Convective Heat Transfer
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Lecture – 05
Scaling Analysis – Momentum

So, in the last class what we did was that we said that to answer the engineer's application engineers needs to answer 2 important questions, what is the drag? What is the heat transfer coefficient, right? The drag in this case means in friction. So, that is in friction and this heat transfer coefficients both are very important let us see how they are related to the boundary layer, right.

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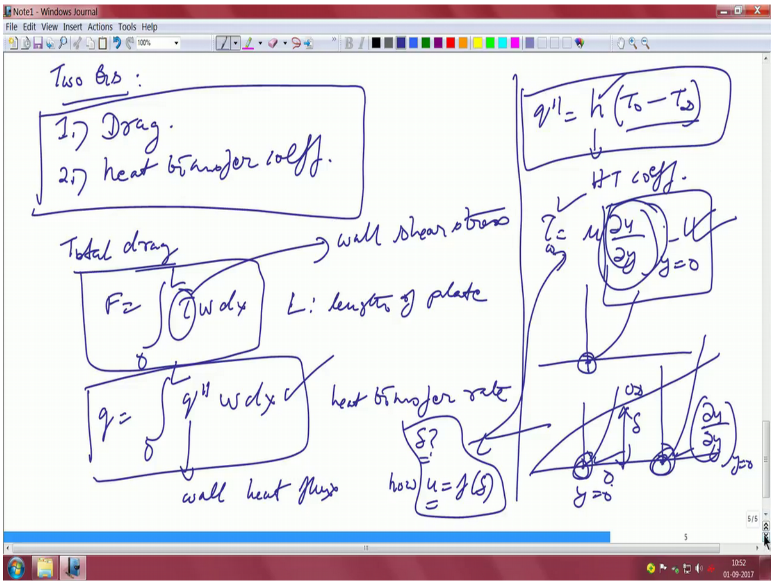

So, let us say that total drag that is encountered by this plate. Now the plate here is a is a canonical system it can be any complicated device that you can think of, right. As I said flight whatever your aeroplane your submarine it can be anything, right. So, the total drag F is given if the length of the plate is L it is given by tau w d x, where L is the length of the plate, right. Tau is basically the shear stress. The total heat transfer on the other hand q is given as 0 to L q double prime w into d x, right.

W into d x so, this is the heat transfer rate, and this is basically the total drag. So, there are 2 questions over here, right. These 2 expressions you know this is how to calculate correct. So, this q double prime, what is this q double prime is nothing but what we call the wall heat flux, right. Heat flux and what is tau called tau is a wall shear stress right. So, that would mean that you need to know that what is the heat flux at the wall you also need to know what is the shear stress at the wall, right.

So, what is the what is q let us look at it what is q double prime, right. Q double prime; that means, if you know q double prime you can easily integrate this fellow over here to get the total heat transfer rate right. So, the wall heat flux, if you talk about it is basically given as h into T naught minus T infinity, correct? This is what the application engineer wants, right. That h and it is T naught minus T infinity T naught is a wall temperature and T infinity is what T infinity is basically the free stream temperature, right. H is some kind of a combined number, right. Which gives you that how much is the heat flux the wall heat flux, right. That is what your aim is right.

Now, this h you would be immediately elated, oh there is a number called h this is like the heat transfer coefficient this is the heat transfer coefficient, right. Now you will be elated to see that oh wow now we have a heat transfer coefficient. So, it is very easy right, but problem here is this heat transfer coefficient h is not a function of the properties this is not like your k not like your thermal conductivity.

So, when you talk about thermal conductivity say for example, you will find that is what you have encountered in conduction mostly because it is k d T d x, right. And on d T d y when you talk about thermal conductivity it is a property right. So, you know the fluid you know what it property going to be say it is there in different handbooks and all over the place. But when you talk about h, h is no longer a function of the property, right. Age depends on the geometry it depends on the flow field it depends on the Reynolds number it depends on a whole gamut of parameters.

