Convective Heat Transfer
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Lecture – 48
Turbulent boundary layer – Viscous sub layer

So, welcome to this lecture. So, if you recall what we did in the last lecture, we covered in details that, how the turbulent boundary layers or the how the turbulent eddy viscosity, and the eddy, basically the eddy diffusivity and the eddy thermal diffusivity actually works, and we also talked about the Prandtl mixing length model which actually showed that how you can basically.

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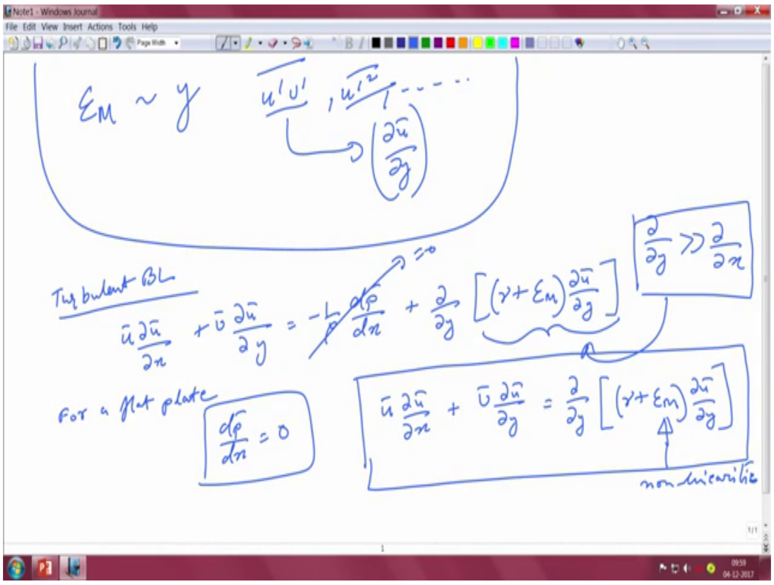

Write this E M all right? In terms of y, essentially that is what we kind of did in the last class, right? Because we showed that the non-linearity of the Navier stokes equation when averaged, actually gives rise to this unclosed terms, in this unclose terms where, if you just recall it was u prime v prime u prime square bar etc, right?

So, these were the unclose terms in the Navier stokes equation when you averaged to the whole Navier stokes equation Reynolds averaging basically. So, once we did that averaging, we saw that these were the unclose terms, and in order to close this unclose terms, we needed some modeling approach which is not basically based on physics, but basically based on intuitions and certain logics. So, essentially that was the whole point that was the time that we spent, in understanding that how these terms can be represented in terms of the velocity gradients. So, that is what we did in the last class, that how this can be represented as velocity gradients, and then bunch them up with the viscous terms, right?

And all them basically the apparent viscosity or the eddy viscosity whatever, there are varieties of nomenclatures that are available, but these are not viscous terms, again we emphasized it in the last class these are basically terms which evolves from the, non-linearity of the Navier stokes equation, right? So, similarly in order to close the equations you needed to close this terms, right? So, in order to close those terms, we advocated a simple model, which is basically called the Prandtl's mixing length model, and that is the mixing length model that we actually are going to use in the subsequent sections.

So, next we took the problem of a flow over a flat plate, this is exactly what we started the course with, right? That external flow over a flat plate so that was the flow that we started with, and in that flow, we saw that, many of the boundary layer assumptions that we made, were actually valid in the turbulent flow also. So, let us rewrite the equation. So, it is a turbulent boundary layer.

So, the equations where this is the averaged Navier stokes equation, and this was the bunching that we did, if you recall this is the bunching that we did. Nothing but the apparent shear stress and of course, if it is a for a flat plate if it is a flat plate then we know that dp bar by dx should be equal to 0, right. And we got this expression by using the same logic, that this gradient is usually more than this, right? That is you see just one term coming over here, right?

