Convective Heat Transfer
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Lecture – 43
Natural convection – Tutorial I

So, welcome. So, in this particular lecture, what we are going to do we are going to look at some of the practice problems not practice, problems which are conceptually based on whatever we taught during the last few lectures. So, these are problems related to only natural convection forced convection problems we already covered a few lectures back.

So, this is just to give an idea, that what kind of problems you would normally need to attack, if you are an engineer engineering student. So, let us look at this particular thing first. So, in this particular problem if you look at. So, there is consider the natural convection if you read the problem let us just put it here.

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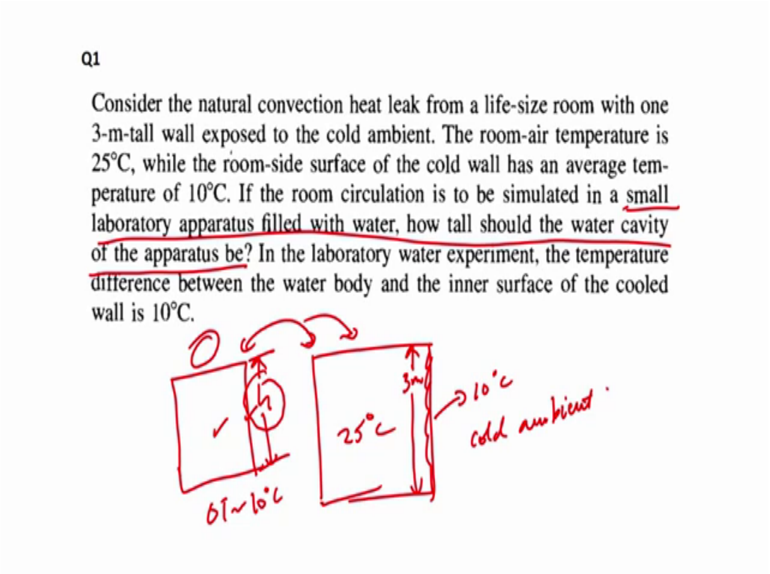

Q1

Consider the natural convection heat leak from a life-size room with one 3-m-tall wall exposed to the cold ambient. The room-air temperature is 25°C, while the room-side surface of the cold wall has an average temperature of 10°C. If the room circulation is to be simulated in a small laboratory apparatus filled with water, how tall should the water cavity of the apparatus be? In the laboratory water experiment, the temperature difference between the water body and the inner surface of the cooled wall is 10°C.

So, consider the natural convection heat leak from a life size room, with a 3-meter-tall wall exposed to the cold ambient. The room air temperature is 25 degree Celsius, where while the room sight surface of the cold wall has an average temperature of 10 degree Celsius, right? So, if the room circulation is to be simulated in a small laboratory apparatus filled with water.

How tall the end apparatus needs to be because, if you told earlier if you remember some of the past lectures we said that, sometimes some of these flows can be represented by water, right? Because, water being a high highly dense fluid you can actually reduce the size of tall buildings because, sometimes if you have to simulate the flows you can need not do it in a exact replica of the building. So, long as you are able to match certain physics, certain non-dimensional quantities you should be still able to do the same problem in a water.

So, in the laboratory water experiment the temperature between the water body and the inner surface of the cold wall is 10 degree Celsius. So, certain things are given. So, for example, if this is the room. So, inside it is 25 degree Celsius, this wall of the room is at 10 degree Celsius just because, it is in to the ambient think about it, that you are trying to build something in say Kashmir or you are trying to build something in Shimla, where outside is very cold, right? And the inside room temperature is 25 degree Celsius.

And this is 3 meter high, correct? So, this is a outside it is basically cold ambient, correct? Now, you want to replicate the same thing in water we do not know what will be the height of this whole thing let us say that, height is h it is a water, it is a water cavity say for example, now in this particular water situation, right? You have a difference between this wall and this. So, that delta T is about 10 degree Celsius. So, that is the max, that you can do this is what your experimental limitations are. So, to say, right? So, in this particular ambient you have to simulate that, what will be the height of this water.

