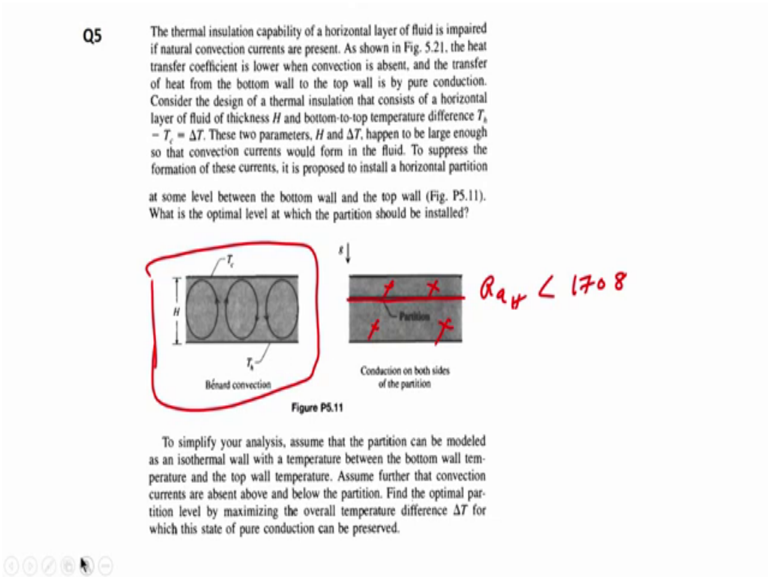
Convective Heat Transfer
Prof. Saptarshi Basu
Department of Mechanical Engineering
Indian Institute of Science, Bangalore

Lecture – 44
Natural convection – Tutorial II

So, last class what we did was that we started doing this problem, if you look at it over here, you will find that this is a typical Bernard convection, which we already covered.

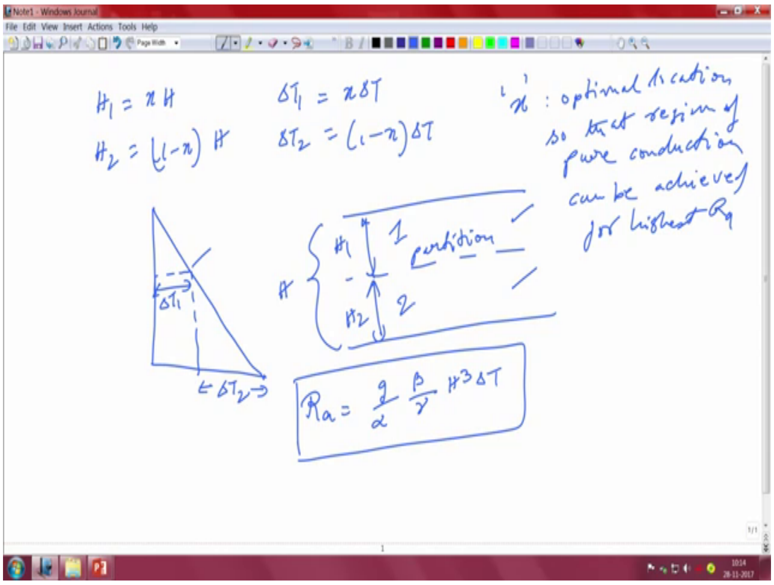
(Refer Slide Time: 00:21)



And now we have put a partition which basically suppresses the Bernard convection at the top and the bottom. So, it essentially translates to that your Rayleigh number must be less than 1 7 0 8.

So, this partition basically freezes the convection on both sides. So, under this pretext we are supposed to do the problem and we started and let me just recap and put the numbers in.

(Refer Slide Time: 01:02)



So, we said that H one is equal to x H and H 2 is equal to one minus x H delta T 1 is equal to x into delta T, delta T 2 2 minus x into delta T. So, the temperature is basically it is a linear profile, because you are having a pure conduction driven problem now.

