Convective Heat Transfer
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Lecture – 45
Introduction to Turbulence

So, the question number this is will be the final question that we will do in this particular natural convection sample problems. So, if you take a look at the problem over here, there is a it is a typical window in a room all right.

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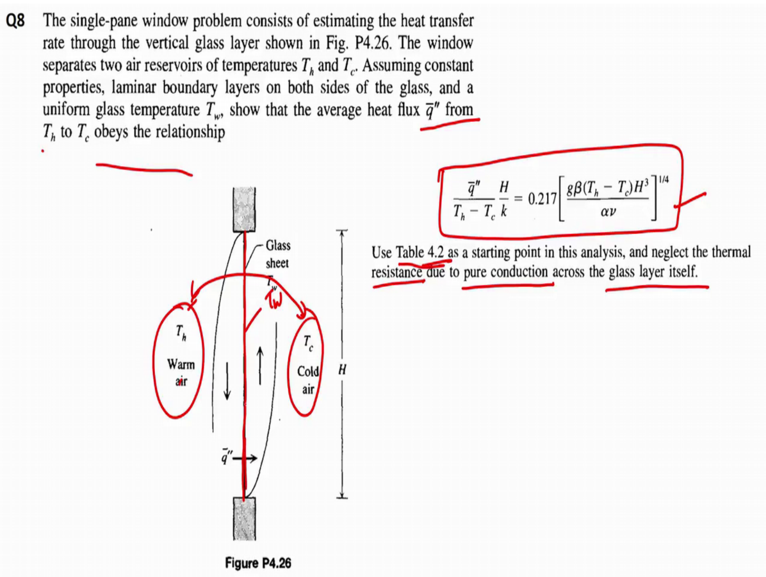

So, it is a single pane window, single pane not double pane; that means, there is no intermediate air layer in between. You can have problems with she has got intermediate air layer as well.

The single pane window consists of estimating the heat transfer through a vertical glass layer. So, this is the vertical glass layer, just shown in this figure. The window separates 2 reservoirs, this one is a warm, and this one is a cold. So, assuming constant properties, the laminar boundary layers on both sides of the glass, and a uniform glass temperature which is TW, show that the average heat flux q double prime from this to this; obeys the relationship which is given by that.

And you are supposed to choose table 4.2 as a starting point in this analysis, and neglect thermal resistance due to pure conduction across the glass layer itself. So, understood so, the glass wall is at a temperature of TW, right hand side it is cold, left hand side it is hot, all right, hot. So, the average heat flux from the warm side to the cold side is given by this particular relationship, this is what we need to prove all right?

we need to prove that neglecting any pure conduction effect that is happening in that the class itself. So, it is typical a cold country problem like, where you have your window pane, and window pane is a source of loss, right like for example, if you have a you know heated room essentially in any cold place. Even in India if you go to Shimla in those kind of places during the winter. So, you have to estimate what is the heat loss that is happening. Because that heat loss is actually costing you the money right.

If you are a design engineer, that is what is costing you the money. Because you want to cut down on that heat loss, correct? So, let us attack this particular problem now. So, what we will do is that, we will take the so, for the so, this will be your question 7 essentially..

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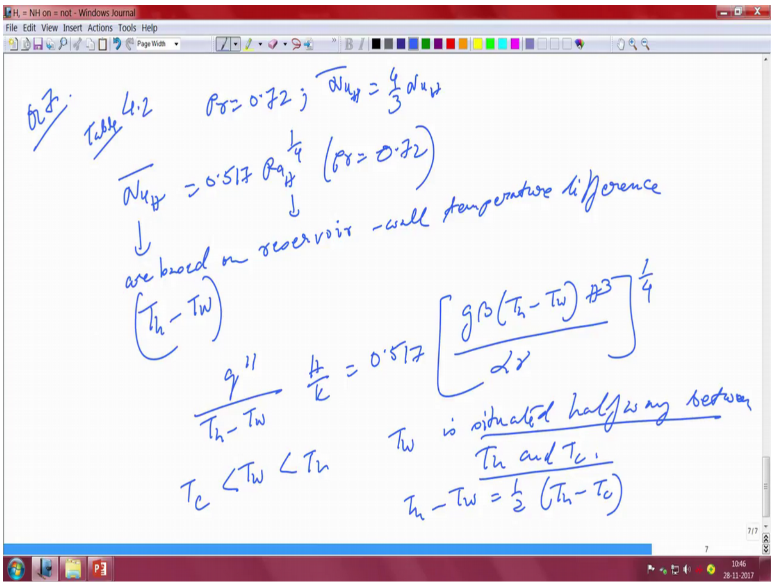