So that, these whatever results you get here, can be directly translated to here, right? Without any loss of generality. So, this one let us go to now the stuff.

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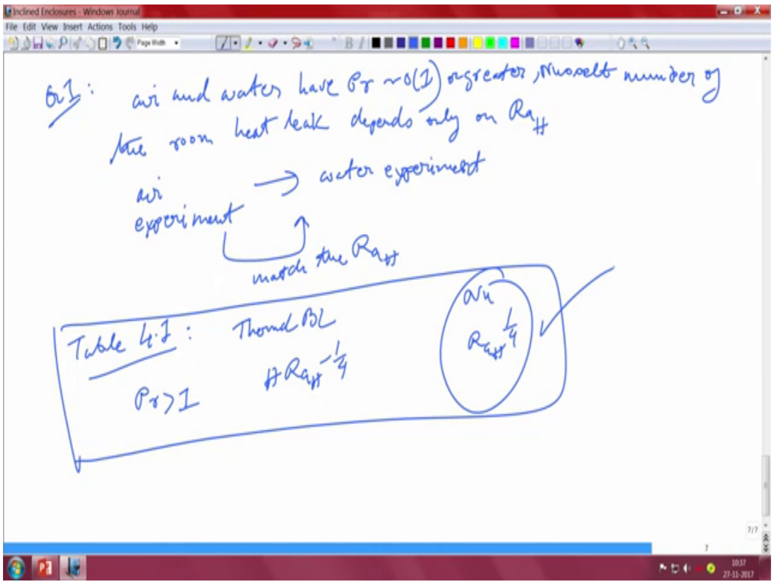

Q1: air and water have $Pr \sim O(1)$ or greater, Nusselt number of the room heat leak depends only on Ra_H

air experiment $\rightarrow$ water experiment

match the Ra_H

Table 4.1: Thermal BL

$Pr > 1$ $\quad H Ra_H^{-1/4}$ $\quad Nu \quad Ra_H^{1/4}$

So, question one both air and water have Prandtl number pr, which are of the order 1, right? Air is about point 72 water is about 6, right? Or sometimes greater. So, the Nusselt's number off the room heat leak is dependent on or rather depends only on Rayleigh number or in other words Rayleigh number based on the height. So, this is the Rayleigh number based on the height that is H. So, this Rayleigh number based on the height is all that we need.

So, if we has to be replicated in a water experiment. So, air experiment if you are going to convert that, to a water experiment, right? All you need to do is match the Rayleigh number, that is all you just need to match the Rayleigh number, right? So, if you look at table 4.1 you can get a very clear idea of what this is like. So, in Prandtl number if you look at table 4.1 and when Prandtl number is greater than 1.

The thermal boundary layer thicknesses are HRaH to the power of minus 1 4th. So, that is all, that you need basically a Nusselt number this is all that you need is a Rayleigh number to the power of 1 4th, correct? This is the part of the table, that you need, right? So, for the room let us calculate, then what is going to be the Rayleigh number, right?

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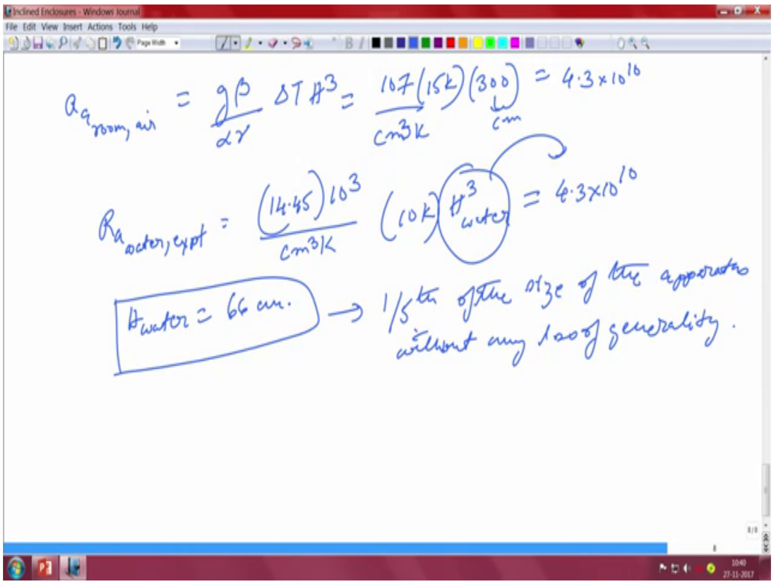