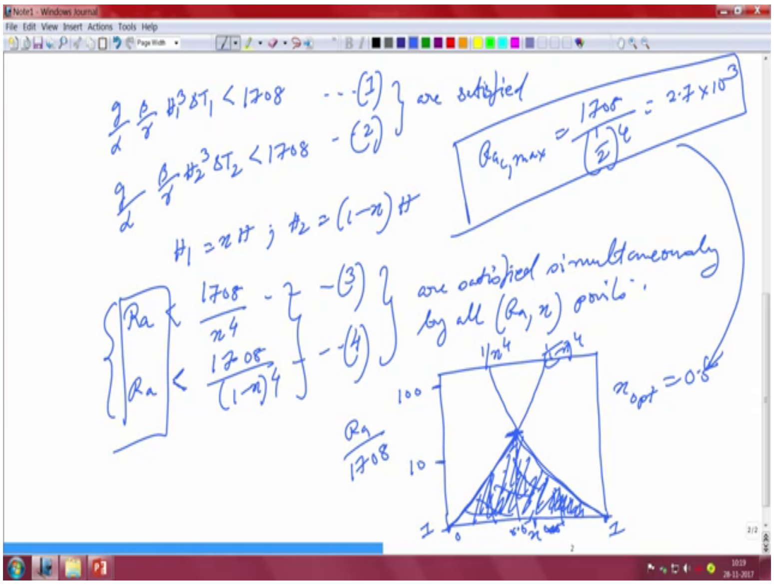
So, this is basically your this particular thing is basically your delta T 1, this is basically your delta T 2 and this is basically the division, this is H 1 this is H 2 and this is the partition and this is the total H. Now our objective is to determine the optimal location x which is basically the optimal location.

So, that the regime of pure conduction extent and regime of pure conduction conduction can be achieved achieved for highest R. So, basically the highest Rayleigh number and Rayleigh number is defined as what R a g by alpha beta by gamma H cube into delta T that is the Rayleigh numbers definition all right ok.

Now, this is the external Rayleigh number based on the overall height and the overall temperature difference, why not actually define Rayleigh number for this intermediate blocks for this as well as this right. Because for the partition as for this entire system each block that is this block 1 and block 2 Rayleigh they almost behave like independent kind of systems well they are strictly not independent, but they behave like one for the purpose of our analysis over here.

So, 1 and 2 basically belongs to we can define 2 Rayleigh numbers based on 1 and based on 2 all right. So, the idea is that if the individual sub layers do not show any convection then there will not be any overall convection in the system as well. So, based on this we can write. So, as long as the Rayleigh number is small.

(Refer Slide Time: 03:51)



So, g by alpha beta and gamma H 1 cube into delta T 1 is less than 1 7 0 8 this we know from the criteria of Rayleigh Bernard convection.

For the lower sub layer so it will be g by alpha beta by gamma H 2 cube into delta 2 that is less than 1 7 0 8. So, that if the convection is suppressed in the entire layer, when these 2 conditions let us name them as 1 and 2 are satisfied, then there will be no conviction in the overall system as well right. So, in other words we can substitute.

So, we substitute the formula for H 1 if you recall H 1 was x into H and H 2 was 1 minus x into H. So, you substitute them over here. So, and we rewrite these conditions as a Rayleigh number less than 1 7 0 8 by x to the power of 4 and Rayleigh number less than 1 7 0 8 1 minus x to the power of 4. So, these are the 2 relationships are given. So, this conditions let us name this as 3 and this is 4.

So, these 2 conditions are satisfied simultaneously are satisfied simultaneously by all Rayleigh number x points under. So, if we draw now the curve you will see what I mean.

So, this is your x basically starts from 0 this 1 is about 0.5 this 1 is 1 and here we are plotting R a 1 7 0 8.

So, this is 1 this is 10 this is 100. So, 1 graph will be something like this. So, this is 1 by x to the power of 4 the other graph will be like this 1 minus x to the power of 4. So, this is the region between the 2 graphs in which you have Rayleigh number less than this as well as less than this on both sides. So, these are the 2 master graphs right. So, on this side you satisfy 1 criteria not necessarily the other. So, in this is the region where you basically satisfy both the criteria's.