So, table 4.2, if you look at table 4.2, it recommends that for prandtl number equal to 0.72 Nusselt number H bar is approximately 4 third of the local Nusselt number. The height averaged heat flux right.

So, Nusselt number h, which is height averaged is 0.517 Rayleigh number h to the power of 4[th], this is for prandtl number equal to 0.72 only. So, both this as well as this are based on reservoir, reservoir wall temperature difference. Difference which in this case it is T h minus TW remember that T h minus TW. So, q double prime T h minus TW into h by k is given by 0.517 g beta T h minus TW h cube divided by alpha gamma raise to the power one forth..

So, TW essentially lies in between. So, it is greater than TC less than T h this we kind of know. And it is usually situated halfway, it is situated halfway as you mean symmetry halfway between T h and TC, halfway means therefore, TW T h minus TW is equal to half T h minus TC. So, it is situated halfway between these 2 limits. That is because comes from the symmetry. And it also can it is evident in this particular picture over here.

Where you see that these are the 2, boundary layers, that you see which are symmetric boundary layers from either side. So, because of this symmetry, you can therefore assume that this TW is basically situated exactly halfway between the 2. So, therefore, the average heat flux based on q double prime bar T h minus TC H by k equal to 0.217 g beta T h minus TC H cube gamma alpha to the power one forth.

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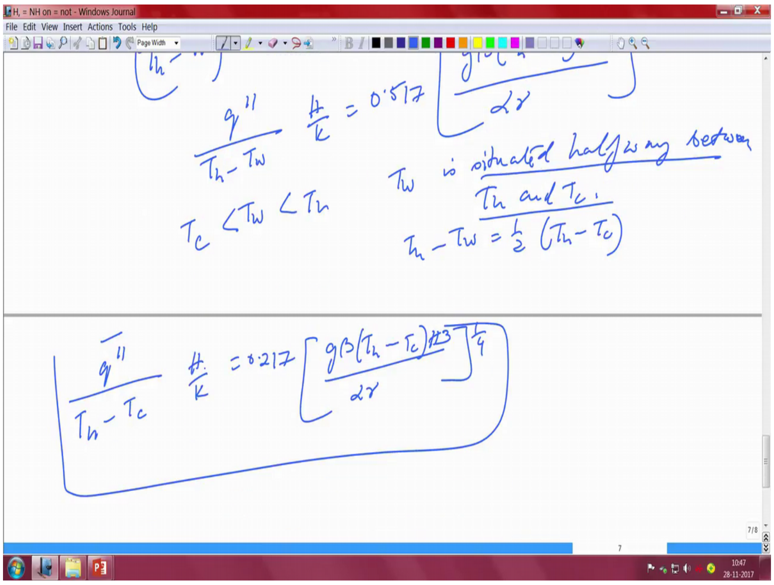

So, this actually proves the question that we had that what will be the so, this was exactly what was asked over here, and we proved it only thing over here that we assumed, because of symmetry the temperature of the wall is just all right in between the 2. But however, all the calculations were performed between T h minus t wall, T h minus t wall that was the that, was how we did the calculation. That was how we did the calculation over here, if you see, and then we just transferred it to the TC value.