$$Ra_{room,\, air} = \frac{g\beta}{\alpha\gamma} \Delta T H^3 = \underset{cm^3K}{\xrightarrow{107}} (15K)(\underset{cm}{300}) = 4.3\times10^{10}$$

$$Ra_{water,\, expt} = \frac{(14.45)10^3}{cm^3K} (10K)\, H^3_{water} = 4.3\times10^{10}$$

$$\boxed{H_{water} = 66\ cm.}$$ → 1/5th of the size of the apparatus without any loss of generality.

So, this is what is needs to be matched, that is what we what we did in our previous class. So, Rayleigh number of the room with air let us put it that way, is basically g beta alpha gamma delta TH cube this is given as 10 to the power of 107 15 Kelvin is a difference in temperature between the 2 and 300 is basically the temp the height, because of the height is 3 meters is about 300 centimeters.

So, all these things have been done in terms of centimeter cube kelvin and this is actually given in terms of centimeter. So, basically what you get is 4.3 into 10 to the power of 10, this is actually to be there in the transition regime, but actually do not be bothered about that one for the time being, that whether it is transition or not. So, similarly the Rayleigh number in for water in the experiment.

So, what you need to do we have to match. So, this is 14.45 this is different 10 to the power of 3 because, 10 is basically the. So, centimeter cube K and this is 10 Kelvin into H tube of water. So, this should be equal to 4.3 into 10 to the power of 10, right? That has to be matched, right? So, from here, you can calculate that what will be the value of H. So, H water comes out to be about 66 centimeter.

So, 0.66 meter essentially, right? So, you can see that, we have been able to reduce the problem to roughly quite a bit in size about 6 times in size. So, or roughly 1 5th in size actually. So, this we have been able to reduce to 1 5th of the size of the room of the apparatus without any without any loss of generality.

So, this shows an interesting problem in which we have used our non-dimensional number, using table 4.1 just to simulate air system with an equivalent water system, that is all that we have d1 over here, right? So, this is an interesting problem which we did and this shows that, how scaling or knowing which numbers are actually responsible for such phenomena is actually very, very important.

So, that you can reduce the problem, because sometimes some problems you cannot do the experiment in a real setup. So, you have to device you know shadow gate kind of a set-up, a set-up which is a scaled version of the set-up, that is how you can you can actually get away with so many problems.

For example, you want to know something in a very large room, you can simulate that in a much smaller space just by using this scaling arguments, that is what we have used over here. So, this particular thing let us look at this particular problem now.

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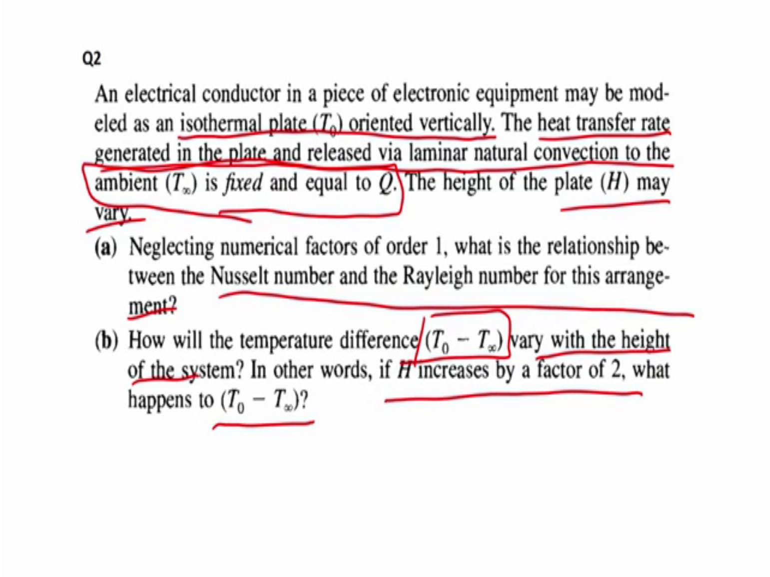

Q2

An electrical conductor in a piece of electronic equipment may be modeled as an isothermal plate (T_0) oriented vertically. The heat transfer rate generated in the plate and released via laminar natural convection to the ambient (T_∞) is *fixed* and equal to Q. The height of the plate (H) may vary.