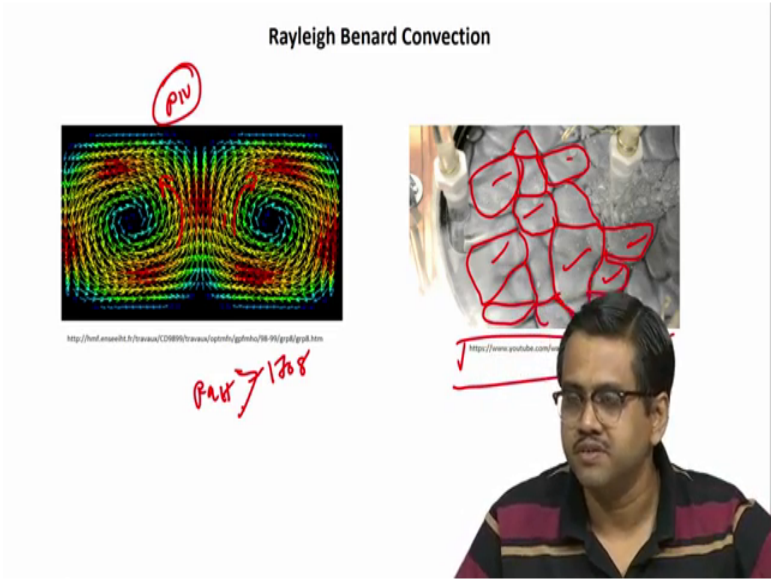
So, this is the point this is the area under the graph, where do you satisfy basically the both the things and this comes exactly as 0.5. So, x optimum is about 0.5 because this is the highest Rayleigh number that you can get. All these regions will be satisfied for any other Rayleigh number try to understand that, if your Rayleigh number is below these limits you will satisfy them regardless.

But the point is that we want the highest Rayleigh number at which this will be satisfied. So, highest will Rayleigh number peaks at around 0.5. So, you can have lower bounds also; that means, you can have x in other locations also. So, long as your Rayleigh number is beyond a is below a certain value. So, this R a c max which is the maximum Rayleigh number is 1 7 0 8 divided by half raised to the power of 4, which is basically 2.7 into 10 to the power of 3 this is the maximum Rayleigh number that you can achieve, based on this x optimum.

So, what we have done we have devised 2 equations, we have plotted the 2 equations. So, whatever that area is enclosed by these 2 graphs, the area underneath those 2 graphs is basically the region where these conditions are automatically satisfied. Then you choose the maxima that can happen in that particular region, which is happens at x equal to 0.5 and for x equal to 0.5 you find out what is your Rayleigh a number?

So, that is all that we have done in this particular problem. So, this is how you would actually solve it in using the partitions.

(Refer Slide Time: 08:51)



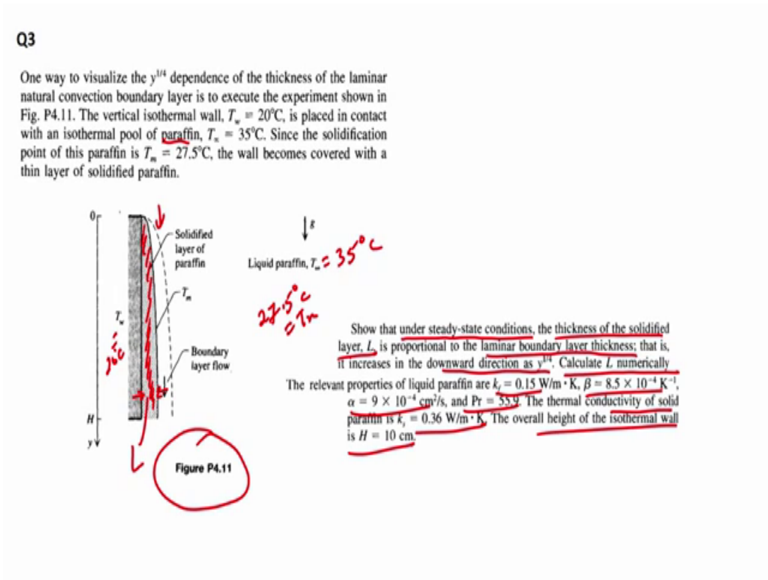
Now, in this slide since, we have said for a long time that we will show the Rayleigh Bernard. So, this is for example, a Rayleigh, a Bernard convection in a cell. So, this is done using PIV. So, you can see that 2 counter rotating cells. So, this is rotating in this direction this is rotating in this direction.

So, p iv rendition which clearly shows the flow vectors and if you go to the following YouTube link that I have marked over here, you can also see that these are basically the Rayleigh Bernard cells that you create see these are basically the cells, I am just marking them out. So, you can see the cells very clearly. So, you can see in this kind of cells also you can clearly see that you can see this Rayleigh Bernard cells and as you go higher and higher in Rayleigh number this these cells becomes more and more chaotic in nature.