So, naturally there is no universal number that actually represents h. So, if I give you the flow say there is a flow of water over a flat plate. And you ask me what is h. I say I cannot tell what is h, right. You need to let me know that what is the velocity you need to know what is the geometry of the plate, you need to give me a whole lot of information before I can calculate what this h is going to be. So, h is not a universal number is highly situation dependent for the same system. You can have different values of h, right. So, but we can see that h somewhere depend do depend on k, there must be some dependence somewhere with k right.

Because if you are dealing with say 2 different completely different materials you will have different age, right. It also depends somewhat on the flow, these are your common-sense intuitions, right. There will be some dependence on the flow; that means, how high the flow field is say for example, going back to the same original problem of your drinking tea, right. If you blow very hard your thing becomes cold, the tea becomes colder real quickly right. So, somehow you are doing something to enhance the heat transfer coefficient correct by blowing hard. So, your intuition tells you that if I blow hard, right. Or blow at a high velocity probably I am doing something which enhances the heat transfer coefficient right.

Now, instead of tea, if it is some other material with a very low thermal conductivity or something like that, you will find that it will not be that easy to cool down this whole thing right. So, you know that there are some property dependences. There are some flow dependences. But there is no universal concrete thing that I can say that I can compile in a in a tabular form which will say this is the value of h you just take it this is the value of h, that is not possible right. So, the heat transfer coefficient over here is not a universal number, that is the first thing ok.

Similarly, the wall shear stress is given by mu this you already know evaluated at y equal to 0; that means, it is evaluated at the wall right. So, what does this mean? This means that the velocity gradient evaluated at y equal to 0; that means, if this is the plate, right. This is the velocity profile how it looks like, I'm concerned about the slope and that particular point, right. To calculate what will be my wall shear stress, got the point right?

Now, imagine this particular situation your boundary layer as I say it looks something like that. So, at each point this is the profile here this is the profile somewhere there, right. You see this kind of a profile right. So, this particular at this particular point in the wall which is basically y equal to 0, right. This slope d u by d y at y equal to 0 will show variations will it not it will show variations, right. Because you can clearly see that the slope should change, right. Because it is going from 0 velocity to u infinity over this length scale delta. So, 0 and u infinity are the same right, but delta is increasing, delta is actually increasing, right. From how I have drawn this particular profile. So, naturally the slope over here will change quite a bit right.

So, as you can see that we need to know that slope in order to evaluate what will be this tau wall, right. This wall shear stress evaluation of this wall shear stress requires a knowledge of what is the slope at the wall what is the velocity gradient at the wall. Now how can you determine the velocity gradient at the wall without knowing what the boundary layer actually looks like, correct? There is no way you need to know what the how the boundary layer is actually growing, you need to know the exact variation of the velocity field before you can evaluate what will be the gradient at the wall right.

So, you can appreciate the fact that if you do not know the boundary layer, right. There is no way you can calculate the shear stress at the wall how can you, right. Because you need to know that slope, right. That slope depends on how your delta and how the velocity profile is. So, both of these the velocity profile in the boundary layer is right. So, both delta and the variation of velocity boundary or the functional variation of velocity boundary layer velocity profile inside the boundary layer both are important right. So, 2 important things that you can take from here one is delta what is delta right.

We need to know something about that and how u, right. Varies with delta, right. Once you know these 2 information, right? Then you can evaluate this tau wall, right. It is a clearly the thing right. So, tau wall. So, as. So now, we have clearly established to know the drag you need to know tau wall to know tau wall, all you need to know the velocity profile at the wall, right. To know the velocity profile at the wall you need to know the velocity field and delta; that means, you need to know about the boundary layer correct. So, as an application engineer unfortunately you do not have an easy way out correct ok.

Now, let us look at the heat transfer part right. So, this you said we agree that drag requires the velocity profile at the wall. So, we need to know about the boundary layer. Let us look at the heat transfer that, how? So, heat transfer we already defined that this is the case, right. It is represented by h into this right. So, let us look at whether this can be..