(a) Neglecting numerical factors of order 1, what is the relationship between the Nusselt number and the Rayleigh number for this arrangement?

(b) How will the temperature difference $(T_0 - T_\infty)$ vary with the height of the system? In other words, if H increases by a factor of 2, what happens to $(T_0 - T_\infty)$?

So electrical conductor in a piece of electronic equipment may be modeled as an isothermal plate, which is oriented vertically isothermal plate oriented vertically. So, these are the important things that you should note.

The heat transfer rate generated in the plate and released via natural convection is equal to Q. So, there is some heat, which is generated in the plate, which is transferred via natural convection to the ambient whose temperature is fixed is basically a large reservoir with a temperature of T infinity. The height of the plate H may actually vary now, neglecting the numerical factors of order 1.

What is the relationship between Nusselt number and Rayleigh number for this arrangement and how will the temperature difference T naught minus T infinity, vary with height of the system, in other words if H increases by a factor of 2 what happens to T naught minus T infinity. So, the problem is quite clear now, right? So, let us now look at this particular problem in details.

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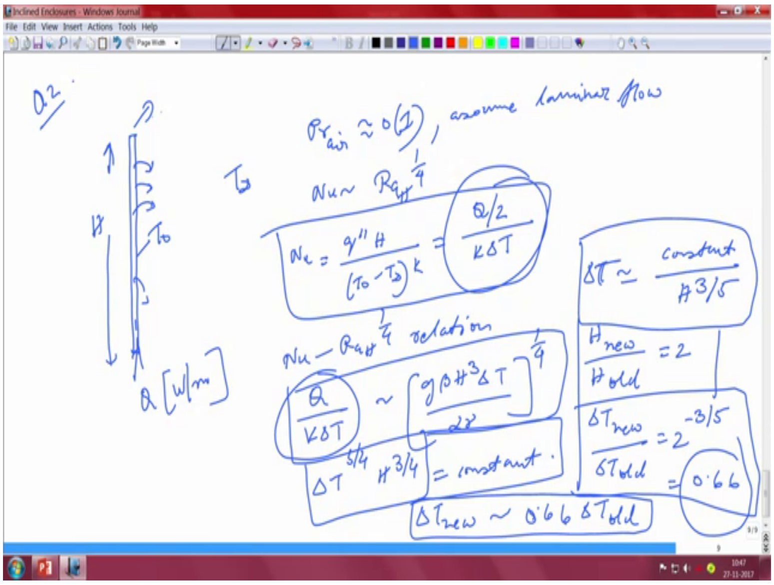

$Pr_{air} \approx o(1)$, assume laminar flow

$Nu \sim Ra_H^{\frac{1}{4}}$

$Nu = \frac{q'' H}{(T_0 - T_\infty) k} = \frac{Q/2}{k \Delta T}$

$Nu - Ra_H^{\frac{1}{4}}$ relation

$\frac{Q}{k \Delta T} \sim \left[\frac{g \beta H^3 \Delta T}{\alpha \nu}\right]^{\frac{1}{4}}$

$\Delta T^{5/4} H^{3/4} = \text{constant}.$

$\Delta T \sim \frac{\text{constant}}{H^{3/5}}$

$\frac{H_{new}}{H_{old}} = 2$

$\frac{\Delta T_{new}}{\Delta T_{old}} = 2^{-3/5} = 0.66$

$\Delta T_{new} \sim 0.66 \, \Delta T_{old}$

So, what we have question 2 now, we have this wall, right? Which is got a height H which can vary. So, you are forcing some Q, which can be at watt per meter, which is forced in electrically. So, it could be the electric heat generation, that is happening and this wall temperature is maintained at T naught this is T infinity; obviously, because of the heat that is being dumped to the fluid leads to that natural circulation, right?