Because wall was basically halfway in between TW and TC. So, this basically concludes our lecture on this natural convection part. So, natural convection we have looked into different types of flows, we have looked into different types of effects, and we are basically now in a situation, that we have been able to solve most of the natural convection problems using scaling arguments, using simple mathematical arguments, we have also seen the analytical suit of products, and we have also done some maths problems just to give you an idea..

Like we did it in forced convection, right. So now, it is a time, that all these flows were limited to laminar flows only. So, we were only concerned with laminar flows. At this point most of these and we kind of stayed away from the flow transitioning to turbulence, we also said for example, in Rayleigh Bernard convection. The flow becomes more and more complicated, and then it goes to turbulence. We did not spend much time in illustrating what is turbulence. And even in the in the boundary layer forced convection boundary there he said after some time the flow transitions into turbulence right.

So, and we know kind of heuristically, that when you have a pipe flow if the Reynolds number exceeds a value of about 4,000, all right you get turbulent flow. In the case of our external forced convection, when the Reynolds number exceeds somewhere relay around 10 to the power of 5 ish, right you get to have turbulent flow. Similarly, here the Rayleigh number about 10 to the power of 9, if you exceed you have turbulence turbulent flow appearing into the system.

So, but we never really said anything about the nature of the turbulent flow, all right. In the most of the flows that you see in your real-life applications are actually turbulent flows, all right. So, any problem which is related to heat transfer most likely their turbulent flows, unless you are dealing with micro fluidics and micro heat exchangers and those kind of applications, there the flow still may be in the laminar regime. And turbulence is kind of interesting, and it is important because it promotes mixing all right it does a lot of lot of cool things right so to say.

So, in this particular chapter, what we are going to start to do is basically, we will start to look into the turbulence from a bird's eye perspective. We are not going to go into the details of turbulence, as I said in the last class that that demands a course in itself. So, this is just to give you an idea that what turbulent flows are how does it effect the heat transfer right.

So, that later on you have a kind of a feel, when you take a more advanced course maybe on turbulent heat transfer or turbulence, I mean in the fluid mechanics perspective or if you are doing combustion then turbulent combustion all right. So, there is an entire family gamut of courses that one can take. But this is just to give you an idea that how turbulence can be attacked what is turbulence, how it can be attacked right.

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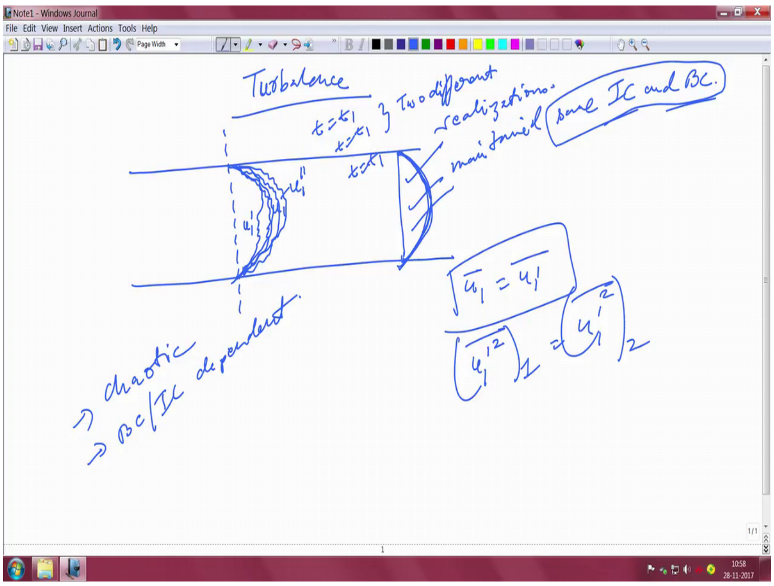

Now, in the case of a turbulent flow. So, we first do our lecture on turbulence. So, turbulence by the name, that we say it is these are the turbulent times things are turbulent. So, it means something that is a little bit chaotic. That is what we mean right essentially. So, when you say turbulent time when it is a political situation is turbulent; that means, we say that there is a lot of chaos, all right.