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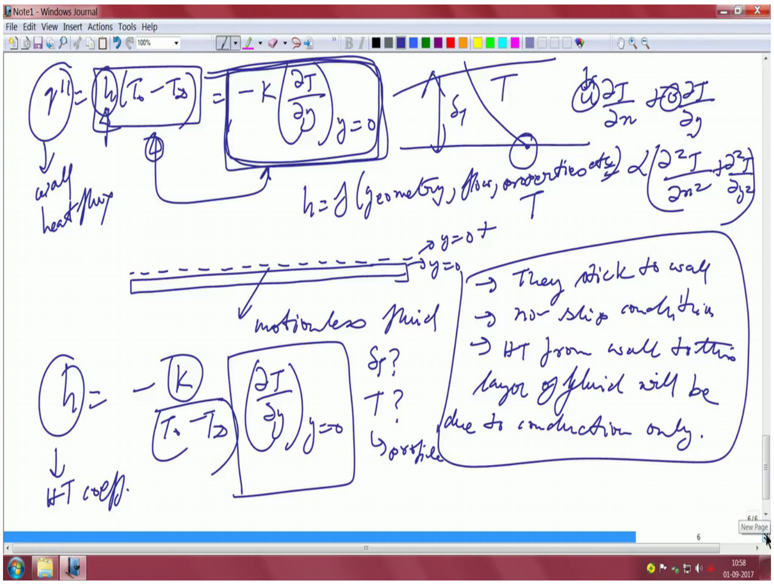

Now if I say that q is equal to h T naught minus T infinity is equal to minus k d T by d y at y equal to 0; that means, just think about it. What I have written I have written that the heat transfer rate or the heat transfer or the wall heat flux q double prime is given by the heat transfer coefficient multiplied by some delta t.

This we already know, right. This is equal to equal to what we call the conduction at the wall. That is how this is written k d T by d y at y equal to 0 right. So, this means that it once again if you have the slope at the wall, right. This is the T variation of T once again. We are concerned about what is the value of that slope of the temperature profile at the wall. And this is nothing but the conduction heat flux correct minus k d T by d y, this you are very familiar with, right. Now the question is oh you are doing convection so far. How come this conduction comes into the picture? How come these 2 things are actually close to each other, right? How are these 2 things equal.

So, the natural question that I will try to explain it that if you look at the plate, you know that if you look at a layer which is very close to the plate which is so, this is basically y equal to 0 this is basically say y equal to 0 plus. Just here the fluid which is adjacent to this wall is basically motionless, right. Why that is because of the no slip, right. Motionless fluid so, it is almost like this this layer of fluid is actually sticking to the surface there is no motion of the fluid. So, they stick to the wall to wall, correct? There is no slip; obviously, no slip condition hence if there is no motion right.

The heat transfer from the wall to this adjacent layer of fluid will be only due to conduction. From wall to this layer of fluid, right? Layer of fluid will be will be due to conduction only, am I right? So, it will be only due to conduction. So, therefore, the equation that I have written for the wall heat flux is the wall heat flux, right. Right that is perfect. Because this particular heat transfer coefficient is therefore, related to the wall conduction right. So, that is what I have written by minus k d T by d y at evaluated at y equal to 0, right. So, once again here the same problem, right. In order to know h right. So, h is what then h is minus k T naught minus T infinity, right. D T by d y evaluated at y equal to 0, correct?

So, in order to know h what do you need to know you need to of course, know the property which is k which is easy that is what I said that the property is there, right. From common sense the property is there you also need to know what is the temperature gradient at the wall, right. It is the same thing like your wall heat flux. You need to know the temperature gradient at the wall. You cannot know the temperature gradient till you know what is this delta T right. So, this requires what is your delta T, and what is the temperature profile. So, temperature profile is important delta T is also important, correct?

So, once again back to square one you need to know what is the temperature gradient at the wall for that you need to know how delta T is vary how the temperature is varying within the boundary layer. And as you know the temperature field is coupled to the velocity field, right. That we established already in the conservation equations, right. Remember that d u d T d x plus v d T d y, right. Is equal to alpha delta square T d x square plus delta square T d y square, right. This is you already know right. So, in order to evaluate T you need to know u and v. So, it brings about the question now that you need to know the velocity boundary layer, in addition you need to know about the temperature boundary layer as well to evaluate this wall heat flux.

Or the heat transfer coefficient, right. This is your heat transfer coefficient right. So, part e is essentially over right. So, as an application engineer, you cannot actually get, right. You cannot actually get away not knowing about the boundary layer, right. Because there are no universal ways there is no one fixed number like k, you cannot quote a number like k..