So, airs Prandtl number of air is of the order 1, right? And we assume that, the flow is laminar, right? So, your Nusselt number scales it as RaH to the power of 1 4th again via table 4.,1 which we already saw what is the definition of Nusselt number, Nusselt number is Q double prime by H divided by T naught minus T infinity into K, right?

So, therefore, this becomes Q by 2 K into delta T because, that would be the heat that will be transferred now, what we do is that, this is the expression for Nusselt number that we got. So, the Nusselt number and RaH to the power of 1 4th relation, if we kind of recast that whole thing Q by K delta T scales as g beta. So, that we are neglecting whatever numerical factors which are of the order 1, that is why the half and other things are excluded 1 4th. So, this actually if you work out this part of the problem. So, you will find that, delta T raised to the power of 5 by 4, H raised to the power of 3 4th is equal to constant, right?

Because, the Q that is dumped is constant. So, in other words delta T scales as some constant divided by H raised to the power of 3 5^{th}, right? So; that means, delta T and H they are actually related, but not related in a linear way. So, the problem actually says that, H new by H old is of the order 2, right? So, therefore, delta T new by delta T old is 2 raised to the power of 3 by 5, which gives you 0.66.

So, the delta T new is actually giving you a temperature, which is 0.66 times the delta T old. So, this should be critical. So, delta T new and delta T old. So, this using very simple analysis now, that what did we do we cast the Nusselt's number, then we found out the relationship between Nusselt number and Rayleigh number using an order and we found that, this is the relationship that naturally evolves; that means, delta T 5 by 4 H to the power of 3 by 4th is a constant.

So, therefore, using the constant scaling we have now, H new by H old if it is 2, then that delta new will actually decrease in magnitude compared to the delta old by about 33 percent essentially, right? Because, it is 0.66 of the, right? So, this actually gives us another good idea by which, you know you can you take a problem, which is very simple in nature, right? What we said that, where the total heat that is dumped is given.

So, that is all that we have done and we have used the Nusselt number, Rayleigh number scaling to answer the question. It is a very simple exercise, which was which gives you an idea, that how to do the scaling in case you face a problem. So, this is like an electronic chip. So, you can think of it like an electronic chip, you can think of it about a lot of other things, but electronic chip is one of the key applications here.

So now, let us look at this problem, because we do not want to go to question number 3, which is a little bit long drawn. So, if you read this particular problem you will find, that rely on purely scaling argument.

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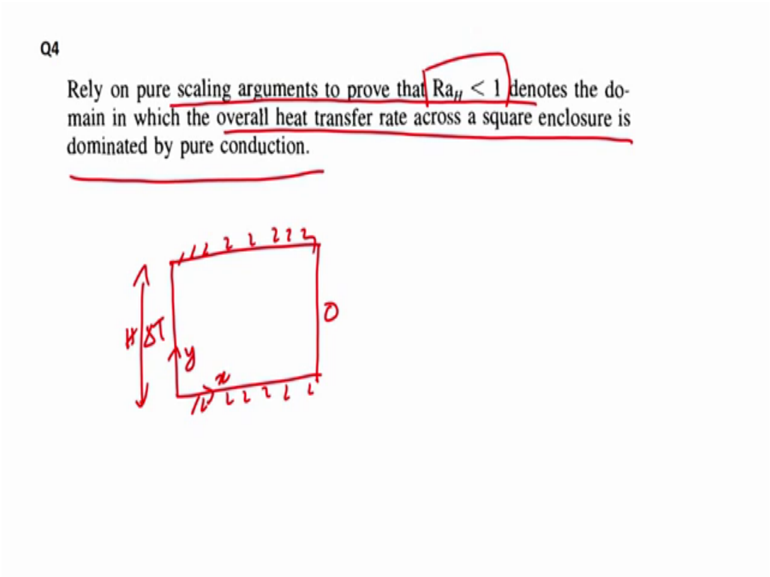
Q4

Rely on pure scaling arguments to prove that $Ra_H < 1$ denotes the domain in which the overall heat transfer rate across a square enclosure is dominated by pure conduction.