So, essentially turbulence is something that we associate with chaos, all right. Some kind of a chaotic thing unpredictable things that are not going according to a nice plan, all right these are the things that we normally say. So, if you just for example, take a simple pipe flow and this time we are dealing with a turbulent pipe flow. And you look at any particular cross section, right it is a turbulent pipe flow.

So, what you do is that you have a very accurate measuring device; which is basically measuring this flow profile, right you do not know how, but you are basically measuring the if you recall we showed this nice parabolic profile, right in the case of a laminar flow which we said in the fully developed regime does not change that the flow is non-accelerating in nature. But in the turbulent flow now you have an instrument by which you can visualize this profile, that somebody is drawing the profile for you.

Let us assume that you can actually do it. So, if you look at nicely at this particular profile, you first see that at one particular instant this profile shows something like this. So, there is a lot of oscillations, there is one particular snapshot. So, this is the profile which was taken at time t equal to t 1 say for example. So, at time t equal to t 1, that is the profile that you get. So, let us assume that this is the profile that you get at time t equal to t 1, for a flow through a pipe, that is what we have.

Now, what you do next is that this same experiment. So, this particular profile depends on whatever is your initial condition, whatever is your boundary conditions are, all right. So now, you do this experiment once again right. So, this experiment is over because time marches. So, next experiment add that time t equal to t 1, right? You take another snapshot of that particular profile right.

So, what do you get that particular snapshot will show something like this, right. Remember, this is say u 1 this is u 1 once again. So, say prime this prime does not mean differentiation, right both were taken at time t equal to t 1. But 2 different realizations, 2 different realizations. And you have maintained, maintain the same initial condition and boundary condition. So, it is a very controlled experiment that you are doing in the best possible facility of the world, with the best possible sophisticated equipment's that you have, you are doing this nice little flow you are measuring the flow at a particular cross section at different time at the same time instants but multiple experimental realization..

In the laminar flow if you take the measurements, the profiles will simply fall on the top of each other, right because it is laminar flow it will just fall on the top of each other right. So, you take a next realizations. So, that gives you something like this. So, this is say u prime double prime. So, these are not derivatives, these are not anything else these are not fluctuating quantities. This is all once again at time t equal to t 1 a third realization you get a completely different profile.

So, initially your idea is oh my experiments should be wrong. So, I must be doing something very wrong in my experiments. Let me look at the initial conditions and the boundary conditions maybe there something is fluctuating. So, you look at the initial and the boundary conditions very carefully. You look at each and every parameter space very carefully and you find no, there are no variations.

So, even if I take another measurement I will get something like this. If I take some other measurements it will be something like this. So, all in all, at the same time instant at the same location, I am never able to match my velocity profile. But in the case of a laminar flow, if this was a laminar flow all your profiles will just fall on the top of each other. Absolutely very little variation right. So, for the same initial condition and boundary condition, you have maintained everything to the best possible ability. You still find that your profiles are drastically different to each other. And they can be drastically different, it is not like small fluctuations.

So, you say let us forget about the instantaneous profiles. Now let us do the average right. So, therefore, you start to compare u bar. U bar instead of a doing the average of you know the fluctuating, instead of measuring each and every profile, right? You are doing an average, right? And you try to see whether this average profile matches between experiments 2 experiments, right? And you find that they do the average profiles actually fall on the top of each other.

So, if you take the average profiles, the average profile of u 1 bar and u 1 prime bar, this prime is once again not the fluctuating quantity this is another experimental realization. You find that they match perfectly. So, u 1 bar and u 2 bar or u 1 prime bar basically match to the dot of each other, right? If you do the moments like for example, if you measure the u prime square bar, now this time it is a fluctuating component.

Between realization one, and this is realization 2, they do match also. So, any of the moments, average is a moment first order moment, and the second order moments. They all match, they seem to line up with each other, right. They seem to give you this, profile regardless of whatever is going on. But the individual profiles do not match; when you take the instantaneous snapshots, and you try to compare a laminar flow both matches, right. Here it does not match, right.