For this h and for this the velocity as well. So, drag and heat transfer coefficient if your boss ask you evaluate those, right. You need to know what is your boundary layer equations, right. How the boundary layer is right. So, h in a nutshell is a function of geometry flow properties etc. you need to know all those things got it. So, this brings about the point. So, I have established why we need to do all these things.

So, let us look at now the boundary layers properly and try to see that if there is a way by which we can find out answers to our queries. Basically, we need 2 gradients, one is a velocity gradient, one is the temperature gradient at the wall. If we can get those 2 numbers is basically the gradient at a point. So, it is basically that value if we know that value we are done. So, all this all this conservation equations is all going through all this problems is basically to get 2 values right. So, the basic governing equations, let us write it once again..

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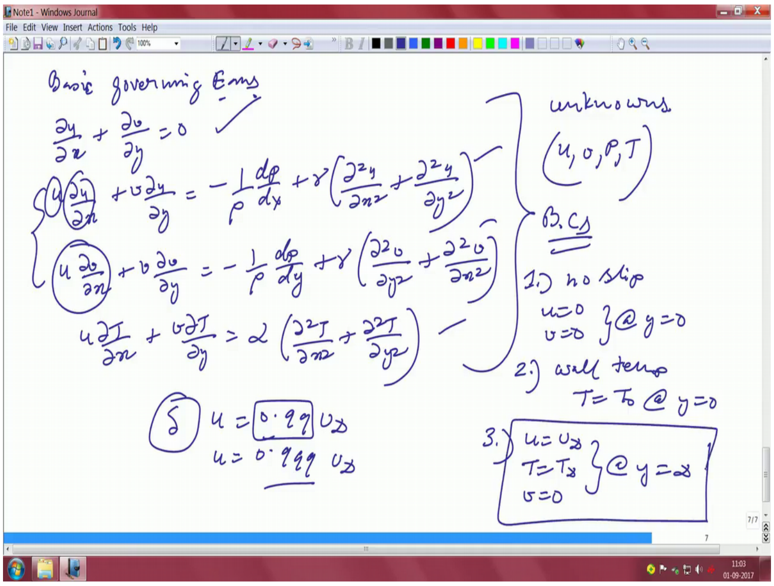

You already know this from the last few lectures and I’m writing it in 2 d anything can be projected to 3 d also. So, this is the continuity the first equation right.

Momentum actually has got 2 equations now. This is because of the x and the y v d u d y, that is the convective derivative part d P d x. There is the Laplace that is a viscous term the last term that I wrote. Similarly, there is a y momentum equation. One is x and one is y for the configuration that we showed, you got it? So, that is the 2 momentum equations, then comes the energy equation. Once again this is the conduction or the diffusion part of the term, got it?

So, basically there are 4 sets of equations that we have. We have unknowns as u v P and T, right. These are unknowns, got it? What are the boundary conditions in order you can see that these are all P d s, right? P d s means partial differential equations, the momentum equations are non-linear because of the presence of these kind of terms over here. There is non-linear that is the reason why you cannot solve most of them.

So, based on this so, but still in order to pose the problem we need to know what are the boundary conditions. So, the first boundary condition is basically the no slip; that means, u is equal to 0, v is equal to 0, at y equal to 0, correct? So, that is a no slip no slip no penetration boundary conditions. I mean this plate is not porous also; that means, there is no aid the flow cannot go through the plate. So, this is a perfectly impervious plate to begin with we will see porous examples also later. Wall temperature say it is defined you get T equal to T naught that is at.

Once again y equal to 0. The third boundary condition is u is equal to u infinity, T is equal to T infinity, v equal to 0 at y equal to infinity, why? This is infinity and not delta, that is because the boundary layer always asymptotes, right. So, it will asymptote to the value of u infinity or T infinity, it will never become it will only become equal to that at infinity essentially.