Pure scaling argument to prove that, Rayleigh number less than 1 denotes the domain in which, the overall heat transfer across a square enclosure is dominated by pure conduction. So, that is the problem that we are trying to address over here, right?

So, it is a problem in which the domain we have to prove the overall heat transfer across a square enclosure is dominated by pure conduction. So, essentially the problem is something like this, insulated at the top and the bottom this is delta T, this is basically 0, this is your x, this is your y, this is the height of the cavity that is the problem, right? So, we have to show that, this is a pure conduction driven problem, right?

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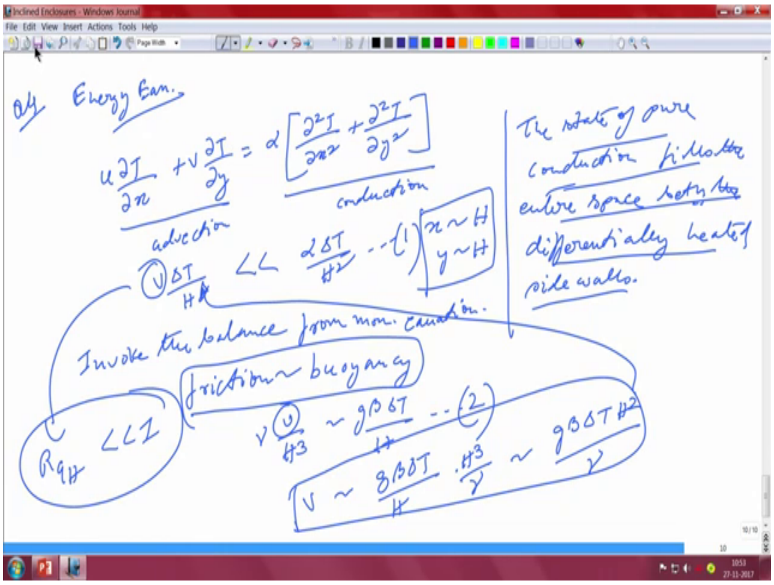

Q4 Energy Eqn.

$$\underbrace{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}}_{\text{advection}} = \underbrace{\alpha\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right]}_{\text{conduction}}$$

$$\textcircled{v}\frac{\Delta T}{H} << \frac{\alpha \Delta T}{H^2} \quad --(1) \quad \boxed{\begin{matrix} x \sim H \\ y \sim H \end{matrix}}$$

The state of pure conduction fills the entire space betn the differentially heated side walls.

Invoke the balance from mom. equation.

$\boxed{\text{friction} \sim \text{buoyancy}}$

$$\nu\frac{\textcircled{v}}{H^3} \sim \frac{g\beta\Delta T}{H} \quad --(2)$$

$$\boxed{v \sim \frac{g\beta\Delta T}{H}\cdot\frac{H^3}{\nu} \sim \frac{g\beta\Delta T H^2}{\nu}}$$

$Ra_H << 1$

So, let us go to our little scribble pad. So, we write the energy equation first. So, this is your question 4 essentially, u dt dx plus v dt dy alpha. So, this is what you call your advection term and this is basically your conduction term, right? So, our motive is to show that Rayleigh number is less than 1 for this, right? So, in this particular case this part can be represented as by H right v dt by dy and the other terms both are of the same order essentially..

So, this is alpha delta T by H square, correct? These are the 2 terms we have done it. I mean enough number of times where basically your x scales as H and y scales as H for the square region both are very similar, because the state of pure conduction fills the entire space this is because, the state of pure conduction fills the entire space between the differentially heated side walls between the differentially heated side walls, correct?

So, that is why both the skills are H, because it fills the whole cavity, right? So, that is why, the scales are H. So, we have a v which is unknown over here, right? So, what we invoke is that, we invoke the balance which is friction balances buoyancy got it. So, in other words, so friction balances buoyancy means v comes from the momentum equation.