So, if this term therefore, goes away you are left with this is the total equation, right? This is the total equation that you have now, in this particular equation now you see these are the basically the convective derivatives, all the nonlinearities are actually hidden here, got it. That is the most important thing to take care now, we are going to make a leap of faith and try to see something.

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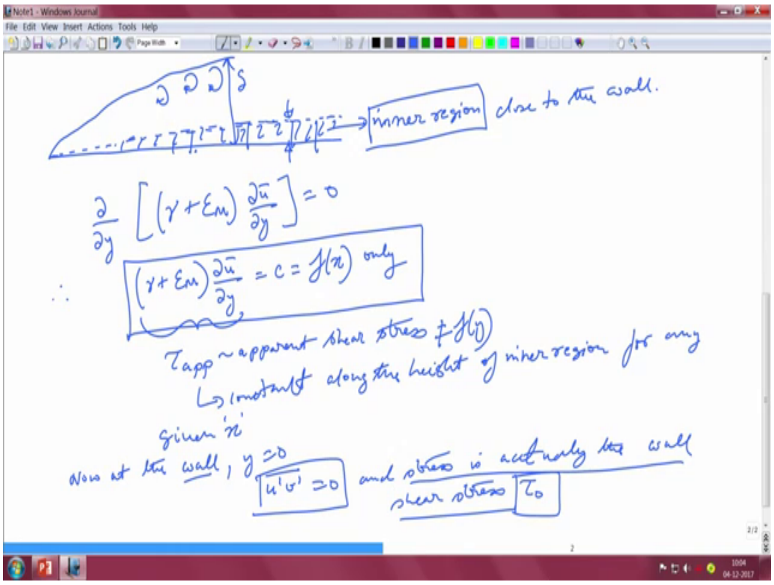

So, if you consider this to be the flat plate all right and this is the turbulent boundary layer, right? Which has got all the eddies that we talked about and these are various kinds, right? In the region very close to the wall let us, envisage that there is a small layer. Where which we can call as the inner region, it is called the inner region it is a region which is very, very close to the wall, right?

And where, your advection terms would not be that important, it is a region which is very, very close to the wall, it is a small fraction of the total boundary layer thickness. So, if this is the total boundary layer thickness delta this is some region, which we do not know why? What it is at this particular point, but this inner region we call it the inner region, and in this inner region it is very close to the wall, where advection can be actually neglected.

So, in that particular region what we can do is that we can write this is equal to 0, right? Because, the advection term we have completely neglected. This actually leads to since, the differential of this with respect to, or the partial differential of this with respect to y is actually equal to 0, gamma plus epsilon M C which is now a function of x only got it.

So, this is now a function of x only got it. So, it is gamma plus epsilon M. Which we, basically call the apparent shear stress is basically as a function of x only so; that means, if this guy is now, called tau apparent which is basically nothing but, the apparent shear stress, right? This is nothing but, the apparent shear stress is not a function of y.

So, in this particular region in this inner region, we talked about this is a inner region tau apparent is basically constant, along the height of that particular layer. So, this tau apparent is not a function of y means, it is not a function of the height, from the along the transverse direction, right? So, in other words this means, that this is constant along the height, off the inner region for any given x, right? It is a function of x only, right? So, along this along any height if you consider it this, will be kind of constant. So, this makes our job very, very easy actually to analyze the problem.

Now, at the wall that is at y equal to 0, right? Absolutely at the wall, right? At the wall whether, we are considering this y equal to 0, you can expect your u prime v prime to be equal to 0, it is very close to it is at the wall and the stress is actually, the wall shear stress tau naught. So, if you call the wall shear stress tau naught, which is basically once again mu into d u bar by dy this, has got no effect of E M our epsilon M, right?

So, at the wall not away from the wall at the wall your u prime v prime is equal to 0, which essentially translates to that, your E M term you are basically neglecting, the stress there is actually the wall shear stress tau naught, it is the actual wall shear stress at that particular point. So, if you evaluate this quantity at y equal to 0 it will be, it will give you the actual wall shear stress, that is the point that we are trying to make here, right?