So, we will see that we have to find a prudent definition of delta, right. In some cases, people say delta is that value when u becomes equal to something like 99 percent of u infinity, right. Or if you want to be more accurate you can go and make this 9 9 9, whatever it is that that is your choice this is a rather arbitrary number, which you can cater to your own way. Say for example in some cases, you are interested in very accurate estimation of your tract. In that case you might have to take numbers which are very I mean large places of decimal, right. But if you are interested in just a casual calculation and it is may not it depends on the requirement of your problem. You might deal with about 99 percent. Most of the cases it is about 99 percent of the free stream. But in reality, this delta actually is infinity that is because it asymptotes is never quite reaches you infinity it only does that at u at y equal to infinity.

So, that is why this validates this particular judgment. None of these equations are actually valid close to the leading edge of the trailing edge of the plate because of the reason that there will be other kinds of problems. So, which you are not going to go into the details of those problems, but when you actually encounter when suddenly you encounter an obstacle, there are a lot of complicated fluid structures that are created at the leading edge of the plate.

So, leading edge effects are kind of ignored so; that means, we are looking at a slightly downstream distance. So, therefore, we are making certain assumptions, which may not be valid at the leading edge of the plate, but we are going to live with that for the time being because that is the only way that we can cast or get some semblance of good analysis done on this particular problem.

So now that we have established the governing equations and the boundary conditions. Let us try to see if we can introduce some kind of a scaling argument to the whole thing. So, let us see that what the scaling actually means. Scaling means that it is an order of magnitude kind of an analysis; that means, I can I do not need to solve the problem, but I need to know what are the scales of the problem. Like for example, in certain cases fluid dynamics is a very complicated beast right. So, for example, there may be certain things which are which are relevant in a small scale, but; however, if you do not want the small scales to come into the picture you may only deal with the large-scale physics.

So, there are certain scales that are present in the flow field. And we may be interested or we may show that only certain scales are important for the present study. So, it will be only evident if we give you some examples which is related to our case. So, let us redraw this boundary layer..

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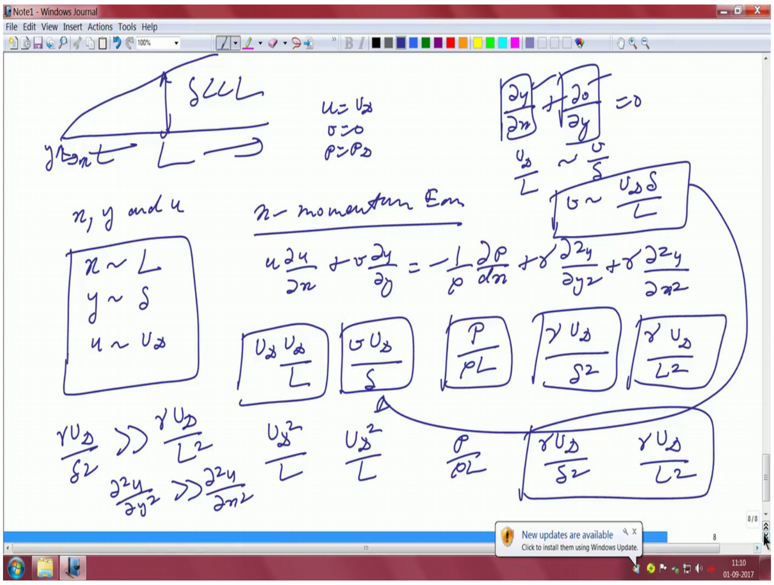

So, that is the boundary layer profile once again you have your x and y marked out. This is the length of the plate L free stream. As we said u equal to u infinity v equal to 0 P equal to P infinity, right..

Now, the scales let us establish what are the scales of x y and u right. So, here x scales as L, right. This is the length of the plate as you can see right; that means, the relevant scale for x, that is the distance in the I mean in the x direction is the length of the plate. Because that is how you can see it readily; however, why the relevant scale is delta. Because delta is the distance over which the velocity varies from u 0 equal to u infinity right. So, that y is not infinity. It is over that distance delta that is most relevant right. So, that is the relevant variation of delta. And u is of course, dependent on u infinity, right. That is the scale of u, right..