So, from the momentum equation, so v u by H cube g beta delta T by H, right? Now, if you eliminate v from these 2 equations. So, this equation can be our equation 1 this equation can be our equation 2, right? So, if you substitute v over here. So, v will scale as g beta delta T by H into H cube divided by gamma, right? So, this will give as g beta delta T into H square by gamma.

So now, if you substitute the same thing over here. So, if you substitute this expression of v take it here, right? So, that will give you the expression, then that Rayleigh number H is actually is much less than 1. So, if you do that kind of an expansion you will see that, Rayleigh number will be necessary you have understood the process. So, it is friction with buoyancy from the momentum invoke the balance, this is from the momentum equation and this we have done thousand different times.

So, friction with buoyancy. So, you get the expression and then you actually find out what v is and then, you substitute it in the energy equation because, we say it that conduction is much, much more than advection and the state of pure conduction basically fills the entire space between the 2 differentially heated sidewalls, that is why x and y scales as H in both these cases, right? So, that is also another interesting problem.

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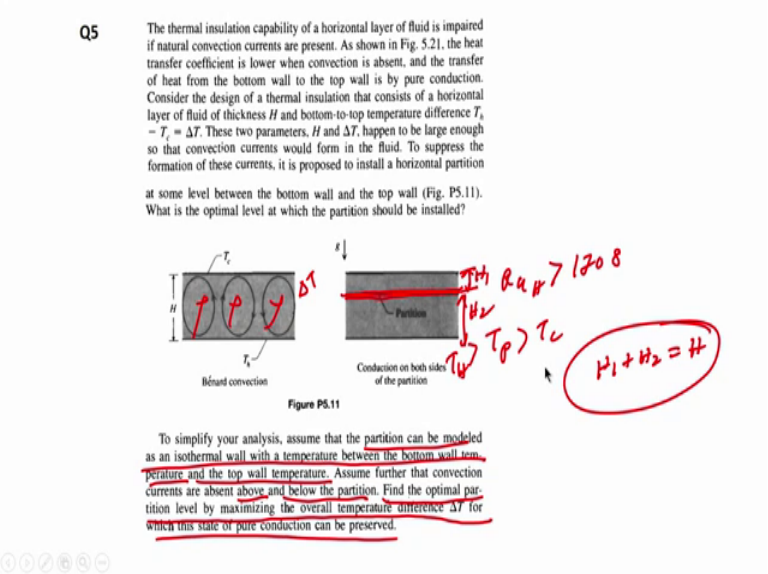

Q5 The thermal insulation capability of a horizontal layer of fluid is impaired if natural convection currents are present. As shown in Fig. 5.21, the heat transfer coefficient is lower when convection is absent, and the transfer of heat from the bottom wall to the top wall is by pure conduction. Consider the design of a thermal insulation that consists of a horizontal layer of fluid of thickness *H* and bottom-to-top temperature difference $T_h - T_c = \Delta T$. These two parameters, *H* and ΔT, happen to be large enough so that convection currents would form in the fluid. To suppress the formation of these currents, it is proposed to install a horizontal partition at some level between the bottom wall and the top wall (Fig. P5.11). What is the optimal level at which the partition should be installed?

Figure P5.11

To simplify your analysis, assume that the partition can be modeled as an isothermal wall with a temperature between the bottom wall temperature and the top wall temperature. Assume further that convection currents are absent above and below the partition. Find the optimal partition level by maximizing the overall temperature difference ΔT for which this state of pure conduction can be preserved.

Let us, look at this particular problem now, which is a long one actually and let us see, and how this problem is actually. So, that you can think about it a little bit. So that, we can take it up. So here, the problem is that you have if you can see them properly, that there is a heated wall and a cold wall and you have the typical Rayleigh Bernad out convection, because your Rayleigh number with respect to H is greater than 1708.

The heat transfer coefficient is lower when convection is absent, which is obvious and the transfer of heat from the top to the bottom wall is by pure conduction. So, what we have what people have designed is that, you get this Rayleigh Benard convection, right? So, they have inserted this partition which you can see over here, if you can see that there is a partition that is over there.