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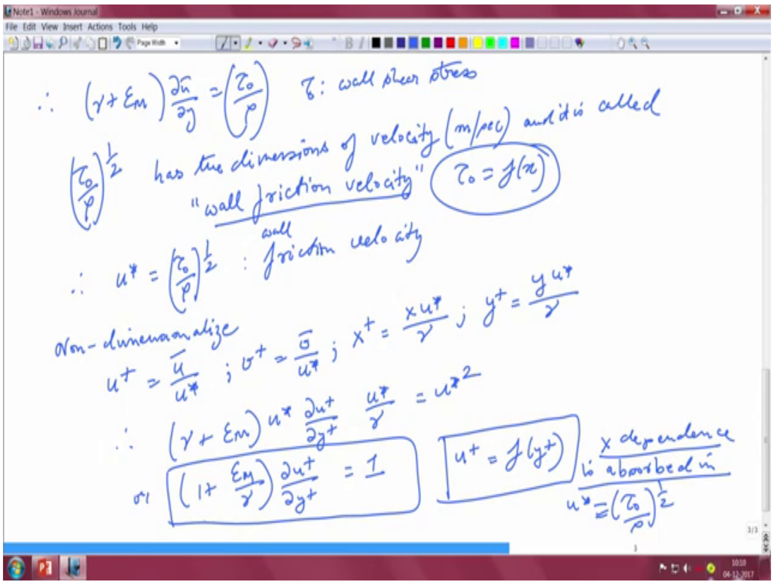

So, therefore, your gamma plus epsilon m du by dy is basically equal to tau naught by rho this is basically the constant that we are talking about because, remember we said that, this is supposed to be a constant. So, this is the constant that we are talking about in this particular case. So, having remember always tau naught is the actual wall shear stress, this is not the apparent shear stress, these 2 things must be clearly spelled out.

So, tau naught is the wall shear stress now interesting feature about tau naught by rho raised to the power of half, right? This if you look at, it has the dimensions of velocity in the sense, it is given as meter per second if you look at it, and therefore, and it is called wall friction velocity got it. So, it is called the wall friction velocity against a misnomer, is just a dimensional matching that we have done.

And of course, as we know that, tau naught is a function of x, this we know from your laminar boundary layer theory also, right? You know that the wall shear stress is a function of your x, that is what we derived earlier. So, it is a very similar thing is valid over here also. So, therefore, let us, call this friction velocity as u star, and it is given as tau naught by rho raised to the power of half, this is basically the friction velocity or rather wall friction velocity.

Now, using this option let us, try to non dimensionalize some of the equations. So, non dimensionalize. So, first one is your u plus is u divided by u star, v plus is v bar this is u bar v bar divided by u star or r bar quantities these are averaged quantities, x plus is given as x u star by gamma similarly, y plus they are given as y u star by gamma this is fine because, gamma is basically meter square per second y is meter. So, it becomes meter per second and meter per second on the top and the bottom. So, they basically give you a dimensionaless length scale.

So, therefore, gamma plus epsilon M u star du plus by dy plus multiply it by u star by gamma given as u star because, remember this is nothing but tau naught by rho because, we have taken the square of that essentially u star is root over of that. So, this should be the square of that, or in other words 1 plus epsilon M by gamma into d u plus by dy plus is actually equal to 1. It is actually equal to 1 this, is one of the most important equation, this is only in the inner region remember, once again we have done all these things in the inner region.

So, basically your u plus is now, a function of y plus, right? From this equation where, the x dependence is absorbed in u star because, u star is nothing but tau naught by rho, right? Raised to the power of half correct, right? So, therefore, tau naught is a function of x we have seen it here, right? So, therefore, the x dependence is basically absorbed in the u star.

So, u plus is basically a function of y plus, very similar to that similarity transformation that we did, many classes back, right? So, it is very similar in principle or in essence to that.