Go to this YouTube link to see the full video and if you want more quantitative data these are basically particle image velocimetry data, which basically shows that how the flow actually takes place between hot and cold essentially it is a hot and cold and this is of course, Rayleigh number is greater than 1 7 0 8 so, it kind of matches with the problem that we just now did all right ok.

So, the problem therefore, it is very easy when I say that there is really there is nothing much to say about it except that this was the kind of problem that can be easily done without doing much of a math. So, that is the whole point.

(Refer Slide Time: 10:40)



Now, let us move to the question number 3 which we kind of said that we will do all. In the last class and we did not. So, see if you read the question you will find that 1 way to visualize the y to the power 4 1 forth dependence of the thickness of the laminar natural convection boundary layer is to execute this experiment, which is shown in this particular figure.

So, what we have is that you basically have a vertical isothermal wall, which has got a temperature T w is equal to 20 degree Celsius that is the wall. It is not placed in contact with an isothermal pool of paraffin, paraffin is like wax and the T infinity is equal to thirty 5 degree Celsius. Now the solidification of the point of the wax is about 27.5 degree Celsius, well the solidification point of this wax which is called as tm which you see over there.

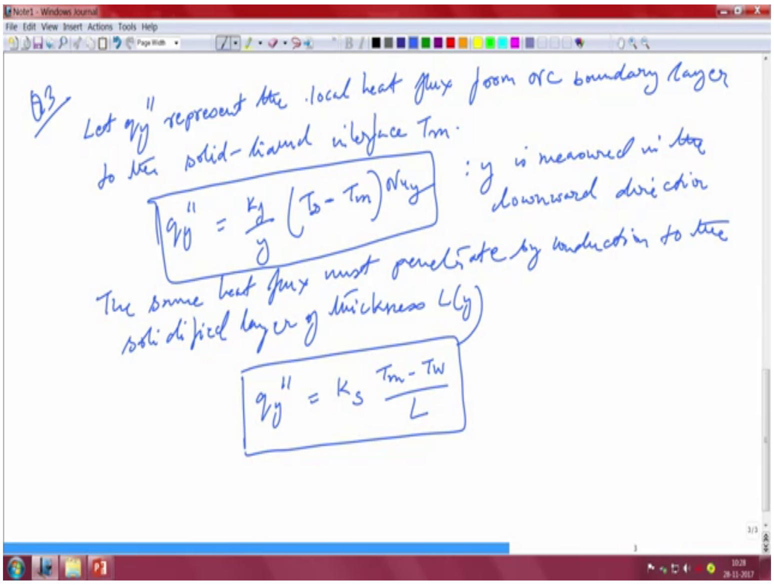
So, the wall becomes covered with a thin layer of this paraffin that is what will happened? So, you have a wall which is at a temperature of 20 degree Celsius the ambient essentially is at a temperature of 3 5 degree Celsius. This wall is covered with an isothermal pool of paraffin all right paraffin, because it is temperature lies somewhere between this ambient and a wall. So, it naturally freezes right freezes and it covers the wall, it covers the wall with a thin layer of paraffin not completely melted and therefore, and on the top of this you have the general airflow. So, this is the problem.

So, this particular problem is used to visualize the y to the power 1 forth dependence of the thickness of the laminar natural boundary layer that is the dependence that we have. Now under studies what we have to show is that under steady state condition, the thickness of the solidified layer L, this is the length L, it is proportional to the laminar boundary layer thickness that is it increases in the downward direction as y to the power of 1 forth.

So, calculate L numerically. So, some of the relevant properties are given over here prandtl number is 55.9 the thermal conductivity of paraffin is that and the overall height of the isothermal wall is H equal to 10 centimeter. So, this total height is about 10 centimeter. So, the idea is basically to show that the L, which is the thickness of this solidified layer of paraffin this is the L, basically increases in the downward direction in a similar way as a laminar boundary layer, which is a y to the power 1 forth.

So, this is the basic context of this particular problem that we are going to be. So, let us do this. So, this is question 3.