So, you kind of initially you thought that this could be some experimental error or some computational error, whatever you may be doing. You find that is not the case, no matter what you do you get this kind of very disparate profiles. So, this brings to the most fundamental question of turbulence, that it is basically chaotic, first and it is highly boundary condition and initial condition dependent.

So, the idea is no matter what you do, in your experiments or in your simulations, right? You will have small errors inadvertently that will be there in your system. These errors may be in the third place forth place fifth place of decimals right; so when you because this pipe flow is usually maintained by a pressure field. So, if the pressure just fluctuates just a little bit, because that those are experimental hard facts, right.

You can have the best control, but each instrument also has got it is own you know error. So, even in the third place forth place of decimal you are encountering a small error. Even the fastest and the best computers will have some truncation error at some point, right? Turbulence basically amplifies these errors. Amplifies the small fluctuations in the initial and the boundary conditions, it amplifies them. And it amplifies them to the extent that you are getting going to get this kind of chaotic flow. So, it is not something that you can do. No matter what you do, these errors will be there. Inevitably these errors will be there, right.

These maybe of very, very small in origin right. But these errors are therefore, going to be multiplied or amplified by turbulence because of the chaotic nature, right? And that is why you have these kind of fluctuations. These kind of fluctuations in the instantaneous profiles right, but if you take the average profiles, you still get very similar match of what you would have got, if you just dealt with a laminar flow, say for example.

So, that is one of the highlighting features of turbulence, right? That any small things are amplified. Though the navier stokes equation is a dissipative equation in general. But turbulence will locally amplify. Any small errors or disturbance that you may have in your system. And that is the whole point that makes turbulence such a difficult subject matter to study. Because your instantaneous profiles absolutely means nothing over here. Because from realization to realization, it does not match.

So, if I say I have measured the flow velocity profile at time t equal to t 1. And I give it to you. So, this is the profile that I get. And you get a completely different profile at the same time, instant using the same experimental setup using the same personnel using the best of your ability. These 2 profiles will be completely different from each other, right. And we cannot blame each other also. I cannot say oh I am I am better my profile is correct your profile is wrong, right. That also you cannot say simple reason is that we know that in spite of the best of the efforts, the small changes in that initial condition and the boundary conditions has actually led to this, right.

But what we can compare is basically I can compare my averaged velocity, with your average velocity. I can compare the second moment of my which is basically the variance of the velocity with your variance of the velocity. Those statistics will match. So, this also gives an idea that whatever theory that comes out of turbulence has to have some statistical component, it has to be based on some statistics, right. And we will see that what those statistics are.

But basically, the understanding is that you can compare statistics, you can compare some other things also. But instantaneous comparison is not a very good idea over here, because from realization to realization it does not match. It is as simple as that. So, let us look at the first set of things. So, you know that though the navier stokes equation is a closed equation, it is a well posed problem. Turbulence somehow adds a new dimensionality of chaos is a multi-degree of freedom chaos, that you have added to the system basically.

So, if you look at for example, you take a cylinder or a sphere in a flow field, turbulent flow field..

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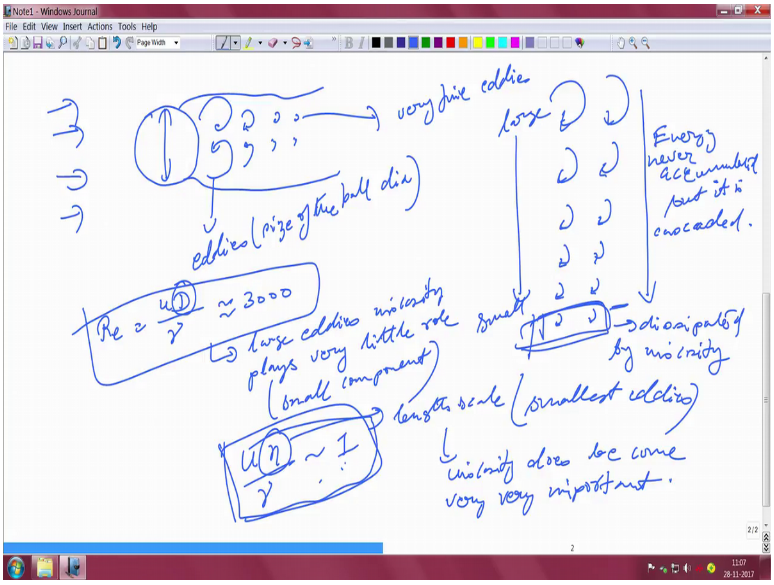