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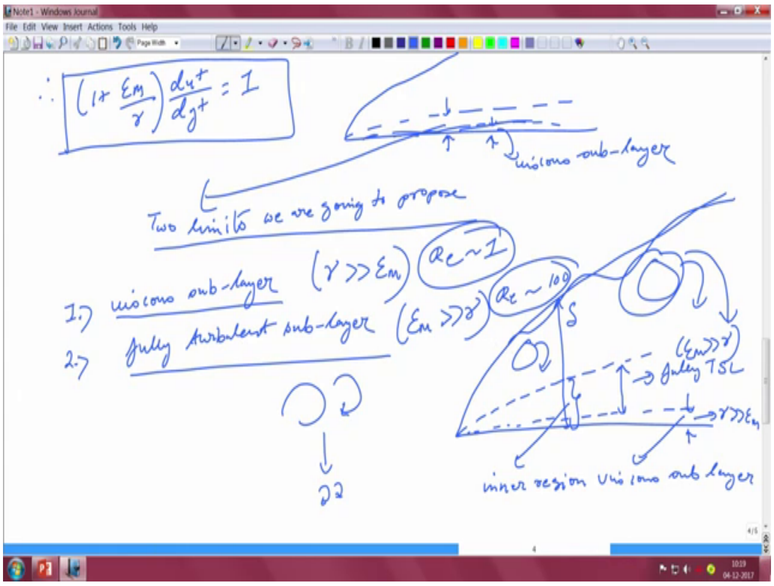

So, therefore, 1 plus in terms of the converted units, right? This is the expression, right? Now, let us see, in the inner region also whether, we can have sub regions.

Remember we said that there is this inner region and then of course, you have the parent boundary layer. So, within this inner region let us see, there are 2 limits. We are going to propose there are 2 limits, that we are going to propose first one is called the viscous sub layer. Where this is gamma is much greater than epsilon M so; that means, it is a region, it is a sub region, very close to the wall. Which is basically called the viscous sub layer got it.

Why we are doing all these things inner region outer regions etcetera because once again we are interested in the shear stress all right and later on the wall heat transfer rate and for that we need to know the nature of the flow not just in the entire flow field, but very close to the wall this was the question that we stated when he started to do convective heat transfer that for any engineering problem the 2 important things that one needs to know is basically the wall shear stress and the wall heat transfer coefficient, right?

In order to know those 2 things, we needed to know in the laminar counterpart what is the shear stress at the wall; that means, the velocity gradient at the wall and the what is the temperature slope at the wall, right? So, these 2 things we needed to know and essentially the same thing we are trying to apply over here that you have may have a very turbulent flow field, but our interest may be if we can know that what the slopes are the profiles are behaving very close to the wall we might have a good idea about what will be the wall shear stress and what will be the heat transfer coefficient all right. So, that is one of the reasons that we are following all these motions.

So, the viscous sub layer we got it that this is should be greater than. So, basically the kinematic viscosity is much much higher than eddy viscosity because in this, but and that particular layer the viscous as we if you recall we say it that when the Reynolds number is very low; that means, when the eddy sizes are very low dissipation becomes very important.

So, basically we are in that particular regime, right? Very, very where viscosity is highly dominant then you have the fully turbulent sub layer once again this is the sub layer this is all a part of the inner region, right? Fully turbulent sub layer, right? Fully turbulent sub layer where your epsilon m is much greater than gamma, right? So, in the fully turbulent sub this is once again a part of the inner region.

So, if you just blow the whole thing. So, this is say is your inner region this is basically the full boundary layer, right? Out of this inner region there is this small region here, which we are calling as the viscous sub layer, right? Correct this part we are calling it as the fully turbulent sub layer all right and together this entire thing is basically called the inner region orbit hm.

The inner region is further divided into 2 in this particular part you have your epsilon m is much greater than gamma; that means, you know that your Reynolds number will be a little higher more than one all right and in this particular region you know that your gamma is much greater than epsilon m all right. So, these are the 2 limiting conditions that we are trying to put forward here all right and of course, then the entire thing this entire thing is basically your delta which is the thickness of the total turbulent boundary layer got it.