So, in view of these 3 numbers or 3s, 3 scalings let us write the x momentum equation right. So, the x momentum equation only the x momentum equation. We are writing v d u d y or in this case. Let us put it as this now let us sport the scaling arguments inside this thing. So, for example, you scales as u infinity. So, the first term is u infinity into u infinity by L. Because x scales as l. So, we are put just putting the relevant scales over here, right. This will be v we do not know the scale for v as of now. U is u infinity y is delta. So, that is the second term over here.

Then comes pressure let us take it as P over here for the time being given as rho into L because x is once again scales as L, right. Then there is gamma u infinity divided by delta square over here right. So, that is given as delta. Then there is u u infinity divided by L square, right. Because x scales as L square. So, these are the relevant scales or the relevant orders of each of the individual term of the momentum equation, right..

So, in this case it is quite evident that if we can we simplify the problem a little bit based on this right. So, if we look at the continuity equation here, that is equal to 0. So, if you if you look at the continuity it is u infinity by L. That should be the same as v by delta. So, v will be given as u infinity delta by L, right? Correct? That comes from the continuity equation, because u and v has to be of the same scale of the same order. This 2 terms has to be of the same order, right, because otherwise you can actually neglect the other. So, if one term is like it is like a balance, right. If one term is very small one term is very big you can; obviously, throw away the small term, right. In certain approximate cases.

So, in this particular case now if you back substitute this over here, you will find that both the terms. Will now become u infinity square by l; that means, none of these 2 terms are actually irrelevant; that means, the 2 terms the convective terms in the momentum equation there are the same order, right. And then you have P by rho L, then you have this by delta square and gamma u infinity by L square, correct? These are the terms that you have.

Now, already inspection between these 2 terms will reveal a interesting story, what is that u infinity by delta square is; obviously, much much greater than u infinity by L square, why? That is because your delta as we said initially, this delta is much much smaller than the length of the plate, correct? Because it varies the boundary layer varies over a very small distance right. So, in an order of magnitude analysis.

What we are trying to say is therefore, d square u by d y square is much much greater than the d square u by d x square. So, basically the axial variation of the u velocity or rather the viscous term due to that is negligible compared to the viscous term that you generate because of the transverse direction right. So, the equation becomes very simple.

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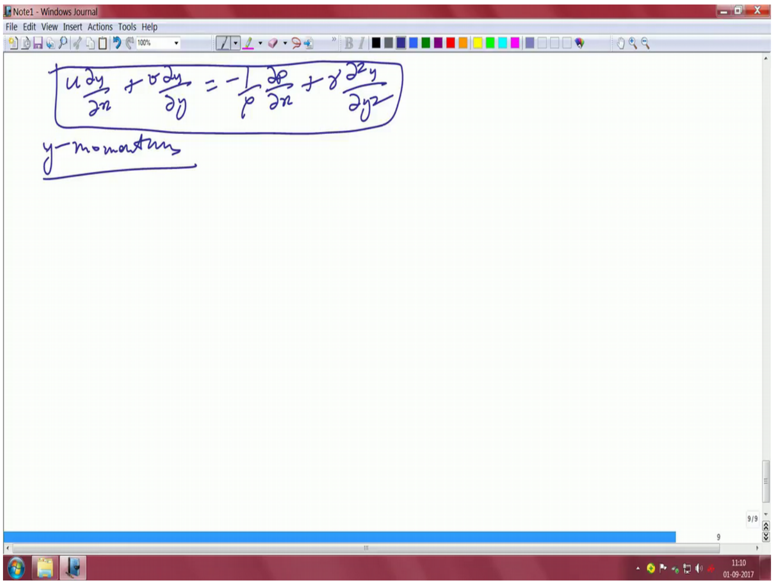

In that case, you get u d u d x plus v d y minus 1 over rho d P d x plus gamma del square u del y square, right. Because we have got rid off the x term now, got it?

So, similarly now let us look at the we should look at the y momentum equation now also, right. The. So, y momentum equation now needs to be looked at. So, in the next class what we are going to do? We are going to look at the y momentum equation and try to see that what we can get out of it.

So, in the next class; so the x momentum is very clear we have got rid off one term. Now let us look at the y momentum equation and see that what we can extract out of this, because still there is a pressure term which is hanging around over there, right. About which we do not have much of an idea. So, let us see that how the y momentum equation can come in handy.