Now, the top and the bottom of this partition. So, there is a delta T difference between the 2, right? So, that delta T partition between the top and the bottom, that is still there. So, you are inserting a partition which is at some level. So, at some distance between the top and the bottom wall and not only that, the temperature of this partition is in between the TH and TC, right? So, that is how the partition has been designed.

So, what happens is that, this partition is like an isothermal wall. So, that is what we have said is can be modeled as an isothermal wall with a temperature between the bottom wall temperature and the top wall temperature, assume further that conviction currents are absent above and below the partition, right? Find the optimal partition level by maximizing the overall temperature difference delta T, for which the state of pure conduction can be preserved right?

So, we want to basically preserve the state of this pure conduction; that means, between the top and the bottom partitions in the top half and the bottom half, there cannot be any convection cells in other words the Rayleigh number has to be less than 1708 between the top and the bottom, right? Not only that, we have to find out at what level this partition has to be inserted, that is the main question, right? At what level the partition should be installed. So, that you basically get rid of this convection cells.

Because, we want a thermal insulation we do not want convection cells to originate because, that will equilibrate out things. So, you want to. So, imagine this to be a problem in which there is a hot tub bottom and a cold top and you want to preserve it that way. So, pure conduction is what we want and because, your heat transfer coefficient increases when you actually have convection which is present, right?

And so, this is the proposed problem, right? As far as what we can see. So, a partition is at an isothermal level. So, the partition temperature is definitely greater than TC, but it is less than TH, right? And the height of this partition this you can say it is H1 this you can say it is H2 or vice versa, right? So, we want to know and total H1 plus H2 is; obviously, equal to H, right? So, we want to know that, how this partition will actually work.

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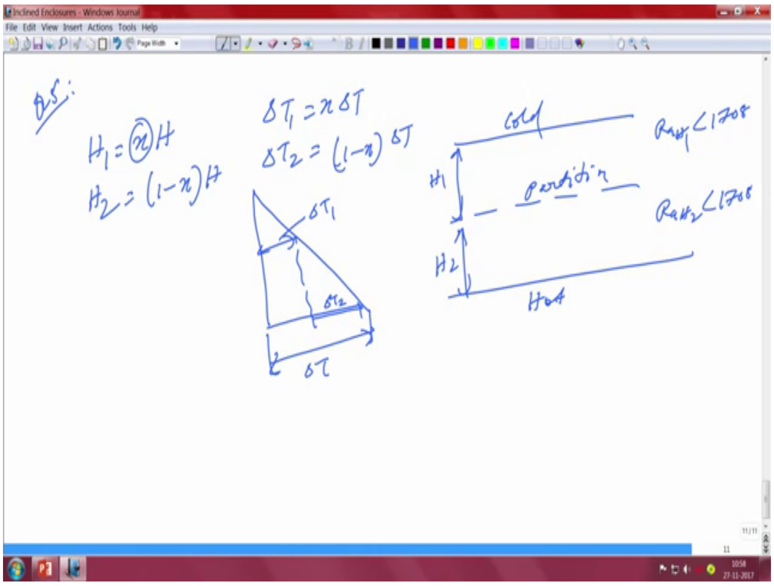

Now, let us look at this problem Q 5. H1 let us say, it is some x into H and H2 is given as 1 minus x into H where, delta T1 is given as x into delta T and delta T 2 is given as 1 minus x into delta T. So, the problem is basically I will pose the problem and we will solve it in the next class. So, this is H1 this is H 2 for you this is cold this is hot and this is the partition, where the partition is given as a percentage factor which is x, right?

And the temperature difference if it is a pure conduction limit problem then therefore, this will be like wherever the partition is installed this will be delta T1 and the rest will be your delta T2 and this being your total delta T. So, we have posed this problem now, we will see how to solve this problem in the next class along with a couple of other problems, that we will solve before we wrap up natural convection completely. So, think about this problem and we will pose it in the next class. So, you are given the variables how to attack and we have also told that partitions are such that, the Rayleigh number has to be less than 1708 in both sides.

So, Ra1 and Ra 2, right? Both has to be less than 1708 and then we have to solve this problem by some kind of an optimization way see you in the next class.