So, in the inner region we have this in the outer region we have that. So, the under this considerations we can see that here of course, the scale you can target that Reynolds number will be of the order one here we will see that the Reynolds number can go up to about 100. So, beyond that. So, these are basically the limits of the 2. So, what we would do is that let us take the first one which is basically the viscous sub layer..

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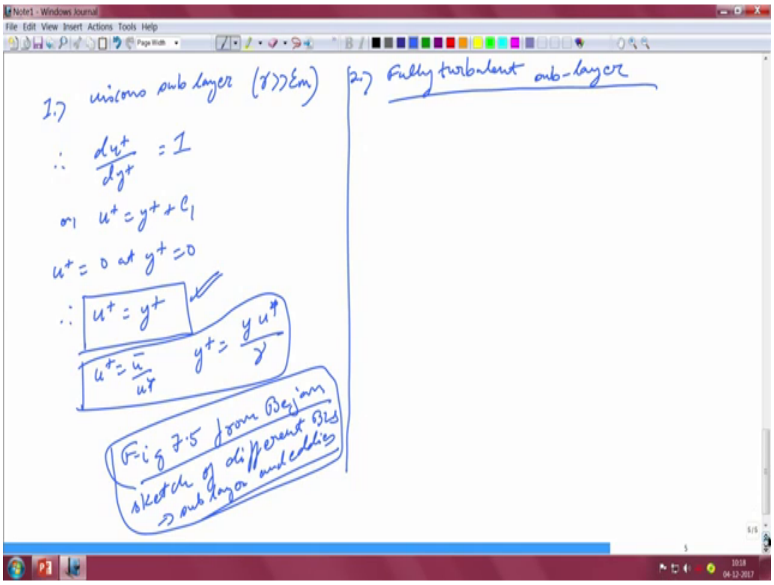

So, the viscous sub layer where. So, therefore, what you have is that u plus y dy plus you have it as equal to 1, right? Or in other words u plus is equal to y plus plus C one we know that u plus is equal to 0 at y plus is equal to 0 at the wall therefore, your u plus is equal to your y plus, right? So, the first viscous sub layer expression inside the viscous sub layer you have a linear function of u plus and y plus, right? U plus remember was u bar by u star all right y plus remember.

If you recall that what was the expression for y plus it was y u star by gamma, right? Always recall these 2 if you lose perspective, but this is a linear relationship that, we have got in that particular region all right in the viscous sub layer of the problem, right?

So, in the viscous sub layer of the problem this is what you get now. So, the viscous sub layer will be linear, but it will scale up to a very small distance only now, we have to look at the scale of the fully turbulent sub layer once again is called sub layer, that is the that is a whole point. So, the second point is basically you have the fully turbulent sub layer sub layer.

If you want to look at the look at the schematic what we drew you can look at figure 7.5 from bejan, where we can where you can actually see that how the 2 boundary layers I mean a sketch basically this is a sketch basically of different boundary layers and it will also mark what is the sub layers and the eddies all those things.

So, that may be consulted and we will also put one of these things as a I will show you this slide at the end of this particular lecture where we will see that how this different sub layers are, right? What are the different sub layers that are associated with it and it is just I have drawn it kind of like this if you look at it here I have drawn it already this just gives you a better idea all right how is the eddies for example, this surface will be a little corrugated because of the general nature of the eddies ok.

There will be like fast moving fluids which will descend that will try to impinge close to the wall. So, this is basically represents the inner scales all right at which your eddies if you remember the cascade there are those large eddies which ultimately went down to the smallest eddies. So, this will be something equivalent to that large blobs of fluid will be carried down towards the towards the wall.

So, this we would put up as a part of this particular lecture, but in the next lecture, we are going to look at what will be the fully turbulent sub layer, what would be the equations for that.