And you start to look at the wake region, right. You will find that a wake will have very large eddies. Eddies means rotating flow components, right. You will have slightly smaller eddies. We will have very fine eddies. You will have lots of these kind of structures. So, basically some eddies, call this eddies, maybe from the size of the ball, if this is a ball. Ball diameter, and these can be very fine eddies. So, in the wake region you can if you are a camera. And if you have that insight, you can actually see eddies of different sizes, from the size of the ball to very small sizes, right.

So, the idea is this leads to the idea of this cascading theory of turbulence right. So, what happens is that the large eddies. So, if you look at the large eddies, then you get slightly smaller eddies, then you get slightly smaller eddies, maybe that is even smaller eddies, then you get even smaller eddies, and then you get to this particular point.

So, if energy is transferred in this direction. So, the large eddies, transfer the s transfers the energy to the immediately, slightly smaller eddies, that gets transferred to the yet smaller eddies, and this way it goes down to the smallest eddies possible, where they are basically dissipated by viscosity correct.

So, the energy is cascaded from the large eddies to the small eddies, right energy is cascaded from the very large eddies, to the very small eddies, large to small, right till viscosity takes care of the dissipation right. So, this is an interesting concept energy is never accumulated in any particular eddies. It is always cascaded away right. So, you do not act as a reservoir of energy your energy. You are transferring it is like a baton changeover you give your baton to the to the next guys, the next guy gives to the baton to the yet, another set of guys till the point where it is you go down to the scale where viscosity is very important. And it dissipates out the whole thing right.

So, the eddies are large to small, and this is how the cascade actually works right. So, if you have a Reynolds number for this kind of a flow say for example, it is u D by gamma say that is of the order of 3,000, you have calculated. Where u is basically the approach velocity right.

So, usually speaking, when the Reynolds number is that high, viscosity cannot be important, right. Viscosity cannot be important because Reynolds number by definition is inertia or viscous. So, it is 3,000 times higher. So, viscosity cannot be important. So, essentially what we mean is that, the eddies which are of this dimension D. Remember, we say that the eddies can be of the size of the ball.

So, eddies which are of the size of the d, where for large eddies essentially, large eddies viscosity plays very little role; which is basically small, small component, small component. But the viscosity will become important, when your scale will become of the order one. So, that means, this particular at that particular length scale, which is basically the length scale of the smallest eddies, smallest eddies, right.

Viscosity does become very, very important. So, here another thing that we never energy is never accumulated, these are some of the principles that you should take down is never accumulated, what it is cascaded. So, viscosity does not become important till you get that scale; which is equal to 1, right. So, energy is not never cascaded.

So, these are the 2 key concepts that we can take out, that large eddies do not are largely immune to the viscosity. Smallest eddies when you actually go down to the scale, where this scale is of the order 1. Viscosity does play a very important role, which is the scale here. The energy is never accumulated, but it is cascaded down. And turbulence is a highly initial and boundary condition dependent phenomena, right is an initial condition and boundary condition dependent phenomena.

And it amplifies any small differences. And it amplifies them; however, the statistics. If it is a statistically stationary kind of systems, statistics will actually match. So, we will stop here, in the next class we will look into the more details of turbulence.

Thank you.