the boundary layer to be consistent with this particular formulation right.

So, once you do this, so, what do you do? We start with the generic profile, then you substitute the parameters; that means, u bar becomes equal to U infinity at y equal to delta, then you substitute it and at the end, you get a profile which is essentially like this right. Similarly, for the temperature part, if you set T equal to T bar equal to T infinity and y equal to delta, you get this particular expression from the from the total equation solution that we actually did, ok.

So, this will be the 2 expressions. However, the temperature in itself ok, if you write the temperature in 2 parts, the temperature in itself will be Prandtl number y plus for y plus is less than y plus CSL and it will be Prandtl number y plus CSL plus Prandtl number T by K ln y plus y plus CSL when y plus is greater than y plus CSL right ok.

So, from as you can see from this expression itself ok, if you push it there, you can actually extract that that will be the total value right. Because, this part is constant right and if you substitute your y plus ok. By substituting y equal to delta u basically get that right. So, in other words and we already know that t plus we defined T plus right here right here ok. So, that was our T plus ok.

Only difference is that, T plus evaluated at y equal to delta means that, T bar should be equal to T infinity right. That is only change that you will have ok. So, it is basically that, it is T naught minus T infinity, then you divide it by q naught square and then you have this. And then, you have on the right-hand side, you have the full expression ok.

So, similarly, the same thing we have done it for velocity. Remember, this is not Prandtl's assumption assumed velocity profile. Because, you remembered that does not follow the law of the wall velocity profile correct ok. So, these are the only 2 changes that we have made ok. Now, if you look at these 2 expressions clearly, ok, what you will find is that, from these expressions, we can eliminate a few things ok. We can eliminate a few things and basically what we need to eliminate.

So, let us write, eliminate delta u star by gamma ok. If you eliminate these 2 expression from equations 1 and 2, we get u infinity by u star is equal to 2 by C f x that is to the power half. That is what you get keeping in mind.

So, we eliminate this, sorry does not we get and this is the definition of U infinity by u star.

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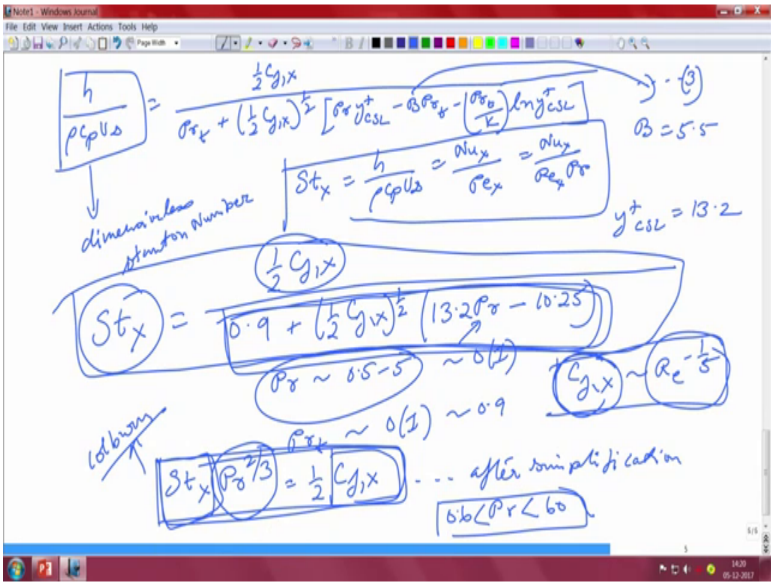

So, using these 2 expressions, the total result becomes, h by rho CP into U infinity this is a long expression a half C f x divided by Prandtl number T plus half C f x raised to the power of half ok.

So, what we have done let us just recap very quickly. So, we have 2 expressions, 1 and 2 right, which are basically the wall laws, which are applied at the edge of the boundary layer ok. So, equations 1 and 2 are wall laws right, applied to edge of boundary layer, edge of boundary layer, which is basically your delta right. So, you and we also have the definition of C f x which we got earlier.