(Refer Slide Time: 14:18)

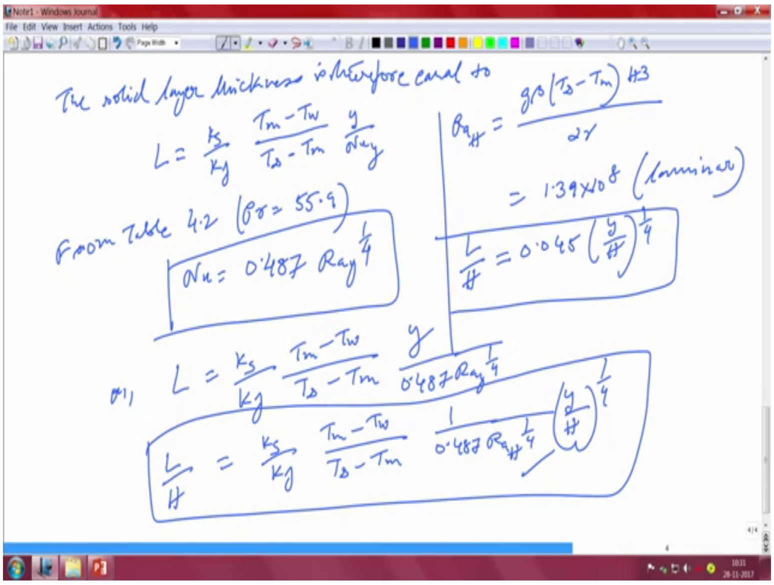


So, let right q y double prime represent the local heat flux heat flux from right natural convection boundary layer right to the solid, liquid interface right which is maintained at a temperature of T m all right this is the solid liquid interface. So, q y right double prime is given as K f by y T infinity minus T m nusselt number y correct.

So, y is measured in the downward direction in the downward direction. So, we can write that y is measured direction. So, the same heat flux. So, whatever heat flux is coming from the natural convection boundary layer, the same heat flux should penetrate by conduction to the solidified layer which has got a local thickness of L y. So, the idea is the same heat flux heat flux.

The same heat flux must penetrate by conduction conduction to the solidified layer, layer of thickness L y. So, q y double prime basically given as K s tm minus T wall by L. So, what we can do basically equate these 2 this and this right, basically you equate those 2 because whatever heat is coming from outside has to penetrate the solid wall.

(Refer Slide Time: 16:57)



So, solid layer thickness therefore, equal to is by K f T m minus T w T infinity minus T m y by N u y. So, from table 4.2 which is from bejan and prandtl number is 555.9 that was given the conditions 4.2. So, you can see that the nusselt number roughly scales as or the relationship of the nusselt number is about by linear extrapolation on 4 8 7 into Rayleigh number to the power of 1 fourth this is the same table, because I interpolated it between at 10 and 100.

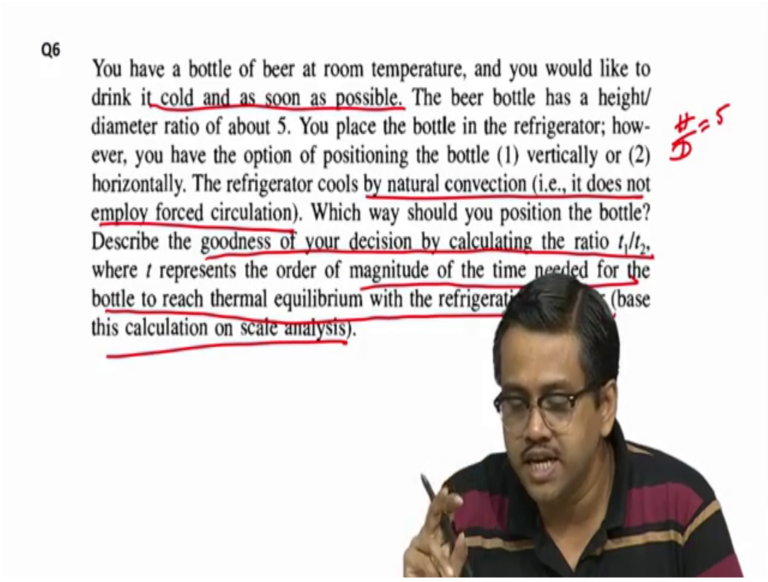
Because 55 is there in the middle. So, you can do with some kind of a linear interpolation at that particular stage. So, therefore, or in other words. So, L K s by K f T m minus T w T infinity minus T m y, then you substitute 0.4 8 7 Rayleigh number to the power of 1 fourth. It is just a substitution that is the substitution that you can do or in other words this particular thing shows, you can do a little bit of more math l by H will be K s by K f tm minus T w T infinity minus T m 1 by 0.4 8 7 Rayleigh number H to the power of 1 fourth y by H to the power of 1 fourth.