So, what we do is that, we eliminate this from equations 1 and equation 2 and apply this definition, so that, we get the new quantity, which is given as this ok. Now, this particular number, that you see over here is dimensionless ok. So, if this is actually a dimensionless number and it has got a name, it is called the Stanton number ok. It is called a Stanton number ok

So, it is one of the dimensionless way of reporting the heat transfer coefficient in turbulent flows. So, Stanton number basically, if you write it with in terms of X it is a rho CP into U infinity it is basically a Nusselt number divided by the peclet number right or in other words it can be Nusselt number divided by Reynolds number into Prandtl number ok. That is what stanton number is ok.

So, now if you look at, let us call this equation 3. Now, if you look at this equation what we get is that, your Stanton number is therefore, given as half over C f x, I am writing it a little large. So, that you have no problem in understanding. Now, before I write the denominator ok, what we can do is that, people have found out this empirical constants. There are quite a few of this empirical constants which are lying around here.

So, our finding those empirical constants people have plugged in some numbers ok. So, for example, it becomes 0.9; that is, the turbulent Reynolds a turbulent Prandtl number is 0.9 this C f x raised to the power of half. So, as you can see, 13.2 into Prandtl number minus 10.25. So, you can see, your y plus CSL is approximately 13.2 right. It is approximately 13.2 and B, this constant B over here has been usually it is found out to be about 5.5 something of that that order ok.

And Prandtl number is assumed to be in the range of 0.5 to about 5. So, it is of the order 1 essentially and Prandtl number turbulent Prandtl number is also of the order 1. It is about 0.9 ok. So, that is the expression that we get. So, as you can see, if you look at it very closely, this entire expression the stanton number expression, you find that it is not very sensitive to the changes in Reynolds number; particularly the denominator part ok. It is dependent only on the Prandtl number. So, to say, ok.

It is only dependent on the Prandtl number and it has got the same order of magnitude and the factor as the Prandtl number over here. So, that. So, indeed. So, if you if you do it like that, so, all out of the order 1 actually. So, therefore, if it is since, it is dependent only on Prandtl number.

So, Prandtl number $2/3^{rd}$, if you just take the things out ok, 2/3rd is equal to half C f x. This is the expression that you get after simplification ok. After simplification. So, this particular expression that you see over here is, originally was suggested by Coburn ok. So, it was suggested by Coburn ok.

And, this is an interesting quantity, in the sense that, we are relating the skin friction coefficient with the heat transfer coefficient using this particular relationship ok. So, there is an analogy basically that happens between the 2, ok. It is valid for a range of Prandtl number basically less than 60 and greater than about 0.6 ok. So, of the order 1 to 10, it is kind of a kind of valid ok.

And this happens only because, this is not very sensitive, the denominator of this particular expression is not very sensitive to the change in Reynolds number y. That is because, if you recall your C f x variation with Reynolds number, it was a rather poor, right? It was Reynolds number to the power of minus 1/5th right. So, even if your Reynolds number changes quite a bit, your C f x changes just by a little bit, ok.

If your Reynolds number changes quite a bit, your C f x just changes just by a small bit. So, that is why, this denominator is largely insensitive to your Reynolds number variation. But it is sensitive to the Prandtl number variation ok. So, because it is sensitive to the Prandtl number variation, that this skin friction coefficient and this stanton number are basically proportional to each other. They are of the same order of magnitude with the only function being the Prandtl number which fills in as a constant of this proportionality ok. And that is exactly what we have we have seen over here, that this is the expression which will come in very handy, that stanton number into Prandtl number 2/3rd is basically equal to half of the skin friction coefficient.

So, this completes a nice little thing. So, we know that the temperature also has got a laminar sub layer and then a full fully turbulent sub layer ok. At the same time, we found out that there is an analogy between the heat transfer and the skin friction coefficient for Prandtl number turbulent Prandtl number of around 1 and Prandtl number to the order of a also varying in that range 0.5 to about 50. It agrees actually increase.

So, in the next class, what we are going to do? We are also going to see that, how this the same relationship can be worked out in a slightly different way ok. And we will see to that, that how that particular thing can be incorporated in this particular analysis of us.

Thank you.