So, that is the scaling that we have established now. So, l scales basically as that as y to the power of 1 fourth. So, the Rayleigh number based on the overall height will be because that is what we have done here as overall height and that was given to us T infinity minus T m H cube alpha gamma. So, if you just plug in the numbers over here it will be 1.3 9 into 10 to the power of 8 this is still in the laminar regime, which is most of the time that is what we have done.

So, L by H therefore if you back substitute it over here this will translate to something like 0.0 4 5 y by H raised to the power of 1 fourth. So, that will be the expression. So, as you can see what important concept we used was that, whatever heat flux is coming through the natural convection boundary layer the same heat flux is transmitted through the solidified layer. And we have just equated the 2 that is all that we have done and we have shown that this L is basically proportional to y to the power of 1 fourth.

So, it exactly grows like the laminar natural convection boundary layer. So, that is the important step that we have taken over here. So, this completes 1 of the set now we will look at an interesting problem. So, let us look at the nature of this problem first. So, once again say you have a bottle of beer at room temperature and you would like to drink it cold and as soon as possible.

(Refer Slide Time: 21:18)



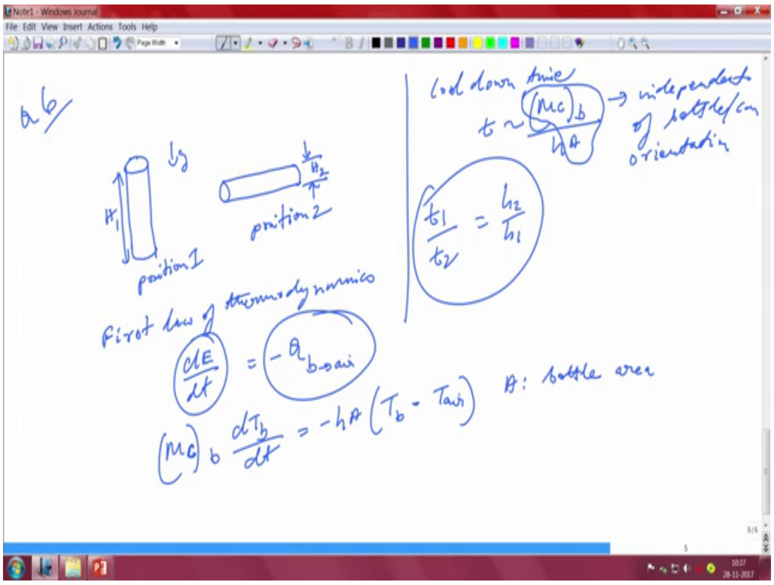
So, that is the basic statement the beer bottle has a height to diameter ratio; that means, H by D is approximately about 5 right you are placed the bottle in the refrigerator.

Now, you have 2 options you can place the bottle vertically or you can place the bottle horizontally. The refrigerator cools by natural convection that is does not employ forced recirculation, which way should you place the bottle will it be vertical or will it be horizontal. Describe the goodness of your decision by calculating the ratio t 1 by t 2 where t represents the order of magnitude of the time needed for the bottle to reach thermal equilibrium with the refrigeration temperature.

So, this calculation is supposed to be done based on scaling analysis. So, it is not just beer any other soft drinks that you might have you always have, if you buy a can either you can place, it vertically or you can place, it horizontally these are a high aspect ratio remember the cans are usually high aspect ratio; that means, the height is more than the diameter in this case it is given as 5 you can have other situations where it can be a lot different.

So, it cools by natural convection. So, the idea is that how would you place it and this you have to justify by taking the ratio of the time t 1 and t 2 which is needed to cool the bottle down to it is thermal equilibrium. So, that is the problem that we have in this particular case. So, let us look at the solution. So, this equation 6 for us.

(Refer Slide Time: 23:03)



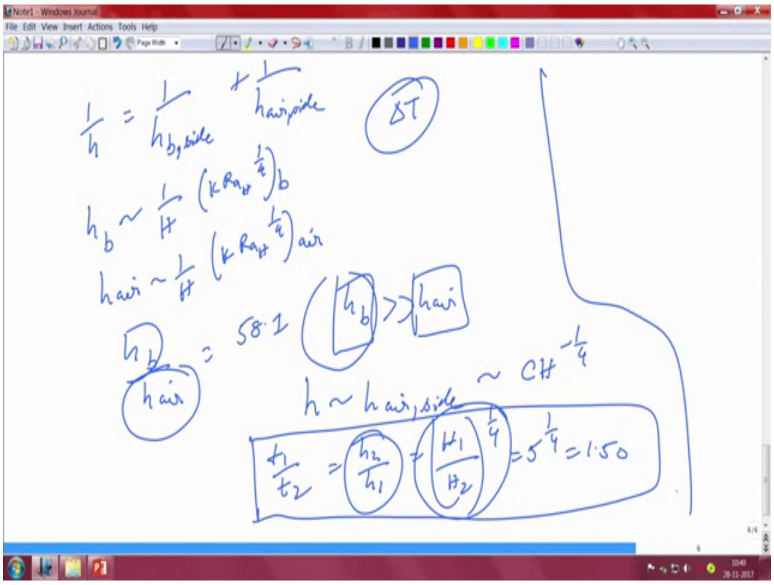
So, the scale of the cool down time. So, 1 case this is the bottle this is the can rather can bottle whatever you can think of otherwise this right. So, this is your height H 1 and g is acting in this direction this is your height H 2, which is placed in the horizontal position. So, this is position 1 this is like position 2.

So, the scale of cooldown time. So, first is resort to the first law of thermodynamics which states that d E by dt equal to minus Q b Q b to air, the rate of internal energy transport. So, the cool down time is essentially. So, if you convert this equation now M c b that is the mass and so this is d T b by d t is equal to minus h A T b minus T air.

Where basically a is basically the bottle area. So, this is basically the rate of change of internal energy is whatever is a heat that is lost or the heat that is in this particular case dumped, I think in this particular case yeah because the bottle comes to an equilibrium with the refrigerator temperature and this is M c into b. So, the cool down time if you write it is t scaling as M c b divided by h A this from just from the scaling analysis that a t if you just take everything to the other side that is what you are going to get.

So, in other words so this particular factor this M c b by A only these part, it is basically independent of the bottle or the can orientation. So, the t 1 by t 2 is basically a ratio of h 2 by h 1, that is the conclusion in general that will be the conclusion. Now what will be this h 1 and what will be this h 2.

(Refer Slide Time: 26:04)



So, 1 by h is 1 by h b that is the bare sight plus 1 by h air this is the air side. So, h b here. So, there is air and then there is the liquid inside the bottle inside the can. So, 1 by H k R a H to the power of 1 forth b and h air is given as 1 by H k R a H 1 forth air, because both are by natural convection.

So, the same for the same delta T because your delta T is basically the same, because your bottle and your refrigerator is kept at the same temperature difference so, h b by h air. So, it is basically the ratio of the 2 if you take the ratio of the 2 it comes out to be around 58.1. So, this means that hb is much much greater than h air. So, your 1 by h is almost the same as 1 by hb.

So, essentially that is the case well I am sorry it is not also your hb is basically much much greater than your h air. So, your h the overall heat transfer coefficient is basically therefore, h of the air side all right because it is a 1 by h ratio.

So, this is basically therefore, a constant into H raise to the power of 1 fourth. So, in conclusion your t 1 by t 2 the time scale is h 2 by h 1, which is basically H 1 by H 2. So, therefore, it is a ratio of the 2 heights essentially all right it is 1 forth it is about 5 to the power of 1 fourth it is about 1.5 0.

So, the cool down in the vertical position requires a time that is 50 percent longer and the cool down in the horizontal position. So, therefore, if you would like to drink the drink it cold, you should choose your horizontal position essentially that is all right that it means all right just through simple scaling argument. What we have done is basically you have converted the heat transfer coefficient ratio into a geometric height ratio that is all that we have done over here, by arguing that the h b and h air; that means, the hb is much much greater than your h air.

So, therefore, the h is governed by the air side. So, that is all that we have done in this particular problem. So, this particular problem is interesting, because it uses a simple scaling argument to nail down the same issue. We have a couple of 1 more problem to deal with and after that we will go to the turbulence see you next class.